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Developing Asymmetric $\mathbb{Z}_2$ Orbifold Actions In Pati–Salam Heterotic String Vacua

Dr Ijaz Durrani

University of Education, Lahore, Pakistan

Abstract: ***We develop a theoretical framework for asymmetric*** $\mathbb{Z}_2$ ***orbifold actions in four-dimensional Pati–Salam heterotic string vacua. The central aim is to understand how left–right asymmetric operations on internal worldsheet degrees of freedom can reduce moduli, induce doublet–triplet splitting, and generate phenomenologically viable spectra. Recent work by Detraux, Faraggi, and Percival explicitly classifies Pati–Salam asymmetric[1]*** $\mathbb{Z}_2 \times \mathbb{Z}_2$ ***heterotic orbifolds in the free-fermionic formulation, identifying asymmetric breaking vectors, residual Narain lattices, and classes with*** $12, 8, 4,$ ***or*** 0 ***real untwisted moduli. Building on this structure, we present a compact but rigorous account of how such asymmetric actions are engineered, constrained, and interpreted.***

Keywords: Pati–Salam Model; Heterotic String Theory; Asymmetric Orbifolds; $\mathbb{Z}_2 \times \mathbb{Z}_2$ ***Compactification; Free Fermionic Construction; Narain Lattice; GGSO Projections; Moduli Stabilisation; Doublet–Triplet Splitting; Exophobic Vacua.***

I. INTRODUCTION

The Pati–Salam gauge structure,

$$SU(4)_C \times SU(2)_L \times SU(2)_R,$$

occupies a privileged position between Standard Model phenomenology and grand unification[4]. It embeds quarks and leptons into unified multiplet s while avoiding the full rigidity of a minimal $SO(10)$ scheme. In heterotic string theory, such models frequently arise through compactification of the ten-dimensional theory on internal six-dimensional spaces, with gauge symmetry and chirality controlled by orbifold actions, Wilson lines, and generalized GSO phases.

The novelty of the asymmetric programme lies in allowing the orbifold action to treat left- and right-moving internal degrees of freedom differently. Symmetric orbifolds act geometrically on compact coordinates; asymmetric orbifolds act more stringily, exploiting the full Narain lattice of momenta, windings, and gauge charges. This is why asymmetric orbifolds are not merely decorated geometry: they are intrinsically string-theoretic constructions. A general Narain-orbifold framework treats such models as finite-order T-duality operations acting on compactified heterotic degrees of freedom [5,6] .

FREE-FERMIONIC FOUNDATION

In the free-fermionic formulation, the internal bosonic degrees of freedom are represented by world sheet fermions. A model is specified by basis vectors

$$v_i = \{\alpha_i(f_1), \alpha_i(f_2), \dots, \alpha_i(f_N)\},$$

where $\alpha_i(f)$ denotes the boundary condition of worldsheet fermion f around non-contractible loops of the string worldsheet. The physical spectrum is then selected by generalized GSO projections,

$$e^{i\pi v_i \cdot F_\xi} \mid \xi\rangle = \delta_\xi \, C\binom{\xi}{v_i}^* \mid \xi\rangle,$$

where F_ξ is the fermion-number operator and $C\binom{\xi}{v_i}$ are GGSO phases.

The free-fermionic construction is known to be closely related to $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds at special points in moduli space

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[2,3] . Faraggi and collaborators used this relation extensively in classifying chiral heterotic vacua and Pati–Salam landscapes [3,4].

THE ASYMMETRIC $\mathbb{Z}_2$ ACTION

Let the compact internal degrees of freedom be encoded in left- and right-moving coordinates,

$$X^I(\sigma,\tau) = X_L^I(\tau+\sigma) + X_R^I(\tau-\sigma).$$

A symmetric $\mathbb{Z}_2$ action acts as

$$X_L^I \to -X_L^I, X_R^I \to -X_R^I.$$

An asymmetric $\mathbb{Z}_2$ action instead permits

$$X_L^I \to -X_L^I, X_R^I \to X_R^I,$$

or more generally combines twists and shifts differently in the left and right sectors. This asymmetry is consistent only if modular invariance is preserved [5,6]. In free-fermionic language, this means the basis vectors and GGSO coefficients must satisfy the modular constraints

$$N_{ij}\, v_i \cdot v_j = 0(\mathrm{mod}4), \qquad N_i v_i \cdot v_i = 0(\mathrm{mod}8),$$

where N_i is the order of v_i, and N_{ij} is the least common multiple of N_i, N_j.

The recent Pati–Salam asymmetric classification begins from symmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ vacua with an $SO(10)$ GUT and then allows the Pati–Salam breaking vector to act asymmetrically on internal degrees of freedom [1].

BREAKING $SO(10)$ TO PATI–SALAM

The relevant breaking pattern is

$$SO(10) \to SU(4)_C \times SU(2)_L \times SU(2)_R.$$

In free-fermionic models, this is often implemented by a breaking vector α, whose boundary conditions separate the $SO(10)$ roots into retained and projected sectors. The asymmetric refinement is to let α act not merely as a gauge-breaking vector, but as a combined gauge–internal operation:

$$\alpha = \alpha_{\text{gauge}} + \alpha_{\text{internal}}^{L,R}.$$

This is where the physics becomes delicate. The same vector that breaks the observable gauge group may also freeze geometric moduli and split Higgs multiplet s.

MODULI FREEZING

Untwisted moduli arise from marginal operators of the form

$$J_L^I \bar{J}_R^J,$$

where J_L^I and $\bar{J}_R^J$ are left- and right-moving internal currents. Under an asymmetric $\mathbb{Z}_2$ action, some of these operators are odd and therefore projected out:

$$J_L^I \bar{J}_R^J \to -J_L^I \bar{J}_R^J.$$

Thus the number of surviving moduli depends on the precise asymmetric twist. Detraux, Faraggi, and Percival report six inequivalent classes of geometric moduli spaces with $12, 8, 4,$ or 0 real untwisted moduli [1], and identify 24 inequivalent cases after combining asymmetric twists with compatible asymmetric shifts.

This is not a decorative feature. Moduli are massless scalar fields; uncontrolled moduli are phenomenological nuisances. Their removal by asymmetric action is therefore a major structural virtue.

DOUBLET–TRIPLET SPLITTING

A persistent problem in unified theories is why electroweak Higgs doublets remain light while dangerous colour triplets are absent or heavy. In Pati–Salam models, the Higgs content descends from representations such as

$$\mathbf{10}_{SO(10)} \to (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2}).$$

The colour triplet lies in $(\mathbf{6},\mathbf{1},\mathbf{1})$, while the electroweak bidoublet lies in $(\mathbf{1},\mathbf{2},\mathbf{2})$. An asymmetric projection can remove the triplet while preserving the doublet:

$$(\mathbf{6},\mathbf{1},\mathbf{1}) \notin \mathcal{H}_{\text{phys}}, (\mathbf{1},\mathbf{2},\mathbf{2}) \in \mathcal{H}_{\text{phys}}.$$

The 2026 classification emphasizes that this doublet–triplet splitting occurs for any asymmetric action, including pure asymmetric shifts that preserve all geometric moduli; hence the splitting is not simply a by-product of moduli stabilization [1].

EXOPHOBICITY AND PHENOMENOLOGY

A viable heterotic vacuum must avoid massless fractionally charged exotic states, or at least confine or decouple them. Models with no massless fractional exotics are called **exophobic**. The classification programme searches GGSO phase spaces for vacua satisfying [4,7]:

$$N_{\text{gen}} = 3,$$

$$N_{\text{exotic}}^{\text{massless}} = 0,$$

$$G_{\text{obs}} = SU(4)_C \times SU(2)_L \times SU(2)_R.$$

The Pati–Salam asymmetric analysis explicitly constructs representative basis sets admitting three chiral generations and identifies phenomenologically viable exophobic models, including both $\mathcal{N} = 1$ and $\mathcal{N} = 0$ vacua.

PARTITION FUNCTION AND VACUUM ENERGY

The one-loop partition function has the schematic form

$$Z(\tau,\bar{\tau}) = \frac{1}{\tau_2^2} \sum_{\xi,\eta} C\binom{\xi}{\eta} Z_L[\xi,\eta](\tau)\, Z_R[\xi,\eta](\bar{\tau}).$$

The associated one-loop vacuum energy is

$$\Lambda = -\frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau,\bar{\tau}).$$

A striking reported feature is that as geometric moduli decrease, the number of distinct partition functions collapses to a smaller set [1], showing degeneracy under GGSO phase variations. This suggests that asymmetric constraints do not merely prune the landscape; they organize it.

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II. RESEARCH FINDINGS

- ✓ *Asymmetry is constructive, not pathological:* left–right asymmetric actions are consistent when embedded in modular-invariant free-fermionic data [1,5,6].
- ✓ The Pati–Salam breaking vector becomes multifunctional: it breaks $SO(10)$, acts on internal coordinates, and controls moduli.
- ✓ *Doublet–triplet splitting is intrinsic:* asymmetric action can project out unwanted colour triplets while preserving electroweak bidoublets.
- ✓ *Moduli reduction is classifiable:* inequivalent classes arise with $12,8,4,$or 0real untwisted moduli.
- ✓ *Phenomenological viability survives asymmetry:* three-generation and exophobic models can be found within the asymmetric landscape.
- ✓ *Vacuum degeneracy emerges:* fewer moduli correlate with fewer distinct partition-function structures.

III. NOVELTY AND SIGNIFICANCE

The central novelty is the conversion of asymmetry into a model-building instrument. In older geometric intuition, asymmetry looks like a nuisance: an operation without simple spacetime interpretation. In heterotic string theory, however, asymmetry becomes a scalpel. It cuts moduli, separates Higgs doublets from colour triplets, and reorganizes the landscape of viable vacua.

Thus the asymmetric $\mathbb{Z}_2$orbifold is not merely a technical embellishment. It is a string-theoretic mechanism by which geometry, gauge symmetry, and phenomenology are made to negotiate with one another.

IV. CONCLUSION

Developing asymmetric $\mathbb{Z}_2$orbifold actions in Pati–Salam heterotic vacua reveals a refined route toward realistic string model building. The asymmetric operation reshapes the Narain lattice, modifies the free-fermionic GGSO projection structure, reduces or eliminates moduli, and induces doublet–triplet splitting. In doing so, it transforms the Pati–Salam vacuum from a conventional descendant of $SO(10)$into a genuinely stringy construction where left and right movers no longer march in lockstep.

The deeper lesson is Penrosian in spirit: geometry alone is not sovereign. In string theory, geometry is accompanied by algebra, duality, and world sheet asymmetry. Reality is not the sum of its visible pieces; it is the hidden consistency condition binding them.

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