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Neutrosophic Fuzzy Matrices Using Algebraic Operations

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***Abstract:** In this paper we have established some neutrosophic algebraic property, and subtraction addition and multiplication operations of these matrices and commutative property, distributive property had been examine. We prove that neutrosophic fuzzy matrices hold associative property with respect to subtraction operation. The results have further been examined with suitable numerical examples. Fundamental operations such as addition, subtraction, multiplication, and scalar multiplication are defined and analyzed within the neutrosophic framework. Additionally, properties such as associativity, commutativity, and distributivity are examined. The study also introduces inverse and determinant concepts for neutrosophic matrices, addressing computational challenges and potential applications in decision-making, artificial intelligence, and complex systems modeling.*

***Keywords:** Neutrosophic fuzzy Matrix; Properties of Neutrosophic Fuzzy Matrices, Associative, Distributive, subtraction of neutrosophic matrices.*

ADDITION OPERATION OF TWO NEUTROSOPHIC FUZZY MATRICES

Let us consider two neutrosophic fuzzy matrices as

A= $\begin{pmatrix} a_1+Ib_1 & a_2+Ib_2 \\ a_3+Ib_3 & a_4+Ib_4 \end{pmatrix}$, B= $\begin{pmatrix} c_1+Id_1 & c_2+Id_2 \\ c_3+Id_3 & c_4+Id_4 \end{pmatrix}$

Then we would like to define the addition of these two matrices as A + B=[D_{ij}]

Where

D_{11}= max(a_1, c_1) + Imax(b_1, d_1)
D_{12}= max(a_2, c_2) + Imax(b_2, d_2)
D_{21}= max(a_3, c_3) + Imax(b_3, d_3)
D_{22}= max(a_4, c_4) + Imax(b_4, d_4).

SUBTRACTION OPERATION OF TWO NEUTROSOPHIC FUZZY MATRICES

Consider two neutrosophic fuzzy matrices given by

A = $\begin{pmatrix} x_1+Iy_1 & x_2+Iy_2 \\ x_3+Iy_3 & x_4+Iy_4 \\ x_5+Iy_5 & x_6+Iy_6 \end{pmatrix}$ and B = $\begin{pmatrix} l_1+Im_1 & l_2+Im_2 \\ l_3+Im_3 & l_4+Im_4 \\ l_5+Im_5 & l_6+Im_6 \end{pmatrix}$

A- B = E, where E_{ij} are as follows

E_{11}= min{x_1, l_1} + I min{y_1, m_1}
E_{12} = min{x_2, l_2} + I min{y_2, m_2}

E_{21} = min{x_3, l_3} + I min{y_3, m_3}
E_{22} = min{x_4, l_4} + I min{y_4, m_4}
E_{31} = min{x_5, l_5} + I min{y_5, m_5}
E_{32} = min{x_6, l_6} + I min{y_6, m_6}
Since min{a, b} = min{b, a}.

MULTIPLICATION OPERATION OF TWO NEUTROSOPHIC FUZZY MATRICES

Consider two neutrosophic fuzzy matrices, whose entries are of the form a + Ib (neutrosophic number), where a, b are the elements of [0,1] and I is an indeterminate such that I_n = I, n being a positive integer, given by

$$A=\begin{pmatrix} p_1+Iq_1 & p_2+Iq_2 \\ p_3+Iq_3 & p_4+Iq_4 \end{pmatrix},\ B=\begin{pmatrix} m_1+In_1 & m_2+In_2 \\ m_3+In_3 & m_4+In_4 \end{pmatrix}$$

The Multiplication Operation of two Neutrosophic Fuzzy Matrices is given by

$$AB=\begin{pmatrix} p_1+Iq_1 & p_2+Iq_2 \\ p_3+Iq_3 & p_4+Iq_4 \end{pmatrix}\begin{pmatrix} m_1+In_1 & m_2+In_2 \\ m_3+In_3 & m_4+In_4 \end{pmatrix}$$

$F_{11} = [\max\{\min(p_1, m_1), \min(p_2, m_3)\}]$ + I max{min{(q_1, n_1),min(q_2, n_3)}]

$F_{12} = [\max\{\min(p_1, m_2), \min(p_2, m_4)\}]$ + I max{min{(q_1, n_2),min(q_2, n_4)}]

$F_{21} = [\max\{\min(p_3, m_1), \min(p_4, m_3)\}]$ + I max{min{(q_3, n_1),min(q_4, n_3)}]

$F_{22} = [\max\{\min(p_3, m_2), \min(p_4, m_4)\}]$ + I max{min{(q_3, n_2),min(q_4, n_4)}]

Hence, $AB=\begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}$.

PROPOSITION

The following properties hold in the case of neutrosophic fuzzy matrix for subtraction

- ✓ A-B = B-A
- ✓ (A - B) - C = A - (B - C) = (B- C) – A = (C – B) – A.

PROOF. Consider three neutrosophic fuzzy matrices A, B and C as follows.

$$A=\begin{pmatrix} a_{11}+Ib_{11} & a_{12}+Ib_{12} \\ a_{21}+Ib_{21} & a_{22}+Ib_{22} \\ a_{31}+Ib_{31} & a_{32}+Ib_{32} \end{pmatrix},$$

$$B=\begin{pmatrix} c_{11}+Id_{11} & c_{12}+Id_{12} \\ c_{21}+Id_{21} & c_{22}+Id_{22} \\ c_{31}+Id_{31} & c_{32}+Id_{32} \end{pmatrix} \quad \text{and}$$

$$C=\begin{pmatrix} l_{11}+Im_{11} & l_{12}+Im_{12} \\ l_{21}+Im_{21} & l_{22}+Im_{22} \\ l_{31}+Im_{31} & l_{32}+Im_{32} \end{pmatrix}$$

$$A-B=\begin{pmatrix} a_{11}+Ib_{11} & a_{12}+Ib_{12} \\ a_{21}+Ib_{21} & a_{22}+Ib_{22} \\ a_{31}+Ib_{31} & a_{32}+Ib_{32} \end{pmatrix} - \begin{pmatrix} c_{11}+Id_{11} & c_{12}+Id_{12} \\ c_{21}+Id_{21} & c_{22}+Id_{22} \\ c_{31}+Id_{31} & c_{32}+Id_{32} \end{pmatrix} = G \text{ (say)},$$

where,

G_{11} = min{a_{11}, c_{11}}+Imin{b_{11}, d_{11}} = $x_{11}+Iy_{11}$

G_{12} = min{a_{12}, c_{12}}+Imin{b_{12}, d_{12}} = $x_{12}+Iy_{12}$

G_{21} = min{a_{21}, c_{21}}+Imin{b_{21}, d_{21}} = $x_{21}+Iy_{21}$

G_{22} = min{a_{22}, c_{22}}+Imin{b_{22}, d_{22}} = $x_{22}+Iy_{22}$

G_{31} = min{a_{31}, c_{31}}+Imin{b_{31}, d_{31}} = $x_{31}+Iy_{31}$

G_{32} = min{a_{32}, c_{32}}+Imin{b_{32}, d_{32}} = $x_{32}+Iy_{32}$

$$G=\begin{pmatrix} x_{11}+Iy_{11} & x_{12}+Iy_{12} \\ x_{21}+Iy_{21} & x_{22}+Iy_{22} \\ x_{31}+Iy_{31} & x_{32}+Iy_{32} \end{pmatrix} \quad \text{and}$$

$$B-A=\begin{pmatrix} x_{11}+Iy_{11} & x_{12}+Iy_{12} \\ x_{21}+Iy_{21} & x_{22}+Iy_{22} \\ x_{31}+Iy_{31} & x_{32}+Iy_{32} \end{pmatrix} = G,$$ [min(a, c) = min(c, a)]

Hence, A – B = B – A.

Now we have, G – C = (A – B) – C

$$=\begin{pmatrix} x_{11}+Iy_{11} & x_{12}+Iy_{12} \\ x_{21}+Iy_{21} & x_{22}+Iy_{22} \\ x_{31}+Iy_{31} & x_{32}+Iy_{32} \end{pmatrix} - \begin{pmatrix} l_{11}+Im_{11} & l_{12}+Im_{12} \\ l_{21}+Im_{21} & l_{22}+Im_{22} \\ l_{31}+Im_{31} & l_{32}+Im_{32} \end{pmatrix}$$

= H (say),

where,

H_{11} = min{x_{11}, l_{11}}+Imin{y_{11}, m_{11}}=min{a_{11}, c_{11}, l_{11}} + Imin{b_{11}, d_{11}, m_{11}}=$n_{11}+Ik_{11}$

H_{12} = min{x_{12}, l_{12}}+Imin{y_{12}, m_{12}}=min{a_{12}, c_{12}, l_{12}} +Imin{b_{12}, d_{12}, m_{12}}=$n_{12}+Ik_{12}$

H_{21} = min{x_{21}, l_{21}}+Imin{y_{21}, m_{21}}=min{a_{21}, c_{21}, l_{21}} +Imin{b_{21}, d_{21}, m_{21}}=$n_{21}+Ik_{21}$

H_{22} = min{x_{22}, l_{22}}+Imin{y_{22}, m_{22}}=min{a_{22}, c_{22}, l_{22}} +Imin{b_{22}, d_{22}, m_{22}}=$n_{22}+Ik_{22}$

H_{31} = min{x_{31}, l_{31}}+Imin{y_{31}, m_{31}}=min{a_{31}, c_{31}, l_{31}} +Imin{b_{31}, d_{31}, m_{31}}=$n_{31}+Ik_{31}$

H_{32} = min{x_{32}, l_{32}}+Imin{y_{32}, m_{32}}=min{a_{32}, c_{32}, l_{32}} +Imin{b_{32}, d_{32}, m_{32}}=$n_{32}+Ik_{32}$

$$(A-B)-C=H=\begin{pmatrix} n_{11}+Ik_{11} & n_{12}+Ik_{12} \\ n_{21}+Ik_{21} & n_{22}+Ik_{22} \\ n_{31}+Ik_{31} & n_{32}+Ik_{32} \end{pmatrix}$$

Next we have,

$$B-C=\begin{pmatrix} c_{11}+Id_{11} & c_{12}+Id_{12} \\ c_{21}+Id_{21} & c_{22}+Id_{22} \\ c_{31}+Id_{31} & c_{32}+Id_{32} \end{pmatrix} - \begin{pmatrix} l_{11}+Im_{11} & l_{12}+Im_{12} \\ l_{21}+Im_{21} & l_{22}+Im_{22} \\ l_{31}+Im_{31} & l_{32}+Im_{32} \end{pmatrix} = K \text{ (say)},$$

where

K_{11} = min{c_{11}, l_{11}}+Imin{d_{11}, m_{11}} = $p_{11}+Iq_{11}$

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$K_{12} = \min\{c_{12}, l_{12}\} + I\min\{d_{12}, m_{12}\} = p_{12} + Iq_{12}$

$K_{21} = \min\{c_{21}, l_{21}\} + I\min\{d_{21}, m_{21}\} = p_{21} + Iq_{21}$

$K_{22} = \min\{c_{22}, l_{22}\} + I\min\{d_{22}, m_{22}\} = p_{22} + Iq_{22}$

$K_{31} = \min\{c_{31}, l_{31}\} + I\min\{d_{31}, m_{31}\} = p_{31} + Iq_{31}$

$K_{32} = \min\{c_{32}, l_{32}\} + I\min\{d_{32}, m_{32}\} = p_{32} + Iq_{32}.$

We have, B – C = K = $\begin{pmatrix} p_{11} + Iq_{11} & p_{12} + Iq_{12} \\ p_{21} + Iq_{21} & p_{22} + Iq_{22} \\ p_{31} + Iq_{31} & p_{32} + Iq_{32} \end{pmatrix}$

A – (B – C) = $\begin{pmatrix} a_{11} + Ib_{11} & a_{12} + Ib_{12} \\ a_{21} + Ib_{21} & a_{22} + Ib_{22} \\ a_{31} + Ib_{31} & a_{32} + Ib_{32} \end{pmatrix}$ - $\begin{pmatrix} p_{11} + Iq_{11} & p_{12} + Iq_{12} \\ p_{21} + Iq_{21} & p_{22} + Iq_{22} \\ p_{31} + Iq_{31} & p_{32} + Iq_{32} \end{pmatrix}$

where

$\min\{a_{11}, p_{11}\} + I\min\{b_{11}, q_{11}\} = \min\{a_{11}, c_{11}, l_{11}\} + I\min\{b_{11}, d_{11}, m_{11}\}$

$\min\{a_{12}, p_{12}\} + I\min\{b_{12}, q_{12}\} = \min\{a_{12}, c_{12}, l_{12}\} + I\min\{b_{12}, d_{12}, m_{12}\}$

$\min\{a_{21}, p_{21}\} + I\min\{b_{21}, q_{21}\} = \min\{a_{21}, c_{21}, l_{21}\} + I\min\{b_{21}, d_{21}, m_{21}\}$

$\min\{a_{22}, p_{22}\} + I\min\{b_{22}, q_{22}\} = \min\{a_{22}, c_{22}, l_{22}\} + I\min\{b_{22}, d_{22}, m_{22}\}$

$\min\{a_{31}, p_{31}\} + I\min\{b_{31}, q_{31}\} = \min\{a_{31}, c_{31}, l_{31}\} + I\min\{b_{31}, d_{31}, m_{31}\}$

$\min\{a_{32}, p_{32}\} + I\min\{b_{32}, q_{32}\} = \min\{a_{32}, c_{32}, l_{32}\} + I\min\{b_{32}, d_{32}, m_{32}\}.$

F = $\begin{pmatrix} n_{11} + Ik_{11} & n_{12} + Ik_{12} \\ n_{21} + Ik_{21} & n_{22} + Ik_{22} \\ n_{31} + Ik_{31} & n_{32} + Ik_{32} \end{pmatrix}$

Therefore, A – (B – C) = F = (A – B) – C.

NUMERICAL EXAMPLE

The following properties hold in the case of neutrosophic fuzzy matrix for subtraction

- ✓ A-B = B-A
- ✓ (A - B) - C = A - (B - C)

SOLUTION

Let us consider three neutrosophic fuzzy matrices as

A= $\begin{pmatrix} 0.2 + I0.7 & 0.2 + I0.4 \\ 0.4 + I0.3 & 0.4 + I0.3 \end{pmatrix}$,

B= $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.7 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$ and

C= $\begin{pmatrix} 0.3 + I0.8 & 0.9 + I0.1 \\ 0.2 + I0.3 & 0.5 + I0.4 \end{pmatrix}$

✓ A – B = $\begin{pmatrix} 0.2 + I0.7 & 0.2 + I0.4 \\ 0.4 + I0.3 & 0.4 + I0.3 \end{pmatrix} - \begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.7 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix} = D(Say)$

where,

$D_{11} = \min\{a_{11}, c_{11}\} + I\min\{b_{11}, d_{11}\} = \min\{0.2, 0.2\} + I\min\{0.7, 0.4\} = 0.2 + I0.4$

$D_{12} = \min\{a_{12}, c_{12}\} + I\min\{b_{12}, d_{12}\} = \min\{0.2, 0.1\} + I\min\{0.4, 0.7\} = 0.1 + I0.4$

$D_{21} = \min\{a_{21}, c_{21}\} + I\min\{b_{21}, d_{21}\} = \min\{0.4, 0.1\} + I\min\{0.3, 0.3\} = 0.1 + I0.3$

$D_{22} = \min\{a_{22}, c_{22}\} + I\min\{b_{22}, d_{22}\} = \min\{0.4, 0.4\} + I\min\{0.3, 0.1\} = 0.4 + I0.1$

D = $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.4 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$ and

B-A = $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.4 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$ = D

Since, min(a, c) = min(c, a)
Hence, A – B = B – A.

✓ Now we have,

D – C = (A – B) – C

= $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.4 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$ - $\begin{pmatrix} 0.3 + I0.8 & 0.9 + I0.1 \\ 0.2 + I0.3 & 0.5 + I0.4 \end{pmatrix}$ = E(Say)

$E_{11} = \min\{0.2, 0.2, 0.3\} + I\min\{0.7, 0.4, 0.8\} = 0.2 + I0.4$

$E_{12} = \min\{0.2, 0.1, 0.9\} + I\min\{0.4, 0.7, 0.1\} = 0.1 + I0.1$

$E_{21} = \min\{0.4, 0.1, 0.2\} + I\min\{0.3, 0.3, 0.3\} = 0.1 + I0.3$

$E_{22} = \min\{0.4, 0.4, 0.5\} + I\min\{0.3, 0.1, 0.4\} = 0.4 + I0.1$

A – (B – C) = E = $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.1 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$ …………(1)

Next We have,

B-C = $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.7 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$ − $\begin{pmatrix} 0.3 + I0.8 & 0.9 + I0.1 \\ 0.2 + I0.3 & 0.5 + I0.4 \end{pmatrix}$ = F(Say)

$F_{11} = \min\{0.2, 0.3\} + I\min\{0.4, 0.8\} = 0.2 + I0.4$

$F_{12} = \min\{0.1, 0.9\} + I\min\{0.7, 0.1\} = 0.1 + I0.1$

$F_{21} = \min\{0.1, 0.2\} + I\min\{0.3, 0.3\} = 0.1 + I0.3$

$F_{22} = \min\{0.4, 0.5\} + I\min\{0.1, 0.5\} = 0.4 + I0.1$

B-C = F = $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.1 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$

A-(B-C) = $\begin{pmatrix} 0.2 + I0.7 & 0.2 + I0.4 \\ 0.4 + I0.3 & 0.4 + I0.3 \end{pmatrix}$ - $\begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.1 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix}$

Where,

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G_{11} = min{0.2,0.2,0.3}+Imin{0.7,0.4,0.8} = 0.2+I0.4
G_{12} = min{0.2,0.1,0.9}+Imin{0.4,0.7,0.1} = 0.1+I0.1

G_{21} = min{0.4,0.1,0.2}+Imin{0.3,0.3,0.3} = 0.1+I0.3
G_{22} = min{0.4,0.4,0.5}+Imin{0.3,0.1,0.5} = 0.4+I0.1

$$G = \begin{pmatrix} 0.2 + I0.4 & 0.1 + I0.1 \\ 0.1 + I0.3 & 0.4 + I0.1 \end{pmatrix} \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

L.H.S =R.H.S
Therefore, A – (B – C) = (A – B) – C.
Hence Proved.

CONCLUSION

According the newly defined addition and multiplication operation of neutrosophic fuzzy matrices, it can be seen that some of the properties of arithmetic operation of these matrices are analogous to the classical matrices. Further some future works are necessary to deal with some more properties and operations of such kind of matrices.

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