

Analysis And Dynamic Simulation Of Two Winding, Single Phase Distribution Transformer With Resistance – Inductance (RL) Load Termination

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Abstract: *This paper presents the analysis of two winding, single phase distribution transformer. The model of the two winding transformer was derived and used to build simulation model of the transformer. From the simulation result, the operational condition of transformer was examined when the secondary terminal is connected to load. The effect of core saturation on the primary current was ascertained using look-up table.*

Keyword: *Two winding transformer, secondary terminal, core saturation, simulation model*

I. INTRODUCTION

Transmitting electricity a long distance at high voltage and then reducing it to a lower voltage for lighting became a recognized engineering roadblock to electric power distribution with many, not very satisfactory solutions tested by lighting companies. The mid-1880s saw a breakthrough with the development of functional transformers that allowed the AC voltage to be "stepped up" to much higher transmission voltages and then dropped down to a lower end user voltage[1-3]

Transformer is an ac machine that transfer electrical energy from one electric circuit to another and does so without a change of frequency. This can be done by principle of electromagnetic induction. It has electric circuit that is linked by common electric circuit[4,5].

Primary distribution voltages range from 4 kV to 35 kV phase-to-phase (2.4 kV to 20 kV phase-to-neutral)[6]

In our country, electrical energy is usually generated at 6.6 or 11 or 33kv, stepped up to 132, 220 or 330kv with the help of step- up transformer for transmission and then stepped

down to 66kv or 33kv at grid substations for feeding distribution transformers, stepping down the voltage further to 440/230v for consumer uses[7,8].

The basic construction of transformer requires no moving parts, so it is often called static transformer and it is rugged machine requiring the minimum maintenance [9,10]. It has no moving part, therefore there are not friction and windage losses. The other losses are relatively low so that the efficiency of a transformer is high. Typical transformer at full load lies between 96% and 97%.

When the primary of a transformer is connected to the source of ac supply, and the secondary is open, the transformer is said to be at no load. When the secondary circuit of a transformer is completed through an impedance or load, the transformer is said to be loaded and current flows through secondary and load. The magnitude and phase of secondary current I_2 with respect to secondary terminal voltage will depend upon the characteristic of load. That is current will be in phase, lag behind and lead the terminal voltage respectively when the load is non inductive and capacitive.

Distribution networks are divided into two types, radial or network.[11] A radial system is arranged like a tree where each customer has one source of supply. A network system has multiple sources of supply operating in parallel. Spot networks are used for concentrated loads. Radial systems are commonly used in rural or suburban areas.

Radial systems usually include emergency connections where the system can be reconfigured in case of problems, such as a fault or required replacement. This can be done by opening and closing switches. It may be acceptable to close a loop for a short time. Long feeders experience voltage drop (power factor distortion) requiring capacitors to be installed.

Reconfiguration, by exchanging the functional links between the elements of the system, represents one of the most important measures which can improve the operational performance of a distribution system. The problem of optimization through the reconfiguration of a power distribution system, in terms of its definition, is a historical single objective problem with constraints. Since 1975, when Merlin and Back introduced the idea of distribution system reconfiguration for active power loss reduction, until nowadays, a lot of researchers have proposed diverse methods and algorithms to solve the reconfiguration problem as a single objective problem [12]. Some authors have proposed Pareto optimality based approaches (including active power losses and reliability indices as objectives). For this purpose, different artificial intelligence based methods have been used: microgenetic, branch exchange, particle swarm optimization and non-dominated sorting genetic algorithm.[13-15]

II. MODEL OF TWO WINDING TRANSFORMER

The flux linkage and terminal voltage equations of two winding transformer can be derived taking into account the resistance and leakage fluxes of the windings and the core.

FLUX LINKAGE EQUATIONS

In terms of flux components, total flux linked by each of the windings can be expressed as [16,17]

$$\phi_1 = \phi_{l1} + \phi_m \quad (1)$$

$$\phi_2 = \phi_{l2} + \phi_m \quad (2)$$

Where ϕ_{l1} and ϕ_{l2} are the leakage flux components of windings 1 and 2 respectively. The mutual flux ϕ_m , is established by the resultant mmf of the two winding acting around the same path of the core.

Assuming that N_1 turns of winding 1 effectively link both ϕ_m and leakage flux, ϕ_{l1} , the flux linkage of winding 1, defines as the turn times the total flux inked is

$$\lambda_1 = N_1\phi_1 = N_1(\phi_{l1} + \phi_m) \quad (3)$$

Equation 3 can be rewritten as

$$\begin{aligned} \lambda_1 &= N_1(N_1i_1 + p_{l1}) + (N_1i_1 + N_2i_2)p_m \\ &= (N_1^2 p_{l1} + N_1^2 p_m)i_1 + N_1N_2 p_m i_2 \end{aligned} \quad (4)$$

Where

$$\phi_{l1} = N_1(i_1 + p_{l1}) \quad (5)$$

$$\phi_m = (N_1i_1 + N_2i_2)p_m \quad (6)$$

$$L_{11} = N_1^2 p_l + N_1^2 p_m \quad (7)$$

$$L_{12} = N_1N_2 p_m \quad (8)$$

Similarly, the flux linkage of winding 2 can be expressed as

$$\begin{aligned} \lambda_2 &= N_2(\phi_{l2} + \phi_m) \\ &= N_2(N_2i_2 p_{l2}) + (N_1i_1 + N_2i_2)p_m \\ &= N(N_2^2 p_{l2} + N_2^2 p_m)i_2 + N_1N_2 p_m i_1 \end{aligned} \quad (9)$$

Where

$$L_{22} = N_2^2 p_{l2} + N_2^2 p_m \quad (10)$$

$$L_{21} = N_1N_2 p_m \quad (11)$$

The resulting flux linkage equations for the two magnetically coupled windings expressed in terms of winding inductances are

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \quad (12)$$

$$\lambda_2 = L_{21}i_1 + L_{22}i_2 \quad (13)$$

Where L_{11} and L_{22} are the self-inductances of the winding and L_{12} and L_{21} are the mutual inductances between them.

The self inductance of winding 1 may be considered as the sum of a leakage, L_{l1} and a magnetizing component, L_m from its own current.

Thus for winding L, with

$$I_2 = 0$$

$$L_{11} = \frac{\lambda_{i_2=0}}{i_1} = \frac{N_1(\phi_{l1} + \phi_m)}{i_1}$$

$$= N_1^2 p_{l1} + N_1^2 p_m \quad (14)$$

Where $\phi_{m1} = N_1i_1 p_m$ is the portion of the mutual flux magnetized by i_1 . Likewise, for winding 2

$$=$$

Where

$\phi_{m2} = N_2i_2 p_m$ is the portion of the mutual flux magnetized by i_2 .

Taking the ratio of L_{m2} to L_{m1} , the relationship between the magnetizing inductances of the two winding are

$$\begin{aligned} L_{m2} &= \frac{N_2\phi_{m2}}{i_2} = \frac{N_2L_{l2}}{N_1} = N_2^2 p_m \\ &= \left(\frac{N_2}{N_1}\right)L_{m1} \end{aligned} \quad (16)$$

The total flux linked by winding 1, expressed in terms of its own magnetizing inductances is

$$\begin{aligned} N_1\phi_m &= N_1(\phi_{m1} + \phi_{m2}) \\ &= L_{m1}\left(i_1 + \frac{N_2}{N_1}i_2\right) \end{aligned} \quad (17)$$

VOLTAGE EQUATION OF THE TRANSFORMER

The induced voltage in each winding is equal to the time rate of change of the winding's flux linkage.

Using the flux linkage expression of equation 12, the induced voltage in the winding 1 is given by

$$e_1 = \frac{d\lambda_1}{dt} = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} \quad (18)$$

Replacing L_{11} by $L_{1i} + L_{m1}$ and $L_{12}i_2$ by $N_2 L_{m1} i_2 / N_1$, the voltage induced in the winding 1 can also be expressed as

$$e_1 = L_{l1} \frac{di_1}{dt} + L_{m1} \frac{d}{dt} \left(i_1 + \frac{N_2}{N_1} i_2 \right) \quad (19)$$

Denoting the referred value of i_2 by i_2' , equation 19 becomes

$$e_1 = L_{l1} \frac{di_1}{dt} + L_{m1} \frac{d}{dt} (i_1 + i_2') \quad (20)$$

Similarly, the induced voltage of winding 2 may be written as

$$e_2 = L_{l2} \frac{di_2}{dt} + L_{m2} \frac{d}{dt} \left(\frac{N_1}{N_2} i_1 + i_2 \right) \quad (20)$$

Multiplying equation 20 through by N_2/N_1 , denoting $N_1 e_2 / N_2$ by e_2' and replacing $N_1^2 L_{m2} / N_2^2$ by L_{m1} , equation 20 can be rewritten into the form

$$e_2' = L_{l2}' \frac{di_2'}{dt} + L_{m1} \frac{d}{dt} (i_1 + i_2') \quad (21)$$

The terminal voltage of a winding is the sum of the induced voltage and the resistive drop in the winding. That for winding 1 is given by

$$\begin{aligned} v_1 &= i_1 r_1 + e_1 \\ &= i_1 r_1 + L_{l1} \frac{di_1}{dt} + L_{m1} \frac{d}{dt} (i_1 + i_2') \end{aligned} \quad (22)$$

The terminal voltage equation of winding 2 can be written in terms of equations referred to winding 1's side, as

$$\begin{aligned} v_2' &= \frac{N_1}{N_2} v_2 = \left(\frac{N_2}{N_1} \right)^2 r_2 + e_2' \\ &= i_2' r_2 + L_{l2}' \frac{di_2'}{dt} + L_{m1} \frac{d}{dt} (i_1 + i_2') \end{aligned} \quad (23)$$

EQUIVALENT CIRCUIT REPRESENTATION OF THE TRANSFORMER

The form of the voltage equations in equation 22 and 23 with common L_{m1} term suggests the equivalent T-circuit shown in figure (1) for two winding transformer.

To establish the mutual flux, a finite magnetizing current $i + i_2'$, flows in the equivalent magnetizing inductance on the winding 1 side, L_{m1} .

The values of circuit parameters of winding 2 referred to winding 1 are determined by relation given in equation 24 and 25 below.

$$r_2' = \left(\frac{N_2}{N_1} \right)^2 r_2 \quad (24)$$

$$L_{l2}' = \left(\frac{N_1}{N_2} \right) L_{l2} \quad (25)$$

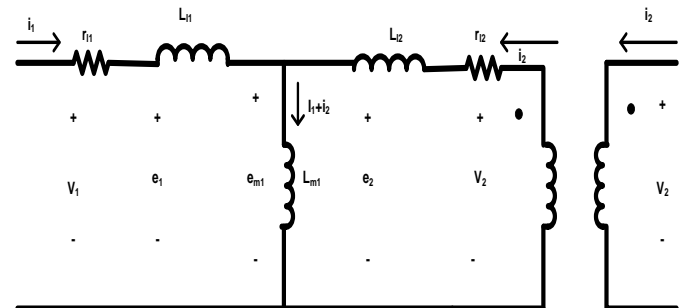


Figure 1: Equivalent circuit of two winding transformer

III. SIMULATION OF TWO WINDING TRANSFORMER

The total flux linkages of the two winding can be used as state variable. In terms of these two state variables, the voltage equations can be written as

$$v_1 = i_1 r_1 + \frac{1}{w_b} \frac{d\psi_1}{dt} \quad (26)$$

$$v_2' = i_2' r_2' + \frac{1}{w_b} \frac{d\psi_2'}{dt} \quad (27)$$

Where $\psi_1 = w_b \lambda_1$; $\psi_2 = w_b \lambda_2$ and w_b is the base frequency at which the reactance are computed.

The flux linkage per second of the windings can be expressed as

$$\psi_1 = w_b \lambda_1 = x_{l1} i_1 + \psi_m \quad (28)$$

$$\psi_2' = w_b \lambda_2' = x_{l2}' i_2' + \psi_m \quad (29)$$

$$\begin{aligned} \psi_m &= w_b L_{m1} (i_1 + i_2') \\ &= x_m (i_1 + i_2') \end{aligned} \quad (30)$$

From equation 28 and 29,

$$i_1 = \frac{\psi_1 - \psi_m}{x_{l1}} \quad (31)$$

$$i_2' = \frac{\psi_2' - \psi_m}{x_{l2}'} \quad (32)$$

Substituting equation 31 and 32 into equation 30,

$$\frac{\psi_m}{x_{m1}} = \frac{\psi_1 - \psi_m}{x_{l1}} + \frac{\psi_2'}{x_{l2}'} \quad (33)$$

From equation 33,

$$\psi_m \left(\frac{1}{x_{m1}} + \frac{1}{x_{l1}} + \frac{1}{x_{l2}} \right) = \frac{\psi_1}{x_{l1}} + \frac{\psi_2}{x_{l2}} \quad (34)$$

Letting

$$\frac{1}{X_M} = \frac{1}{x_{m1}} + \frac{1}{x_{l1}} + \frac{1}{x_{l2}} \quad (35)$$

Equation 34 can be written more compactly as

$$\psi_m = X_M \left(\frac{\psi_1}{x_{l1}} + \frac{\psi_2}{x_{l2}} \right) \quad (36)$$

Using equation 31 and 32 to replace the current in equation 26 and 27, integral equation of the two total flux linkages can be expressed

$$\psi_1 = \int \left\{ w_b v_1 + w_b r_1 \left(\frac{\psi_1 - \psi_m}{x_{l1}} \right) \right\} \quad (37)$$

$$\psi_2 = \int \left\{ w_b v_2 + w_b r_2 \left(\frac{\psi_2 - \psi_m}{x_{l2}} \right) \right\} \quad (38)$$

Equation 31, 32, 36, 37 and 38 are used for model of two winding transformer to which magnetic non linearity and iron losses may be added if necessary.

The simulink simulation of the two winding transformer can be set up using the voltage input, current output.

Core saturation can be handled using piece-wise linear analytic approximation of the saturation curve.

IV. ANALYSIS OF RESULT

The value of the parameters for simulation of the two winding transformer is shown in table 1.

Parameters	Value
Voltage ratio	120/240v
KVA Rating	1.5KVA
Frequency	50Hz
Primary resistance, R ₁	0.24Ω
Secondary resistance, R ₂	0.13 Ω
Primary reactance, x _{L1}	0.056 Ω
Secondary reactance, x _{L2}	0.056 Ω
X _{m1}	708.8 Ω

Table 1: Two winding distribution transformer,

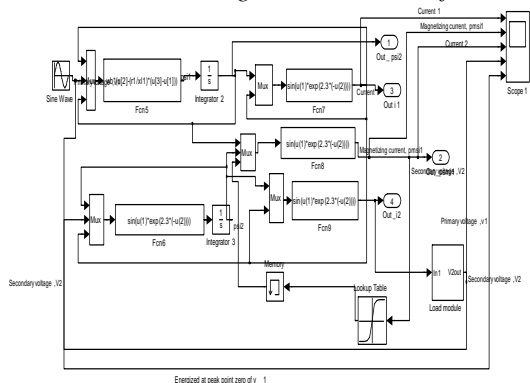


Figure 2: Simulink model of the two winding transformer

The simulink model of figure 2 to is obtained using equation 31, 32, 36, 37 and 38. The core saturation was handled using the look up table.

The simulation result of the two-winding transformer is shown in figure 3. It was obtained by substituting the values of parameters in table 1 into the simulink model shown in figure 2

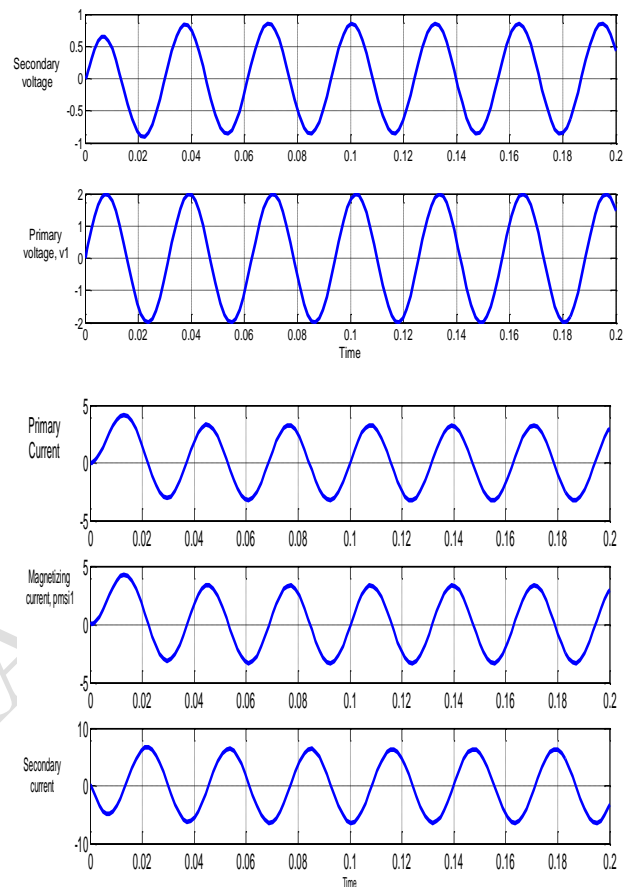


Figure 3: Simulation result of the two winding transformer

V. CONCLUSION

The simulink simulation of the two-winding transformer can be set up using voltage-input, current output model. The core saturation was handled using voltage look up table between ψ_m^{sat} and $\Delta\psi$. The parameter for the simulation of the two winding transformer is shown in table 1. The runs on the simulation was made with the 240v side terminals short circuited. The transformer was energized with the short circuit termination on the secondary terminal with a fixed impedance representing 1.5KVA, 0.8 lagging power factor of loading at rated voltage.

From the result of the simulation shown in figure 2, it is ascertained that the primary voltage, primary current, saturation flux secondary voltage and secondary current are sinusoidal waves. Saturation has serious impact on the decay times of the dc offset in the input current or flux against the values of the time constant for the corresponding terminal condition on the secondary side.

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