International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 7 Issue 6, June 2020

ISSN: 2394-4404

Mathematical Model On The Control Level Of Corona Virus Disease 2019 (COVID-19) In Nigeria, Considering Some Preventive Measures

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Abstract: A mathematical model on the spread of COVID-19 pandemic was developed and studied here. Analysis on the deficiency of medical testing kits which as a result, lead to more infected individual were outlined. The sensitivity analysis and numerical analysis of the model suggested that: increasing the lock-down with the provision of some basic needs by the government, will help cub the spread of the virus.

Keyword: Mathematical model, COVID-19, sensitivity analysis, reproduction number. AMS 2010: 02H13, 34L30, 34D35.

I. INTRODUCTION

The corona-virus disease popularly known as COVID-19 was named by the World Health Organization (WHO) on the January 10, 2020 after the health authorities of China and Centre for Disease Control Prevention (CDC) discovered the pathogen as a new type of corona-virus. As at December 2019, this virus for some period of time has been the major cause of pneumonia, severe cough, fatigue, dyspnea and fever in Wuhan, China [1]. The number of deaths recorded Worldwide from COVID-19 is more than three hundred and fifty-two thousand (352,069) as at 27th May 2020. It was revealed by National Health Commission China, that most of the occurred death were elderly people and about eighty percent of deaths were those over 60 years of age. Where 75% of them had underlying conditions including diabetes and cardiovascular disease. On 25 May 2020, WHO Director-General mentioned in his speech that Africa has just 1.5 percent of the world's reported cases of COVID-19, and less than 0.1 percent of the world's deaths. Although, these numbers don't paint the full picture in the case of Africa [2].

Nigeria With a population of about 206,139,589 people as at June 2020, being one of the most densely populated Africa countries draws the basis for the quest of this research. Just as with the rest of the world, Nigeria is currently battling with the coronavirus disease

2019 (COVID-19) pandemic. The first case as recorded in Nigeria was announced on the 27th February 2020, when it was claimed that a 44-year old Italian citizen tested positive to this virus in Lagos. Due to the un-seriousness of the government and the medical parastatals in the early checkmate of this virus, no account was kept on the number of people the Italian citizen came in contact with. After the incidence, there was travel ban on legal migration into the country.

On the other hand, the porous nature of the Nigerian borders contributes in making it impossible for a proper account of immigrants into the country. Another case of a Nigerian citizen who was believed to had contacts with the Italian citizen was recorded in Ewekoro (Ogun state) on the March 9, 2020 [3]. As of May 30 2020, it was recorded that 268 stranded Nigerians was evacuated from china to Abuja airport in Nigeria and has been sent to the nearest quarantine center where they are expected to stay a minimum of 14-days [4].

As of 22 April 2020, Nigeria Center for Disease Control (NCDC) confirmed 873 positve cases, 197 recoveries, and 28 deaths. WHO advised on several methods to help prevent spread of the virus and to save health systems across the world from a complete collapse eg: *health washing, social distancing, and staying at home*. However, Nigeria's health system before the pandemic was not encouraging. In most of the cities' in the country, health systems are completely dilapidated as they have not received adequate attention, and some government officials have contributed to health system collapse by encouraging medical tourism. In a plight to moderate its spread, the federal government of Nigeria enforced an initial 2-weeks lockdown on March 30, 2020, for three of 36 states (Lagos, Ogun, and Abuja) and, on April 13, extended it another 2 weeks [5].

The actual number of infected people are unknown, as testing kits where not available and the available ones lacked accuracy. As such, greater percentage of people (healthy and unhealthy) are not tested unless situation arises where one has to travel to a high-indexed country for a particular time period. As at 13 April 2020, the fatality rate of COVID-19 infection in Nigeria is approximately 0.03 (10 cases of death) out of 323 confirmed cases [6].

This work never considered the road map that can be taken to reduces the high rate of inflow of strangers into the country through its local borders. The table below displayed a few estimation on the rate of COVID-19 in the Nigerian population.

	Total confirmed cases	Total recovered cases	Total deaths	Recovery rate	Fatality rate	Mortality rate %100,000
April 13, 2020	343	91	10	0.2653	0.0292	0.0049
April 27, 2020	1,337	251	40	0.1877	0.0299	0.0194
May 11, 2020	4,641	902	152	0.1944	0.0328	0.0737
May 25, 2020	8,068	2,311	233	0.2864	0.0289	0.1130
June 8, 2020	12,801	4040	361	0.3156	0.0282	0.1751

Source: Nigeria Population (2020-02-17). Retrieved 2020-04-13 from http://worldpopulationreview.com/countries/nigeria/
Source: Worldometer and Nigeria Center for Disease Control (NCDC): An update of COVID-19 outbreak in Nigeria from https://ncdc.gov.ng/diseases/sitreps/?cat=14&name=An%20update%20of%20COVID-19%20outbreak%20in%20Nigeria.
Note that there may be unconfirmed cases which were never reported to the public health authorities. bf Source: https://www.macrotrends.net/countries/NGA/nigeria/population

Table 1: Estimated rates from COVID-19 pandemic in Nigeria for every 14-days starting from April 13, 2020

As with some other countries around the world, Nigeria's prevention efforts are limited to screening international travellers at its airports while promoting hand-washing, wearing of nose mask and hygiene among the general populace. A country with fewer airports is relying on temperature screening at its airport only, which means that if their exist an undisclosed

II. THE MODEL VARIABLE AND PARAMETER

The variables and parameters used in the model are:

VARIABLE DESCRIPTION

S(t)Susceptible population at time *t*.
Q(t)Quarantined immigrants at time *t*.
I(t)Infected individuals at time *t*.
H(t)Population of Isolated at time *t*.
R(t)Recovered population through quarantine or through immunity at time *t*.
N(t)Total population

PARAMETER DESCRIPTION

Λ Recruitment rate through legal immigration
$\mathit{\Lambda}$ Incidence rate or force of infection in the population
β Contact rate as the susceptible population interacts with the infected population before and during the lock-down order was given.
θ Degree of adherence to the preventive measures of the susceptible population due to forceful implementation or media sensitization $(0 \leq \theta \leq 1)$. Rate at which individuals move to the nearest quarantine centre.
ω Rate at which quarantined immigrants return to the susceptible population if found negative after 14-days (This depends on the accuracy of the testing machine).
δ_1 Disease induced death of the infected individuals who treats them selves and refuses to be isolated.
δ_2 Disease induced death of the isolated infected people receiving treatment. μNatural death rate of the population.
α Rate at which government or medical parastatals discover and evacuate infected people to the nearest isolation centre.
φ rate of recovery due to treatment offered in the isolation centres $(0 \leq \varphi \leq 1)$.

ASSUMPTIONS OF THE MODEL

Below are some assumptions made for the feasibility of the derivation of the mathematical model.

- ✓ No emigration from the total population.
- ✓ Legal and illegal immigration serves as the only source at which the virus is introduced into the susceptible population.
- ✓ Health workers in the quarantine class are well covered such that infection is impossible.
- ✓ Illegal immigration is possible because of porous borders, and was not accounted for in this model.
- ✓ Because of time delayed and insufficient/poor medical equipments to Scan/detect the virus at its early stage, it is assumed that once an individual contacts the virus, the individual becomes infectious.
- ✓ Infected individual with high immunity and those receiving self treatment can still spread the virus without showing any symptoms.
- ✓ Recovered individuals becomes permanently immunity.

 ISSN: 2394-4404

PREVENTIVE MEASURES (TAKEN IN NIGERIA)

- ✓ Use of sanitizer
- ✓ Wearing of nose mask and
- ✓ Social distancing

A. MODEL DIAGRAM

The following model diagram describes the COVID-19 frame work in Nigeria as at the moment of this research. Here, a susceptible class is that class which is yet to be infected but open to the disease through interaction with infected members, and they observes the possible preventive measures. The infected class are those individuals who have the disease, and can recover through quarantine (if detected early) or personal immunity (for those with strong immunity).

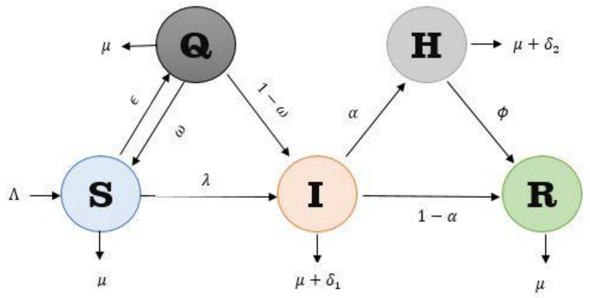

Figure 1: Model diagram

B. MODEL EQUATION

From the model diagram and model assumptions, the following system of equations were gotten:

$$\begin{aligned}\frac{dS}{dt} &= \Lambda - \lambda S + \omega Q - (\epsilon+\mu)S\,, & S(0)=S_0>0\\ \frac{dQ}{dt} &= \epsilon S - [(1-\omega)+\omega+\mu]Q\,, & Q(0)=Q_0>0\\ \frac{dI}{dt} &= \lambda S + (1-\omega)Q - [\alpha+(1-\alpha)+\delta_1+\mu]I\,, & I(0)=I_0>0\\ \frac{dH}{dt} &= \alpha I - (\phi+\mu+\delta_2)H\,, & H(0)=H_0>0\\ \frac{dR}{dt} &= (1-\alpha)I + \phi H - \mu R\,, & R(0)=R_0>0\end{aligned}$$

$S(t)=S, Q(t)=Q, I(t)=I, H(t)=H, R(t)=R$

where

$$\lambda = \frac{\beta(1-\theta)I}{N} \tag{2}$$

THEOREM 2.1. The variables of the model (1) are mathematically well posed and contains in the invariant region for all time t > 0.

PROOF:

We show that the solution of our model is contained in

$$\Omega = \{(S,Q,I,H,R)\in\mathbb{R}^5_+ : N(t)\le \frac{\Lambda}{\mu}\} \tag{3}$$

But

$$\begin{aligned} N(t) &= S+Q+I+H+R & (4)\\ \frac{dN(t)}{dt}=\dot{N}(t) &= \dot{S}+\dot{Q}+\dot{I}+\dot{H}+\dot{R} & (5)\\ &= \Lambda - \mu(S+H+I+Q+R) - \delta_1 I - \delta_2 Q & (6)\\ &\le \Lambda - \mu(S+H+I+Q+R) & (7)\\ &\le \Lambda - \mu N(t) & (8)\\ \dot{N}(t)+\mu N(t) &\le \Lambda & (9)\\ \frac{d}{dt}[N(t)e^{\mu t}] &\le \Lambda e^{\mu t} & (10)\end{aligned}$$

Integrating over the interval $t \in [0,\infty)$, we obtain

$$N(t) \le \frac{\Lambda}{\mu} \tag{11}$$

Hence **Ω** is positively invariant and an attractor so that no solution path leaves through any boundary of **Ω**.

C. POSITIVITY OF SOLUTION

Here, we show that the solutions to system (1) are positive together with the initial condition.

THEOREM 2.2. The solutions [S(t),Q(t),I(t),H(t),R(t)] of the system (2) are positive with nonnegative initial conditions.

PROOF:

System (1) can be written in matrix form

$Y^0 = P(Y)$, (12)

where $Y = (S,Q,I,H,R)^T \in R^5$ and $P(Y)$ is given by

$$P(Y)=\begin{pmatrix}P_1(Y)\\P_2(Y)\\P_3(Y)\\P_4(Y)\\P_5(Y)\end{pmatrix}=\begin{pmatrix}\Lambda-\frac{\beta SI(1-\theta)}{N}+\omega Q-(\epsilon+\mu)S\\ \epsilon S-[(1-\omega)+\omega+\mu]Q\\ \frac{\beta SI(1-\theta)}{N}+(1-\omega)Q-[\alpha+(1-\alpha)+\delta_1+\mu]I\\ \alpha I-(\phi+\mu+\delta_2)H\\ (1-\alpha)I+\phi H-\mu R\end{pmatrix} \tag{13}$$

Then, we have

$$\begin{aligned}\left.\frac{dS(t)}{dt}\right|_{S=0} &= \Lambda+\omega Q>0\\ \left.\frac{dQ(t)}{dt}\right|_{Q=0} &= \epsilon S>0\\ \left.\frac{dI(t)}{dt}\right|_{I=0} &= (1-\omega)Q\ge 0 \qquad (1)\\ \left.\frac{dH(t)}{dt}\right|_{H=0} &= \alpha I\ge 0\\ \left.\frac{dR(t)}{dt}\right|_{R=0} &= (1-\alpha)I+\phi H\ge 0\end{aligned}$$

Therefore

$$P_i|_{Y_t(t)=0,\ Y_t\in C^5_+}\ge 0\,, \qquad i=1,2,3,4,5 \tag{15}$$

From Lemma 2 in [7], any solution of system (1) is such that $Y(t)\in\mathbb{R}^5_+\ \ \forall\ t\ge 0$. With this, the proof is complete.

D. THE REPRODUCTION NUMBER R_0

This is the expected number of susceptible individuals an infectious person will create over the duration of the infectious period. Usually if $R_0 < 1$, it is interpreted as the disease not being able to invade the population. In other words, it an infected individual produces less than one new infected individual over the cause of his infectious period, and the infection cannot grow. If $R_0 > 1$ the number of infected individuals will increase from a generation to the next and the disease will persist. On the other hand, $R_0 = 1$ is a threshold below which the generation of secondary cases is insufficient to maintain the infection in human community [8]. We employ the next generation method in the calculation of R_0.

$$F=\left[\frac{\partial\mathcal{F}_i}{dx_j}(E_0)\right] \text{ and } V=\left[\frac{_i}{dx_j}(E_0)\right],\ 1\le i,j\le m \quad \partial V_i$$

ISSN: 2394-4404

Where F (which is nonnegative) and V (a non-singular matrix) are $m \times m$ matrices (*note m denotes the infectious classes only*), and E_0 denotes at Disease Free Equilibrium (*DFE*) of the system.

$$F = \begin{bmatrix} \frac{\beta\,(1-\theta)\,S}{N} & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

$$V = \begin{bmatrix} 1+\delta_1+\mu & 0 \\ -\alpha & \phi+\mu+\delta_2 \end{bmatrix} \tag{17}$$

Then

$$FV^{-1} = \begin{bmatrix} \frac{\beta\,(1-\theta)\,S}{N\,(1+\delta_1+\mu)} & 0 \\ 0 & 0 \end{bmatrix} \tag{18}$$

At *DFE*, $N(t) = S(t)$, while $Q(t) = I(0) = H(0) = R(t) = 0$. The eigenvalues of FV^{-1} at *DFE* are

$$Eigenvalue(FV^{-1}) = \begin{bmatrix} 0 \\ \frac{\beta(1-\theta)}{1+\delta_1+\mu} \end{bmatrix} \tag{19}$$

But

$$R_0 = \rho\,(FV^{-1}) \tag{20}$$

where ρ is the spectral radius of FV^{-1} (by spectral radius, we mean the maximum eigenvalue of FV^{-1}). Therefore

$$R_0 = \frac{\beta(1-\theta)}{1+\delta_1+\mu} \tag{21}$$

III. SENSITIVITY ANALYSIS

Sensitivity analysis describes how important the model parameters are with respect to the basic reproduction number R_0. It mostly used to determine the robustness of model predictions to parameter values, since there are usually errors in data collection and presumed parameter values. It is used to discover parameters that have a high impact on R_0 and should be targeted by intervention strategies [9].

DEFINITION 3.1. [10] The normalized forward-sensitivity index of R_0, that depends differentiably on a parameter, p, is defined as:

$$\Upsilon_p^{R_0} = \frac{\partial R_0}{\partial p} \times \frac{p}{R_0} \tag{22}$$

In particular, sensitivity indices of the basic reproduction number, R_0, with respect to the model parameters are examined. Using the R_0 we obtained:

$$\Upsilon_\beta^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0} = 1 \tag{23}$$

$$\Upsilon_\theta^{R_0} = \frac{\partial R_0}{\partial \theta} \times \frac{\theta}{R_0} = \frac{-\theta}{1-\theta} \tag{24}$$

$$\Upsilon_\mu^{R_0} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0} = \frac{-\mu}{1+\delta_1+\mu} \tag{25}$$

The sensitivity index (S.I.) of R_0 to a parameter say μ can be determined in same manner, and signs of S.I. will be summarized in the table below. It is good to note that if the S.I. of parameters happens to be positive, it shows an increase (or decrease) in the value(s) of each of the parameter will lead to an increase (or decrease) in R_0 of the model (1). Hence, with sensitivity analysis, one can get insight on the appropriate intervention strategies to prevent and control the spread of the disease described by model (1).

Variable/ Parameter	Value	% rate	Source
N	206,139,589	—	[12]
S	206,119,284	—	Estimated
H	269	—	[4]
I	10,162	—	[11]
Q	6,868	—	[11]
R	3,007	—	[11]
$n\beta$	—	0.0000493	Estimated
θ	—	0.2	Assumed
α	—	0.0000333	Estimated
φ	—	0.4378	Estimated
$\delta 2$	287	0.0418	[11]
$\delta 1$	79	0.007774	Assumed
μ		5.42	[12]

Table 2: Parameter values of the model (as at June 1, 2020) [11]

Parameter	S.I.
β	positive
θ	negative
μ	negative
$\delta 1$	negative

Table 3: Sensitivity indices (S.I.) some parameters of R_0

IV. SIMULATIONS

Here, we display some graphical simulations of the our model (1).

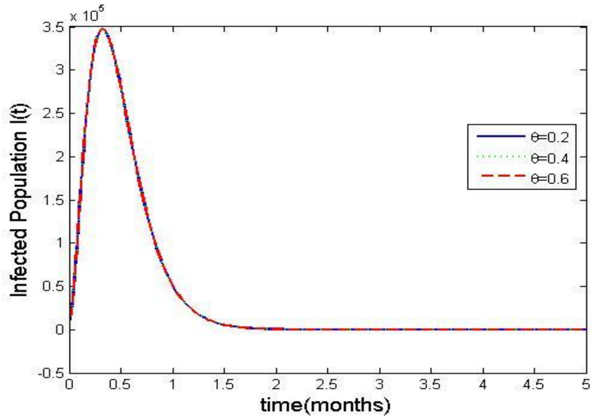

Figure 2: Variation in adherence to preventive measure as it affects the susceptible population

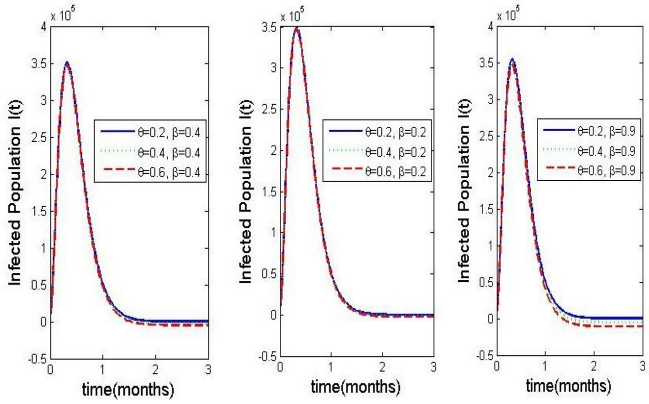

Figure 3: Improved simulation on the parameter β for a constant θ

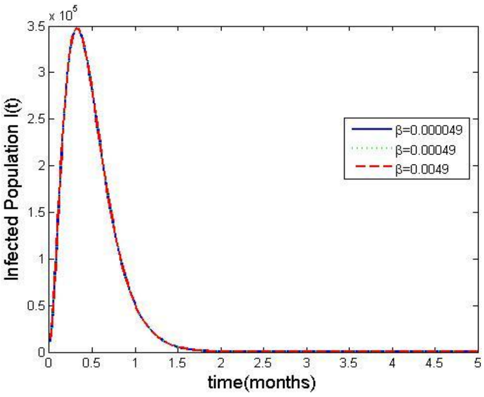

Figure 4: Variation in rate at which the lock-down order was obeyed

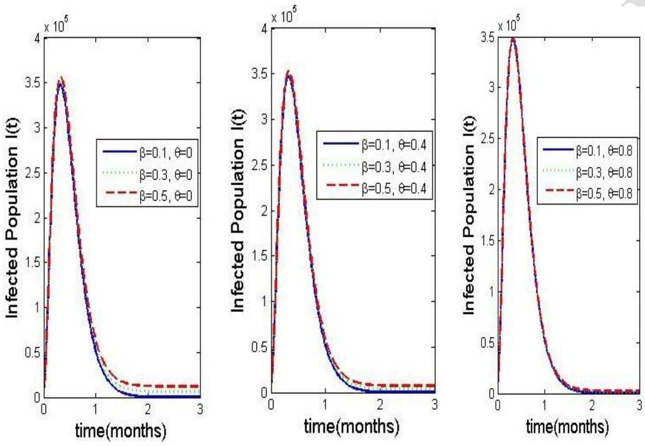

Figure 5: Improved simulation on the parameter θ for constant β

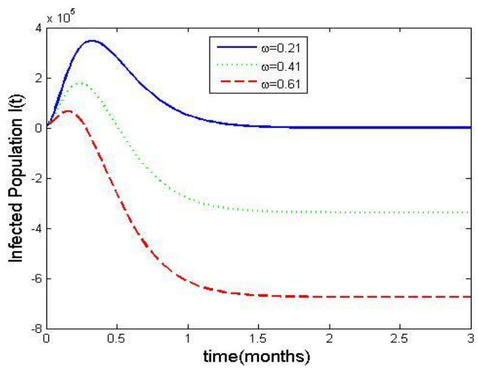

Figure 6: Effect in the efficiency of the test machine(s) on the infected class

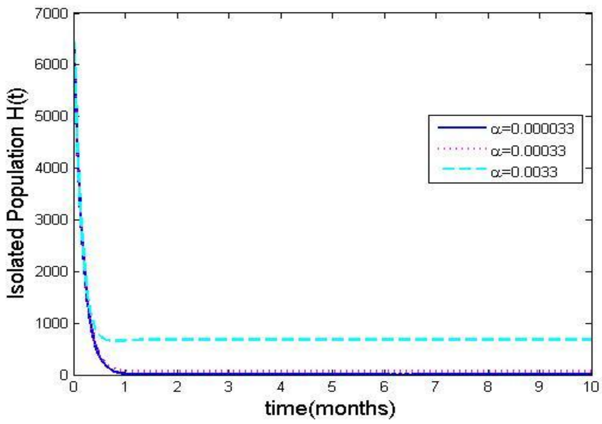

Figure 7: Variation in rate at which government/medical parastatals respond to infected individual

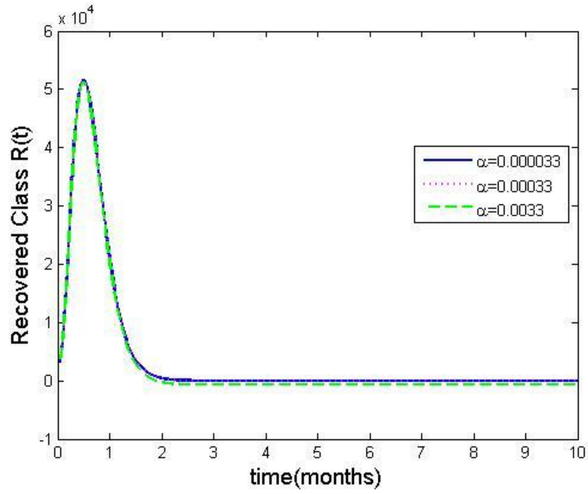

Figure 8: Variation in rate at which individuals recover from immunity or self-medication

V. RESULTS AND DISCUSSION

Figure (2), displayed the increasing rate of information and sensitization by the media towards the prevention of the disease. Clearly as observed, the increasing rate of θ did not improve (positively) the control/prevention of the disease. This in turn means that the government has not made adequate resources available in the society for people to obey the media rules in prevention and control of the spread of the disease. On the other hand, figure (3) displayed the effect when the lock-down order increases. It was observed that increasing the lock-down order to β = 0.9 rate, at θ = 0.6 will reduce the number of people that gets infected (assuming food palliatives and other few basic needs are provided by the government).

In figure (4), it was observed that going by the original rate of the lock-down order, no positive change will be observed towards the control of the disease. Figure (5) tired to display the ineffectiveness of some of the available preventive measures (e.g. Sanitizers and wearing of Nose mask). In other words, none of these preventive measures as listed above could control the spread of this disease, because these measures has low or no efficiency. Therefore, it is advised to

ISSN: 2394-4404

improve on the chemical contents of these preventive measures to check-mate the spread of this disease.

Next, we take a look at the daily chat of COVID-19 in Nigeria up until June 7, 2020.

Figure 9: A 14-day chart of COVID-19 in Nigeria [13]

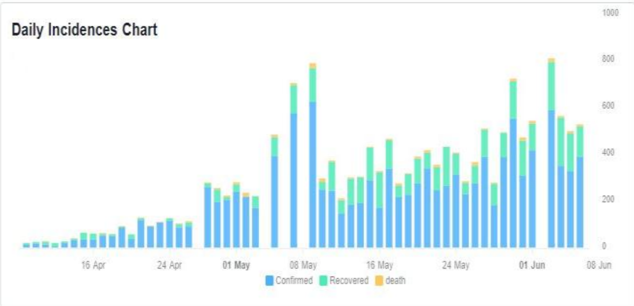

Figure 10: Daily incidence chart of COVID-19 in Nigeria [13]

VI. CONCLUSION AND RECOMMENDATION

This work developed a model of the characteristics of COVID-19 in Nigeria. It was ascertained that with such a number of porous borders in the country, the control of this pandemic may be impossible because of the presence of many undetected cases. On the other hand, with the level of poor facilities to help detect and combat the early stage infected individuals, serves as another challenge to the nation. Our model suggested the improvement in some parameters to enable the virus fizzle out with time. Exposed individuals where not considered in this model because of the delay in early detection of the virus in the susceptible population. The suggestions to cub the increasing rate of this virus, are stated below:

- ✓ High security measure in all the local borders.
- ✓ Increasing the number of isolation centres to most remote areas of the country.
- ✓ The selected medical teams should be well equipped and precautionary measures should be taken to avoid them contacting the virus from their patients.
- ✓ Palliatives and some basic needs should be provided to the communities (especially the poor citizens), to prevent them from coming out during the lock-down order.
- ✓ Quality hand-sanitizers should be made available at a cheaper rate for usage by the poorest citizens of the country.
- ✓ Sanitized nose mask should also be made available.

ACKNOWLEDGEMENT

I want to humbly acknowledge my co-author and supervisor Prof. G.C.E. Mbah for his contributions to the success of this work.

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