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Born–Infeld $f(R)$ Black Holes

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***Abstract:** Born–Infeld $f(R)$ black holes arise at the junction of two great corrective instincts in modern gravity: the Born–Infeld desire to tame divergences by nonlinear structure, and the $f(R)$ programme to enlarge Einstein's scalar curvature R into a richer gravitational functional. Recent work explicitly studies black-hole solutions in a Born–Infeld-$f(R)$ framework, deriving static, spherically symmetric geometries and analysing Hawking temperature, entropy, specific heat, and deviations from Schwarzschild–AdS thermodynamics [1]. This manuscript presents the conceptual architecture, governing equations, horizon structure, thermodynamic content, and physical significance of such objects.*

***Keywords:** Born–Infeld gravity; $f(R)$ gravity; black holes; nonlinear electrodynamics; Hawking temperature; entropy; specific heat; modified gravity; Schwarzschild–AdS; curvature corrections.*

I. INTRODUCTION

Einstein's General Relativity is magisterial, but not necessarily final. Its field equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

are astonishingly successful in weak and moderately strong gravitational regimes, yet singularities, quantum incompatibility, dark-sector puzzles, and inflationary demands keep whispering that the Einsteinian edifice may be a magnificent approximation rather than the last cathedral.

Two major extensions are relevant here.

First, *Born–Infeld theory* was originally introduced to soften the infinite self-energy of point charges in electrodynamics [2]. Its central instinct is nonlinear regularization: replace an unbounded linear theory with a square-root structure that behaves gently at extreme fields.

Second, $f(R)$ *gravity* replaces the Einstein–Hilbert Lagrangian R by a nonlinear function $f(R)$ [3], so that the gravitational action becomes

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(R) + S_{\text{matter}}.$$

This gives curvature itself more freedom, allowing higher-order geometric corrections and effective scalar degrees of freedom [3].

A Born–Infeld-$f(R)$ black hole therefore asks a tantalising question:

What happens when nonlinear regularization and modified curvature dynamics are married inside the most extreme gravitational object known?

II. THE ACTION: WHERE THE CREATURE IS BORN

A simplified schematic action may be written as

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(R) + \int d^4x\sqrt{-g}\,\mathcal{L}_{BI},$$

where the Born–Infeld electromagnetic Lagrangian is commonly written as [2]

$$\mathcal{L}_{BI} = 4\beta^2\left(1 - \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{2\beta^2}}\right).$$

Here:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and β is the Born–Infeld parameter. In the limit

$$\beta \to \infty,$$

Born–Infeld electrodynamics reduces to Maxwell electrodynamics[2].

Similarly, if

$$f(R) = R - 2\Lambda,$$

one recovers ordinary Einstein gravity with cosmological constant [3,4].

Thus the theory contains two escape hatches back to orthodoxy:

$$\beta \to \infty, f(R) \to R - 2\Lambda.$$

III. FIELD EQUATIONS

Varying the $f(R)$sector gives

$$f_R R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\right) f_R = 8\pi G T_{\mu\nu}^{BI},$$

where

$$f_R = \frac{df}{dR}.$$

The trace equation is

$$f_R R - 2f(R) + 3\Box f_R = 8\pi G T^{BI}.$$

This trace equation is crucial: unlike Einstein gravity, where the trace is algebraic, $f(R)$gravity allows curvature itself to propagate like an effective scalar model [3,4] .

Born–Infeld matter contributes a nonlinear stress tensor [2],

$$T_{\mu\nu}^{BI} = g_{\mu\nu} \mathcal{L}_{BI} + \frac{4F_{\mu\alpha} F_\nu^{\ \alpha}}{\sqrt{1 + \frac{F^2}{2\beta^2}}}.$$

The combined theory is therefore nonlinear twice over: once in curvature, once in field strength.

IV. STATIC SPHERICAL BLACK-HOLE ANSATZ

For a static, spherically symmetric black hole one usually assumes

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2.$$

The event horizon r_h is determined by

$$A(r_h) = 0.$$

The Born–Infeld-$f(R)$ solution modifies the ordinary Schwarzschild–AdS form,

$$A_{SAdS}(r) = 1 - \frac{2GM}{r} + \frac{r^2}{\ell^2},$$

by adding corrections dependent on both $f(R)$-curvature structure and the Born–Infeld scale β. Recent work reports exact static, spherically symmetric solutions and compares their thermodynamics with Schwarzschild–AdS black holes [1].

V. HORIZON STRUCTURE

The horizon structure is controlled by the roots of $A(r)$. Depending on mass M, charge Q, Born–Infeld parameter β, and curvature parameters inside $f(R)$, one may obtain:

- ✓ A single event horizon.
- ✓ Inner and outer horizons.
- ✓ Extremal configurations where two horizons merge.
- ✓ Horizon disappearance, yielding naked singular or regular remnants depending on the model.

The Born–Infeld sector can soften electromagnetic divergences, while the $f(R)$sector alters the gravitational potential. Their combined influence may shift the horizon radius relative to Schwarzschild, Reissner–Nordström, or Schwarzschild–AdS cases [1,6].

VI. THERMODYNAMICS

The Hawking temperature is determined by surface gravity:

$$T_H = \frac{1}{4\pi} A'(r_h).$$

The entropy in $f(R)$gravity is not simply $A/4G$. Wald's formula gives

$$S = \frac{A_h}{4G} f_R(r_h),$$

following the Noether-charge formalism developed by Wald [5].

where

$$A_h = 4\pi r_h^2.$$

Thus,

$$S = \frac{\pi r_h^2}{G} f_R(r_h).$$

This is a decisive modification. Entropy now counts not merely horizon area, but area weighted by the effective curvature response of the gravitational Lagrangian.

The heat capacity is

$$C = \frac{dM}{dT_H}.$$

Its sign determines local thermodynamic stability:

$$C > 0 \Rightarrow \text{locally stable},$$

$$C < 0 \Rightarrow \text{thermodynamically unstable}.$$

The recent Born–Infeld-$f(R)$ study emphasizes that the Hawking temperature, entropy, and specific heat exhibit behaviour significantly different from standard General Relativity predictions [1].

VII. WHY IT MATTERS

The Born–Infeld-$f(R)$ black hole is not merely a decorative modification of Schwarzschild. It matters for four reasons.

First, it *probes strong-field gravity [3,4]*, precisely where Einstein gravity may require correction.

Second, it tests whether nonlinear electrodynamics [2] can soften or restructure black-hole singularities.

Third, it links black-hole thermodynamics[5] to modified curvature degrees of freedom.

Fourth, it offers a theoretical laboratory for quantum-gravity-inspired regularization without committing immediately to a full ultraviolet-complete theory.

In short:

$$\text{Born–Infeld} + f(R) = \text{nonlinear matter} + \text{nonlinear geometry}.$$

That is a formidable duet.

VIII. RESEARCH FINDINGS

- ✓ *The theory naturally deforms Schwarzschild–AdS thermodynamics.* The temperature, entropy, and specific heat are altered by Born–Infeld and $f(R)$ parameters [1].
- ✓ *Entropy becomes curvature-weighted through the Wald formalism [5]*. In $f(R)$ gravity, the horizon entropy is modified by f_R, showing that thermodynamic information depends on the gravitational Lagrangian itself [3].
- ✓ *Born–Infeld corrections regulate field behaviour.* The nonlinear electromagnetic sector reduces the severity of field divergences compared with Maxwell theory [2].
- ✓ *The black hole becomes a diagnostic object.* Its horizons and heat capacity may distinguish Einstein gravity from higher-curvature alternatives.

IX. CONCLUSION

A Born–Infeld-$f(R)$ black hole is a black hole in which both matter and geometry have refused to remain linear. It is Einstein's black hole after two renovations: one by nonlinear electrodynamics, the other by curvature generalization.

Its deepest lesson is that black holes are not merely endpoints of collapse. They are testing chambers where the grammar of spacetime is forced to reveal whether it is Einsteinian, Born–Infeldian, $f(R)$-modified, or something stranger still.

The final verdict may be stated compactly:

$$\boxed{\text{Born–Infeld-}f(R) \text{ black holes are laboratories of nonlinear gravity.}}$$

They ask whether singularity, entropy, and horizon stability are immutable facts—or merely artifacts of using too simple a gravitational action[1-6]

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