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Exotic Dyonic Black Holes From Lovelock Gravity

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Abstract: ***This manuscript examines exotic dyonic black holes arising in Lovelock gravity, the most natural higher-dimensional extension of Einstein's general relativity [1,2] yielding second-order field equations [1-4]. A dyonic black hole carries both electric and magnetic charge; a Lovelock black hole carries, in addition, the imprint of higher-curvature geometry. Their union produces a thermodynamic object richer than Reissner–Nordström: its horizon is not merely an area but a curvature-weighted ledger, its charge sector is doubled, and its phase structure may admit multiple branches, extremal limits, and unusual stability windows[5,7]. Recent work has constructed dyonic solutions in Lovelock and related quasitopological electromagnetic settings[8,9], including odd-dimensional black holes and AdS thermodynamic phases.***

Keywords: Lovelock Gravity; Dyonic Black Holes; Higher Curvature Gravity; Gauss–Bonnet Gravity; Wald Entropy; Magnetic Charge; Electric Charge; AdS Black Holes; Exotic Horizons; Black Hole Thermodynamics

I. PRELUDE: WHEN EINSTEIN GRAVITY BECOMES TOO SMALL

Einstein gravity is austere and beautiful, but in higher dimensions its uniqueness is not absolute. Lovelock gravity generalizes the Einstein–Hilbert action while preserving second-order equations of motion[1,2]—an extraordinary property, because generic higher-curvature theories produce ghosts or higher-derivative pathologies [[3,4]. Reviews of Lovelock black holes emphasize that these solutions generalize the Boulware–Deser Gauss–Bonnet family and include topological horizons, branes, charged black holes, and higher-order curvature thermodynamics [3,4,7].

The Lovelock action in D dimensions is [1,2]

$$I = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\sum_{k=0}^{K}\alpha_k \mathcal{L}_k + I_{\text{matter}},$$

where

$$\mathcal{L}_k = \frac{1}{2^k}\delta^{\mu_1\nu_1\cdots\mu_k\nu_k}_{\rho_1\sigma_1\cdots\rho_k\sigma_k} R^{\rho_1\sigma_1}_{\mu_1\nu_1}\cdots R^{\rho_k\sigma_k}_{\mu_k\nu_k}.$$

Here:

$$\mathcal{L}_0 = 1, \mathcal{L}_1 = R,$$

and

$$\mathcal{L}_2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},$$

the celebrated Gauss–Bonnet invariant [3,4].

II. WHAT MAKES A BLACK HOLE "DYONIC"?

A dyon carries both electric and magnetic charge [10]:

$$(Q_e, Q_m) \neq (Q,0), (0,P).$$

In Maxwell theory,

$$F = dA,$$

with electric component

$$F_{tr} \sim \frac{Q_e}{r^{D-2}},$$

and magnetic flux through angular sections,

$$\int_{\Sigma_2} F = 4\pi\, Q_m.$$

Thus a dyonic black hole is a gravitational object whose horizon traps both:

$$\text{electric flux} + \text{magnetic flux}.$$

In ordinary four-dimensional Einstein–Maxwell gravity, this produces the familiar Reissner–Nordström-like structure [7,10]:

$$f(r) = 1 - \frac{2M}{r} + \frac{Q_e^2 + Q_m^2}{r^2}.$$

But in Lovelock gravity, the metric function is no longer simply rational. It is determined by a polynomial curvature equation [1,4].

III. THE LOVELOCK POLYNOMIAL: THE ENGINE ROOM

For a static topological black hol, [4,7],

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma^2_{\kappa,D-2},$$

where

$\kappa = 1, 0, -1$

labels spherical, planar, or hyperbolic horizon topology [7] .

Define

$$\psi(r) = \frac{\kappa - f(r)}{r^2}.$$

The Lovelock field equations reduce schematically to

$$\sum_{k=0}^{K} \hat{\alpha}_k \, \psi(r)^k = \frac{16\pi GM}{(D-2)\Omega_{D-2} r^{D-1}} - \mathcal{E}_{\text{dyonic}}(r),$$

where $\mathcal{E}_{\text{dyonic}}(r)$ encodes electric and magnetic energy densities [8-10].

For Maxwell-like dyons,

$$\mathcal{E}_{\text{dyonic}}(r) \sim \frac{Q_e^2 + Q_m^2}{r^{2D-4}},$$

though in quasitopological electromagnetism or nonlinear electrodynamics the charge dependence may become more exotic [8,9,10]. Recent papers construct dyonic black holes by coupling Lovelock gravity to generalized electromagnetic sectors, including quasitopological electromagnetism and conformal scalar fields [8,9].

IV. WHY "EXOTIC"?

These black holes are exotic for five reasons.

First, they usually require

$D > 4,$

because higher Lovelock terms either vanish or become topological in lower dimensions [1,2].

Second, their horizons can possess nontrivial topology [4,7]:

$$\Sigma_{\kappa,D-2} \neq S^{D-2}.$$

Third, their thermodynamics is not governed by area alone. Jacobson and Myers showed that stationary Lovelock black hole entropy includes intrinsic curvature invariants [5,6] of the horizon cross-section, not merely one quarter of its area.

Fourth, electric and magnetic sectors may cease to be simply dual when nonlinear electrodynamics enters; studies of nonlinear dyonic black holes note that Maxwell electric–magnetic duality can be lost under Born–Infeld-type generalization [10].

Fifth, AdS dyonic Lovelock systems can display rich phase structure, including multiple black hole phases [11] at fixed temperature in some models.

V. ENTROPY: THE HORIZON IS NO LONGER A MERE AREA

For Lovelock gravity, entropy becomes [5,6]

$$S_{\text{Lovelock}} = \frac{1}{4G} \sum_{k=1}^{K} k \, \alpha_k \int_{\mathcal{H}} d^{D-2} x \sqrt{h} \, \mathcal{L}_{k-1}(h),$$

where $\mathcal{L}_{k-1}(h)$ is the Lovelock density built from the intrinsic horizon metric h_{ab} [5].

For Einstein gravity [6]:

$$S = \frac{A}{4G}.$$

For Einstein–Gauss–Bonnet gravity[5,6]:

$$S = \frac{A}{4G}\left[1 + \frac{2\alpha_{\text{GB}}(D-2)(D-3)\kappa}{r_h^2}\right].$$

Thus the black hole does not merely say:

"I have area."

It says:

"I have curvature memory."

This is precisely where the manuscript becomes fortified: a dyonic Lovelock black hole stores electromagnetic charge and geometric charge simultaneously [5-7].

VI. TEMPERATURE AND EXTREMALITY

The horizon r_h is defined by[4,7]

$$f(r_h) = 0.$$

Temperature follows from surface gravity[6]:

$$T = \frac{f'(r_h)}{4\pi}.$$

Extremality occurs when

$T = 0,$

or equivalently,

$$f(r_h) = 0, f'(r_h) = 0.$$

For dyonic Lovelock black holes, extremality is controlled by a delicate balance among:

$$M, Q_e, Q_m, \alpha_2, \alpha_3, \ldots, \Lambda, D.$$

The usual Reissner–Nordström extremality condition becomes a *higher-curvature algebraic constraint [7,10]*.

VII. FIRST LAW

The thermodynamic first law becomes [5,6,11]:

$$dM = T\, dS + \Phi_e \, dQ_e + \Phi_m \, dQ_m + \sum_k \Psi_k \, d\alpha_k + V \, dP.$$

Here:

- ✓ Φ_e is electric potential,
- ✓ Φ_m is magnetic potential,
- ✓ Ψ_k are potentials conjugate to Lovelock couplings,
- ✓ $P = -\Lambda/(8\pi G)$ is thermodynamic pressure.

This is not ordinary black hole thermodynamics. It is *extended curvature thermodynamics [5,11]*.

VIII. PHYSICAL INTERPRETATION

An exotic dyonic Lovelock black hole may be viewed as a threefold bound state [7-10]:

$$\boxed{\text{Mass} + \text{Dyonic Flux} + \text{Higher-Curvature Geometry}}$$

Its electric charge measures radial field strength.

Its magnetic charge measures topological flux.

Its Lovelock couplings measure the capacity of spacetime to respond nonlinearly to curvature[1-4].

Thus the black hole is not merely a collapsed star. It is a *geometric capacitor*, holding both electromagnetic and curvature information[5,6].

IX. STABILITY AND PHASE STRUCTURE

Thermodynamic stability is governed by heat capacity [10]:

$$C_Q = T\left(\frac{\partial S}{\partial T}\right)_{Q_e,Q_m}.$$

Stability requires

$$C_Q > 0.$$

Phase transitions occur when

$$C_Q \to \infty,$$

or when the Gibbs free energy[11]

$$G = M - TS - \Phi_e Q_e - \Phi_m Q_m$$

develops competing branches.

In AdS settings, the analogy with Van der Waals fluids becomes powerful:

$$P = \frac{T}{v} + \cdots,$$

with dyonic charges and Lovelock terms modifying the equation of state.

This is where exoticity becomes experimentally conceptual: the black hole develops *multiple thermodynamic personalities[11]* .

X. RESEARCH FINDINGS

- ✓ Lovelock gravity supplies the natural higher-dimensional framework for black holes with second-order field equations despite higher-curvature terms. [1-4]
- ✓ Dyonic Lovelock solutions can arise when the gravitational sector is coupled to suitable electromagnetic theories, especially quasitopological or nonlinear electrodynamic extensions. [8-10]
- ✓ Lovelock entropy is not purely area-based; it includes intrinsic curvature invariants of the horizon, as established by Jacobson–Myers and related Wald-type arguments. [5,6]
- ✓ Magnetic charge introduces topological structure, while electric charge controls radial flux; together they reshape extremality, heat capacity, and horizon algebra [10].
- ✓ In AdS backgrounds, dyonic higher-curvature black holes may exhibit rich thermodynamic phase behaviour, including multiple branches and criticality [11].

XI. CONCLUSION

Exotic dyonic black holes from Lovelock gravity are not ornamental curiosities. They are laboratories where three deep principles meet [1-11]:

Topology,Charge,Curvature.

In Einstein gravity, a charged black hole is already dramatic. In Lovelock gravity, it becomes polyphonic: the metric solves a curvature polynomial, entropy records horizon invariants [5,6], and electric–magnetic charge sculpts the causal skeleton.

The final moral is sharp:

A dyonic Lovelock black hole is not merely a hole in spacetime; it is a higher-dimensional archive where electromagnetism and curvature jointly write the thermodynamic script [5-11].

REFERENCES

[1] D. Lovelock, “The Einstein Tensor and Its Generalizations,” Journal of Mathematical Physics 12, 498 (1971).

[2] D. Lovelock, “The Four-Dimensionality of Space and the Einstein Tensor,” Journal of Mathematical Physics 13, 874 (1972).

[3] D. G. Boulware and S. Deser, “String-Generated Gravity Models,” Physical Review Letters 55, 2656 (1985).

[4] J. T. Wheeler, “Symmetric Solutions to the Gauss–Bonnet Extended Einstein Equations,” Nuclear Physics B 268, 737 (1986).

[5] T. Jacobson and R. C. Myers, “Black Hole Entropy and Higher Curvature Interactions,” Physical Review Letters 70, 3684 (1993).

[6] R. M. Wald, “Black Hole Entropy is Noether Charge,” Physical Review D 48, R3427 (1993).

[7] C. Garraffo and G. Giribet, “The Lovelock Black Holes,” arXiv:0805.3575.

[8] A. Cisterna et al., “Quasitopological Electromagnetism and Black Holes,” Physical Review D 101, 124041 (2020).

[9] A. Ali et al., “Quasitopological Electromagnetism, Conformal Scalar Field and Lovelock Gravity,” European Physical Journal C (2023).

[10] S. Panahiyan et al., “Nonlinearly Charged Dyonic Black Holes,” Nuclear Physics B (2020).

[11] P. Mou et al., “Thermodynamics and Phase Transitions of Dyonic AdS Black Holes,” 2024.