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Van Hiele's Geometric Thinking Levels And Achievement Differences Of Pre-Service Teachers' And In-Service Teachers' In Ghana

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Abstract: ***The purpose of this study was to investigate the differences in Van Hiele's geometric reasoning levels and achievement scores between In-Service Teachers (ISTs) and Pre-Service Teachers (PSTs). The study was guided by two research questions and six null hypotheses. The sample was 133 (37.89%) ISTs and 218(62.11%) level 300 PSTs. Both purposive and convenient samplings were used in this study. The instrument used was the 25 Van Hiele's Geometry Test (VHGT) items. The combined results from levels 1 to 5 of the independent sample t-test revealed that there was a statistically significant difference in the scores of ISTs (M=11.55, SD=1.79) and PSTs (M=10.98, SD=1.87), (t (349) = 2.80, p=0.005< 0.05) in favor of the ISTs. Also, the findings revealed the following levels of reasoning stages for both ISTs and PSTs. For level 1, (ISTs = 3 (2.26%), and PSTs = 3 (1.38%) ; level 2, (ISTs = 10 (7.52%), and PSTs = 83 (38.07%) ; level 3 (ISTs = 110 (82.71%) and PSTs = 127 (58.26 %) ; level 4 (ISTs = 8(6.02%) and PSTs = 5 (2.30 %) and 2 (1.50 %) for level 5 for only ISTs. It was recommended that college tutors should adopt Van Hiele's phase-based approach in teaching geometry.***

Keywords: Geometry, Pre-Service Teachers, In-Service Teachers , Van Hiele's Geometry Test (VHGT), Significant Difference, Independent Sample t-test

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I. INTRODUCTION

According to Salifu (2018, p1.), "Geometry is so important that it could not have been left out in any school's curriculum throughout the world". Geometry as a topic in Mathematics is linked to so many areas including trigonometry, algebra, measurement, and calculus and the knowledge of geometry is applied by physicists, land surveyors, engineers, and many other professionals (Russell, 2014). (Armah, Cofie & Okpoti, 2017; Alex & Mammen, 2016), in their studies posited that geometry supports students to improve upon their skills in deductive reasoning, conjecturing, visualization, intuition, problem-solving, perspective, logical argument and proof.

Colleges of Education per the new 4-year Bachelor of Education curriculum which just took off in September 2018 is tasked to produce programme based teachers to teach at the basic school level in Ghana. So these colleges are to produce grade based teachers (lower primary teachers, upper primary teachers, and Junior High School teachers) in their programme areas. The pedagogical and content knowledge of these PSTs is so vital that they now have major and minor areas for specialization. For example, mathematics major is compulsorily asked to do Information Communication Technology as a minor. The pedagogical and content knowledge in the geometry of the Pre-Service Teachers (PSTs) and In-Service Teachers (ISTs) is in doubt. Several studies notably (Halat & Şahin 2008; Ndlovu, 2014) have indicated that teachers at all stages need experience in order to attain the content knowledge for effective instruction in Mathematics. According to Halat & Şahin (2008) again, teachers' content knowledge is vital to students' performance because if the knowledge of the teacher is not sufficient then it will lead to poor performance.

According to (Anamuah-Mensah & Mereku, 2005; Armah, Cofie & Okpoti, 2017; Baffoe & Mereku, 2010), several concerns have been raised of the Ghanaian students' geometric thinking. Chief examiners' annual reports 2011 and 2012 for the Colleges of Education have indicated that: (i) PSTs' solutions to a majority of 2D and 3D questions were poorly tackled. (ii) PSTs could not solve questions involving concepts of interior and exterior angles of polygons and their properties (Institute of Education, UCC-Ghana, Chief Examiner's Report on the (2011&2012).

Armah et al (2018), posited that in the past, teachers have tested three models notably (i)Piaget and Inhelder Topological primacy thesis Piaget & Inhelder's (1948), (ii) Van Hiele's levels of Geometric Thinking Van Hiele's (1957) and (iii) Cognitive Mathematics Model (Friedenberg & Silverman, 2006) to correct the imbalance among students performances and geometry classroom instructions. Stipek (1998), contended that the inadequate geometry knowledge of PSTs and ISTs might be an additional significant factor behind students' poor achievement in geometry. The biggest issue is that both ISTs and PSTs teach the basic school pupils before they write their final exams, so the contention is which of these groups (the ISTs or the PSTs) contribute to the abysmal performance of the pupils. Hence to address this argument, this study is undertaken to clearly explain by finding the van Hiele's levels of ISTs and PSTs in geometry.

II. LITERATURE REVIEW: VAN HIELE'S STUDIES IN GHANA

Five Mathematics/Science Colleges of Educations in the three Northern Regions in Ghana research was conducted by Salifu (2018) whose purpose was to investigate Van Hiele's levels of geometric thinking among Mathematics PSTs. The population for that study was 412 whiles the sample used was two hundred and ninety- eight (298). The sample composed of 83.2% male and 16.8% female. Salifu study recorded 50.3% for Van Hiele's Level 0, 23.5% Van Hiele's Level 1, 14.8% for Van Hiele's Level 2, 9.1% Van Hiele's Level 3, 2.34% Van Hiele's Level 4 and 0 % Van Hiele's Level 5. The conclusion by Salifu was that only 11.44% were eligible to teach basic school Mathematics in Ghana. His study again revealed that 88.56 % of Mathematics PSTs attained levels 0, 1 and 2 meaning those MPSTs were not eligible to teach mathematics at the basic schools. Salifu (2018) recommended that Colleges of Education tutors should adopt Van Hiele's phase-based when delivering Geometry lessons.

In a study by Armah et al (2017), concluded that 75.33% of PSTs was lower than their expected future JHS 3 students when Van Hiele's geometry test was administered. Their study focused on Van Hiele's geometric thinking of PSTs' in Ghana before going out for teaching practice. A sample of 300 PSTs in their second-year was drawn from 4 Colleges of Education and three (3) regions of Ghana. From the analysis of the results, the following were recorded. 16.33% of the PSTs were at level 0, 27% for levels 1, 32 % for levels 2, 17.67% for levels 3, 6% for levels 4 and 1% for levels 5.

Asemani, et al (2017), the study used 200 Ghanaian Senior High School final year students selected from three (3) municipalities in the Central Region were used as the subjects. The study sample composed of 44% males and 56% females. The quantitative analysis revealed that students who did not meet any of Van Hiele's Geometric thinking level were 42.5%. Their study further analysis revealed that 33% of the final year students reached Van Hiele's level 1. The records for levels 2, 3 and 4 were 22.5%, 1.5% and 0.5% respectively. Asemani, et al (2017), established that 43% of Ghanaian final year secondary school students did not attain any of Van Hiele's Geometric thinking level.

The purpose of the study by Salifu, Ibrahim and Yakubu (2018) was to determine PSTs geometric thinking level using the Van Hiele's Model. The study population was 473 level 200 PSTs representing both science 82 (17.3%) and general programme 391 (82.7%). The sample for the study was 351 general programme PSTs with female 133 and male 218. The College and the general programme PSTs were selected using both convenient and simple random sampling techniques respectively. The researchers used Van Hiele's Geometry Test (VHGT) as the main instruments. The mean and standard deviation scores were 8.79 and 2.49 respectively. Further analysis revealed that 114 (32.5 %), 73 (20.8%), 28 (8.0%) and 5 (1.4%) attained levels 1, 2, 3 and 4 respectively. Also, 131 (37.3%) did not reach any of the VHGT levels and no PST reached level 5. The following 56.5%, 48.9%, 36.2%, 21.7% and 15.7% were recorded for levels 1,2,3,4 and 5 respectively as the correct response percentage rates. The overall correct response percentage rate for the entire 25 items is 35.8%. The researchers recommended that tutors of colleges of education in Ghana should adopt practical approaches in teaching geometric concepts.

Baffoe & Mereku (2010), study schools were Winneba Senior High and Zion Girls senior high with a sample size of 188 from the Winneba metropolis. Baffoe & Mereku's purpose of the study was to measure Ghanaian Senior High School (SHS), 1 students. When the students were four weeks old in their SHS campuses Van Hiele's levels of geometric thinking were administered. The emanated results of the study showed that Ghanaian SHS 1 students were far behind in achievements when compared to their colleagues from other countries in geometric thinking. When the Van Hiele's test analysis was done quantitatively, it revealed that 59%, 11%, 1% of Ghanaian SHS 1 students attained Van Hiele's levels 1, 2 and 3 respectively.

III. EMPIRICAL STUDIES ON PRE-SERVICE TEACHERS AND IN-SERVICE TEACHERS VAN HIELE THEORY

In Israel, Patkin and Barkai (2014) study investigated the geometric thinking levels of pre- and in-service mathematics teachers whose aim was to examine whether there were

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differences in the participants' mastery of triangles and quadrilaterals, circles and three-dimensional geometric figures according to van Hiele's theory. Their research used a sample of 71 comprising 47 pre- service and 24 in-service mathematics teachers. Their study results indicated that all the participating groups demonstrated higher mastery in triangles and quadrilaterals, circles and three-dimensional. Additional analysis revealed that the participants failed to master (solid) three-dimensional geometric figures. In a similar study in Turkey conducted by Halat and Sahin (2008) entitled "Van Hiele Levels of pre- and in-service Turkish elementary school teachers and gender-related differences in geometry". With a sample size of 186 pre-and in-service elementary school teachers, they investigated their reasoning stages. Their research results indicated that the pre- and in-service elementary school teachers attained the first four van Hiele with different percentiles. Their findings also revealed that there was no difference in terms of reasoning stages between the pre- and in-service elementary school teachers.

STATEMENT OF THE PROBLEM

"It is largely revealed in the literature that students consider mathematics as an abstract subject and irrelevant to their progress in life" (Salifu, 2018, p.3). In the educational setting around the world, the teacher is viewed as a professional who holds the required pedagogical knowledge and content knowledge for teaching all subjects including mathematics.

In Ghana, almost every teacher uses the traditional approach of teaching including mathematics teachers. The said method of teaching and learning geometry focuses more on rote learning instead of relational learning. The traditional method encourages students to recall formula to use rather than improving students' reasoning abilities. Van Hiele's, (1999), Baffoe and Mereku (2010) all posited that over a long period of time teaching geometry in high schools and colleges is still based on formal axiomatic geometry that Euclid articulated or created over 2000 years ago. Van Hiele's (1999), stated that Euclid logical construction of geometry was admirable for mathematical achievement with its axioms, postulates, definitions, theorems, and proofs. But it was also stressed by Baffoe and Mereku (2010), that school geometry that is taught in the old-fashioned Euclidean ways with the goal that the students will think on a formal deductive level is debatable.

According to Hiele's, (1999), Baffoe and Mereku (2010) students find geometry very difficult when it is presented in the Euclidean approach to them. In the traditional teaching methods (i) Students accept whatever they are taught by their teachers, (ii) The teachers assume the authority of the Mathematical knowledge in their classrooms and (iii) Students accept principles, formulas, Mathematical facts, and theorems. These difficulties have been addressed in some Western countries and few African countries that used Van Hiele's theory to improve students' performances in geometry, Asemani, Asiedu-Addo, and Oppong (2017). Those studies have helped improve the performance of students, enlighten curriculum developers and teachers on the benefits of Van Hiele's theory. Those notably studies are (Van Hiele's, 1999; Abdullah & Zakaria, 2013; Armah, et al, 2018).

Again, analysis of reports from the geometry examination by the Institute of Education, UCC- Ghana, (2007) indicated that a total of 9,168 candidates (56.8%) scored below an average of 50%. In 2009, 31.8% of the pre-service teachers scored below the average mark of 50% in the geometry semester examination, (Institute of Education, 2009). For instance, out of the 7, 449 PSTs who took the geometry semester examination in 2013, 1,965 representing 26.4% obtained weak grades (grade D+ and D). In that same examination, 12.4% of the PSTs failed the geometry semester examination. The situation became worst in 2015, where the percentage of the weak grade moved from 26.41% in 2013 to 28.8%. The failure rate also moved from 12.4% to 42.3%. Comprehensive analyses from 2009-2015 of PSTs' examination scripts have demonstrated that there is a big problem in teaching geometry in the University Colleges of Education in Ghana.

Baffoe and Mereku, (2010) posited that despite the widespread usage of Van Hiele's theory in many western countries to improve their mathematics curriculum, only a few African countries have applied the model. (Halat & Sahin, 2008; Halat, 2008) have revealed that PSTs lacked the geometric thinking skills required to teach at the basic schools. According to Armah et al (2017), the situation is no different from the Ghanaian perspective. Recent literature and instructional moves suggest the use of Van Hiele's theory levels of geometry learning in Schools, Colleges and Universities all over the world but unfortunately, the researchers only cited (Asemani, et al, 2017; Armah, et al 2017; Baffoe & Mereku, 2010; Salifu, 2018) as the only papers on Van Hiele's instruction in Ghana. The review of the literature indicates that there has been very little investigation involving Van Hiele's model in Ghana most especially among in-service teachers. Thus, very little studies have been done on Van Hiele's theory to determine the actual levels of geometric conceptualization among PSTs in Colleges of Education in Ghana to improve geometric teaching and learning (Armah, et al, 2018; Armah, et al, 2017). From the review of the literature the gap identified is that there is no single study on Van Hiele models on pre-service teachers and in-service teachers in Ghana, hence the motivation of the researchers to conduct the study. This study was to determine the actual Van Hiele's geometric thinking levels and achievement differences of Pre-Service Teachers' and In-Service Teachers' of basic schools in Ghana.

PURPOSE OF THE STUDY

The purpose of the study was to determine In-Service Teachers (ISTs) and Pre-Service Teachers (PSTs') geometric thinking levels and achievement of the Northern Region and Bono East Region using the Van Hiele's Model. The researchers essentially wants to figure out whether both ISTs and PSTs actually possess the needed content knowledge to be able to teach their junior high schools students, since, they are specifically trained to handle Mathematics topics. In pursuance of this purpose, these research questions below were formulated to guide the study.

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RESEARCH QUESTION

- ✓ What are the geometric reasoning levels reach by Pre-service teachers and In-service teachers?
- ✓ Is there any significant difference in terms of geometric thinking achievements between Pre-service teachers' and In-service teachers'?

RESEARCH HYPOTHESIS

The following hypotheses were formulated to guide the study.

- ✓ Ho: There is no significant difference in achievement scores in VHGT levels 1, 2, 3, 4 and 5 among In-service teachers and Pre-service teachers.
- ✓ Ho: There is no significant difference in achievement scores in overall VHGT levels combined among In-service teachers and Pre-service teachers.

IV. THEORETICAL FRAMEWORK

THE VAN HIELE'S THEORY

The van Hiele's theory of geometrical thinking was developed by a husband and wife known as Pierre van Hiele's and Dina van Hiele's Geldof out of the hindrances the couple and their students experienced during geometry teaching and learning (Armah, et al, 2017; Armah, et al, 2018; Salifu, 2018). As researchers and mathematics teachers, the theory emanated from their thesis at the University of Utrecht in 1957 (Usiskin, 1982). Based on their theory, according to (Haviger & Vojkůvková, 2015) in the 1960s, the Soviet Union carried research on their model. As opined by Usiskin (1982), Americans also carried out several studies which influenced NCTM Standards and Common Core State Standards in the 1970s. Van Hiele's Model proposes that geometrical thinking has five closely related stages.

The theory propounded by Van Hiele's has three parts as (i) the existence of levels, (ii) the properties of the levels, and (iii) the phase based instruction (Vojkuvkova, 2012). Initially Van Hiele's theory was from level 0 to level 4. However, studies by the following researchers (Alex & Mammen, 2016; Howse & Howse, 2015) increased it to level 5 to make room for a sixth level known as pre-recognition for students who will not be able to attain van Hiele's level 1. The following are the five chronological and graded separate levels by Van Hiele's (1986). They are (1) Visual, (2) Analysis, (3) Order, (4) Deduction, and (5) Rigor, arranged in ascending order of difficulty (Usiskin, 1982; Alex & Mammen, 2016).

VAN HIELE'S THEORY LEVELS

Levels	Characteristics
Level I (Visualization)	Students recognize figures by their appearance. They make decision based on intuition not reasoning.
Level 2 (Analysis)	Students recognize figures by their properties. They can analyze and name properties of figures, but they cannot make relationships between these properties.
Level 3 (Informal deduction)	Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning.
Level 4 (Deduction)	Students can construct theorems within an axiomatic system. They know the meaning of necessary and sufficient conditions of a Theorem
Level 5 (Rigor)	Students understand the relationship between various systems of geometry. They can compare, analyze and create proofs under different geometric systems.

Source: Karakus & Peker, 2015, p. 339 as Cited by Armah, et al 2018

Table 1: Characteristics of van Hiele's Geometry Levels

V. METHODOLOGY

RESEARCH DESIGN

The descriptive cross-sectional research design was employed to investigate the PSTs and ISTs' geometric reasoning levels and achievement scores using VHGT in Northern Region and Bono East Region of Ghana.

POPULATION

The population of the study was all Pre-service teachers' and In-service teachers' of Nanumba North Municipality and Atebubu Municipality of Northern Region and Bono East Region of Ghana respectively.

SAMPLE SIZE AND SAMPLING PROCEDURE

The sample for the study was three hundred and fifty- one (351) consisting of 133 (37.89%) ISTs and 218 (62.11%) level 200 PSTs. Out of the total number of the ISTs, the males were 70 while the females were 63. Also, from 218 PSTs, 120 (55.05%) represented male while the female were 98 (44.95%). The sample from the Nanumba North Municipality was 211 (both PSTS and ISTs) and that of Atebubu Municipality was 140 (both PSTS and ISTs). The sample had a minimum age of 18, a maximum of 54 years, modal age was 37 years and a mean age of 28 years. Convenient and purposive sampling was also adopted in selecting the pre-serve teachers and ISTs. The ISTs were drawn from Nanumba North and Atebubu Municipalities while the PSTs were from E.P. College of Education, Bimbilla and Atebubu College of Education.

VALIDITY AND RELIABILITY

Studies by these researchers (Usiskin, 1982; Atebe, & Schafer, 2008; Baffoe & Mereku, 2010; Salifu, 2018), have testified that the VHGT is reliable and valid. Both the face and content validity of the VHGT was done by two experienced mathematics tutors of the college. The tutors concluded that it met the PSTs' standard and hence cleared it for administering having compared it to the Colleges of Education Geometry course outlines. A reliability coefficient of 0.77 was recorded from the pilot test when thirty (37) PSTs took part. Kuder-Richardson formula 20 methods were utilized to determine the reliability coefficient of the instruments.

PILOT STUDY

The VHGT was piloted both in Nanumba North and Atebubu Municipalities to both Pre-service teachers and In-service teachers. Twenty-five (25) each of the Pre-service teachers and In-service teachers from both municipalities took part in the pilot study. The purpose of the pilot was to enable the researcher to determine the reliability coefficient of the instrument.

ADMINISTRATION OF THE RESEARCH INSTRUMENT

The research Instrument used was an achievement test. The entire 25– items multiple choice of VHGT developed by Usiskin (1982) under the Cognitive Development and Achievement Secondary School Geometry (CDASSG) special programme was administered to the PSTs and ISTs to measure their geometric thinking levels. In order to find answers to the research questions for this study, the PSTs took the test in the 2018/2019 academic year, the second semester. The date for PSTs test was 9th May 2018 whiles the in-service date was from 16th to 30th May 2018. The duration of the test lasted 45 minutes. Before administering the test, a written permission request was earlier sent to Prof. Zalman Usiskin, in his reply he gave his approval and advice for the use of his test items. The VHGT questions are into five (5) subgroups, organized chronologically such that it starts from the very cheap to the most difficult item. Each subgroup covered the Van Hiele's geometric thinking level.

NATURE OF THE QUESTIONS

Questions	Levels	Features
1-5	1	It is about visual form. It aims to determine whether the students recognize the shape by looking at the shape of the figure.
6-10	2	It is concerned with the Characteristics of the forms and on the one hand it aims to show that the students do not know the forms and on the other hand they do not know the Characteristics of the forms.
11-15	3	It determines whether students can recognize the relationships between forms. They identify students who respond correctly to questions in this group and have proven that they have knowledge of axioms.
16-20	4	4 It is the question of reasoning and logical deduction. In these questions, it is determined whether the students are at a level of understanding and writing.
21-25	5	The questions at this level are used to determine whether students can reason in Euclidean and Euclidean geometries.

Source: (Hurma, 2011, p.60)

Table 2: Characteristics of van Hiele's Geometry Thinking Test

SCORING SYSTEMS AND LEVEL ASSIGNMENT

✓ The study used the grading system developed by Usiskin (1982) for assigning the various levels. Usiskin reported that a student can score 0 as the minimum mark and a maximum of 31 points from the VHGT. The grading key for Van Hiele's Geometric Thinking Test developed by Usiskin is shown below:
- If at least three questions (between 1 and 5) are answered correctly: 1 point
- If at least three questions (between 6 and 10) are answered correctly: 2 points
- If at least three questions (between 11 and 15) are answered correctly: 4 points
- If at least three questions (between 16 and 20) are answered correctly: 8 points
- If at least three questions (between 21 and 25) are answered correctly: 16 points

Zero point is scored if a student gets 2 out of 5 corrects answers. For a student to pass from one level to another, then the students' needs to answer correctly at least three of previous level questions in order to be assigned a level. For instance, a student who was able to correctly answer three questions from 1 to 5, correctly answer two questions from 6 to 10, correctly answer three questions from 11 to 15, gets 1 point from first level, 0 point from second level, 4 points from third level respectively making a total of 5 points. Even though Van Hiele's level 3 criterion was met by this student, he cannot be placed in Van Hiele's level 3 because the student failed to answer correctly at least three of second level questions (Okumuş, 2011).

VI. DATA ANALYSIS

Quantitative method analysis was utilized by the researchers which is consistent with Usiskin (1982), Armah et al (2017), Asemani et al (2017), and Baffoe & Mereku (2010) studies. For the analysis of this study, Microsoft Excel 2013 and SPSS Version 20 were employed. Both inferential and Descriptive statistics were used to analyze the data.

VII. RESULTS

The purpose of this study was to use Van Hiele's Geometric thinking test to classify ISTs and PSTs geometric thinking levels and also to find out whether differences exist in achievement scores along the levels 1 to 5. The results of the study are organized by means of descriptive statistics and inferential statistics.

RESEARCH QUESTION 1: What are the geometric reasoning levels reach by Pre-service teachers and In-service teachers?

Group	Visualization Level 1	Analysis Level 2	Deduction Level 3	Order Level 4	Rigor Level 5
In-service teachers	3 (2.26%)	10 (7.52%)	110 (82.71%)	8(6.02%)	2 (1.50 %)
Pre-service teachers	3 (1.38%)	83 (38.07%)	127 (58.26 %)	5 (2.30 %)	0 (0 %)
Total	**6 (1.71%)**	**93 (26.50%)**	**237 (67.52 %)**	**13 (3.70 %)**	**2 (0.57 %**

Table 3: The Overall Van Hiele's Levels Attained by both In-service teachers' and Pre-service teachers'. It is represented by means of numbers and percentage

In determining the ISTs and PSTs reasoning levels with the VHGT. Table 3 gives a summary of the frequency and percentage distribution of VHGT level 1 to level 5. For level 1, both the ISTs and PSTs had 3 teachers each representing (2.26%) and (1.38%). The analysis indicates that for level 2, 10 (7.52%) and 83 (38.07%) were recorded for ISTs and PSTs respectively. ISTs and PSTs for Level 3 performance, indicates 110 (82.71%) and 127 (58.26 %) respectively. From the table 8(6.02%) and 5 (2.30 %) were recorded for level 4, while 2 (1.50 %) was recorded for level 5 in favor for ISTs, there no PST in level 5. The most frequent level attained was deduction level. From the analysis, no ISTs and PSTs were at pre- visualization.

Research Question 2: Is there any statistical significant difference in terms of geometric thinking achievements between Pre-service teachers' and In-service teachers'?

Score	Number of students (N)	Cumulative (N)	Percentage (%)	Cumulative Percentage (%)
6	1	1	0.5	0.5
7	3	4	1.4	1.8
8	14	18	6.4	8.3
9	33	51	15.1	23.4
10	37	88	17.0	40.4
11	46	134	21.1	61.5
12	41	175	18.8	80.3
13	23	198	10.6	90.8
14	10	208	4.6	95.4
15	9	217	4.1	99.5
16	1	218	.5	100.0
Total	**218**		**100.0**	

Table 4: In-service teachers achievement detail scores

From table 4, the minimum score of 7 and maximum score of 17 were recorded among the ISTs, giving a range of 10. From the analysis again 62 out of the 133 ISTs obtained less than half of the total score. This represents 46.6% of the total number of ISTs who took part in the VHGT. Further analysis revealed that 34 (25.6%) of the ISTs scored exactly half the total mark of 12. Also, 37 ISTs representing 28% of the total number scored above the half mark of 12. It was observed that the mode was 12.

Score	Number of students (N)	Cumulative (N)	Percentage (%)	Cumulative Percentage (%)
7	1	1	0.8	0.8
8	1	2	.8	1.5
9	17	19	12.8	14.3
10	22	41	16.5	30.8
11	21	62	15.8	46.6
12	34	96	25.6	72.2
13	19	115	14.3	86.5
14	11	126	8.3	94.7
15	5	131	3.8	98.5
16	1	132	.8	99.2
17	1	133	.8	100.0
Total	133		100.0	

Table 5: Pre-service teachers achievement detail scores

From the analysis in table 5, 41(18.8%) scored half the test mark, minimum score of 6 and maximum of 16 were noticed among the PSTs with a range of 10. Also, from the same Table, 43(19.8%) of the PSTs obtained scores above half of the total marks of the test. Again, 134 from the 218 represents 61.5% of the PSTs obtained scores below half of the total score. The mode was 11 among the PSTs. In other to fully understand the main hypothesis, it was broken down into sub-hypothesis according to levels 1 to 5.

Research Hypothesis 1: Ho: There is no statistical significant difference in achievement scores in VHGT level 1 among In-service teachers and Pre-service teachers.

Group	N	Mean	Std. Dev.	DF	t	Sig
In-service teachers	133	3.27	0.45	349	0.181	0 .856
Pre-service teachers	218	3.26	0.47			

Table 6: In-service teachers and Pre-service teachers performance on the VHGT level 1

From Table 6, the independent t-test of both groups were compared and it revealed that, the ISTs attained (M =3.27, standard deviation = 0 .45) against the PSTs of (M= 3.26, standard deviation = 0 .47, t (349) =0 .181, p= 0.856 > 0.05). This implies that majority of the ISTs scores fell within the percent range (2.82% - 3.72 %) at level 1, while majority of the PSTs scores fell within the percent range (2.79% - 3.73%) at level 1. The mean difference is 0.01 which is insignificant. Therefore, there was no significant difference in the achievement scores between the ISTs and PSTs. This suggests that, they are at the same level of thinking in level 1.

Research Hypothesis 2: Ho: There is no statistical significant difference in achievement scores in VHGT level 2 among In-service teachers and Pre-service teachers.

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Group	N	Mean	Std. Dev.	df	T	Sig
In-service teachers	133	3.15	0.54	349	-0.659	0.510
Pre-service teachers	218	3.19	0.50			

Table 7: In-service teachers and Pre-service teachers performance on the VHGT level 2

The findings are that the ISTs attained (M = 3.15, standard deviation = 0.54) and the PSTs had (M= 3.19, standard deviation = 0.50 at t (349) = - 0.659 at significant value 0.510. The mean difference between the groups is 0.04 in favor of the PSTs. This implies that majority of the ISTs scores fell within the percent range (2.61% - 3.69 %) at level 2, while majority of the PSTs scores fell within the percent range (2.69% - 3.69%) at level 2. The result in table 7 is suggesting that there is no significant difference in their scores.

Research Hypothesis 3: Ho: There is no statistically significant difference in achievement scores in VHGT level 3 among In-service teachers and Pre-service teachers.

Group	N	Mean	Std. Dev.	DF	t	Sig.
In-service teachers	133	2.89	0.39	323.45	6.73	0.000
Pre-service teachers	218	2.43	0.89			

Table 8: In-service teachers and Pre-service teachers performance on the VHGT level 3

The results of Independent t-test samples in Table 8 showed a statistical significant difference in level 3 scores ((323.45) = 6.73, p = 0.000 < .05) between ISTs and PSTs. The ISTs recorded high mean performance (M = 2.89, SD = 0.39) than the PSTs with (M = 2.43, SD = 0.89). This implies that majority of the ISTs scores fell within the percent range (2.5% - 3.28 %) at level 3, while majority of the PSTs scores fell within the percent range (1.54% - 3.32%) at level 3. The difference between their means was 0.46. This finding indicates that, ISTs performance is better than PSTs. Further analysis comparing variance using the Levene's Test for equality of variances confirmed significant difference in variance between the ISTs and PSTs.

Research Hypothesis 4: Ho: There is no statistical significant difference in achievement scores in VHGT level 4 among In-service teachers and Pre-service teachers.

Group	N	Mean	Std. Dev.	DF	T	Sig
In-service teachers	133	1.48	1.21	251.337	1.37	0.173
Pre-service teachers	218	1.31	1.06			

Table 9: In-service teachers and Pre-service teachers performance on the VHGT level 4

Table 9 present the geometrical achievement for both groups (ISTs and PSTs) show that the average score obtained by the ISTs was higher than that of the PSTs. At the same time if we look at the column of the standard deviation, we can see that the standard deviation of the PSTs is lower than that of the ISTs. In addition, in table 9, t calculated =1.37 and p-value=0.173>0.05, so we upheld the null hypothesis. Thus, there is no significant difference between the ISTs and PSTs geometrical achievement scores. The t-test results indicate that the ISTs (Mean=1.48; SD=1.21) did well in the geometrical achievement test than PSTs (Mean=1.31; SD=1.06). This implies that majority of the ISTs scores fell within the percent range (0.27% - 2.69 %) at level 4, while majority of the PSTs scores fell within the percent range (0.25% - 2.37%) at level 4.

Research Hypothesis 5: Ho: There is no statistical significant difference in achievement scores in VHGT level 5 among In-service teachers and Pre-service teachers.

Group	N	Mean	Std. Dev.	DF	T	Sig.
In-service teachers	133	0.75	0.85	349	-0.518	0.605
Pre-service teachers	218	0.80	0.79			

Table 10: In-service teachers and Pre-service teachers performance on the VHGT level 5

From Table 10 the results of the independent sample t-test showed that (t (349) = -0.518, p= 0.605>0.05) between the ISTs and PSTs with small mean difference of 0.05 in favor of the PSTs. The descriptive statistically further indicates that the ISTs (M=0.75, SD=0.85) and PSTs (M=0.80, SD=0.79) was not statistically significant when the VHGT level 5 was tested. This implies that majority of the ISTs scores fell within the percent range (-0.1% - 1.6 %) at level 5, while majority of the PSTs scores fell within the percent range (0.01% - 1.59%) at level 5. Therefore, the null hypothesis is upheld for VHGT level 5.

Research Hypothesis 6: Ho: There is no overall statistically significant difference in achievement scores in VHGT levels 1 to 5 among In-service teachers and Pre-service teachers.

Group	N	Mean	Std. Dev.	DF	t	Sig.
In-service teachers	133	11.55	1.79	349	2.80	0.005
Pre-service teachers	218	10.98	1.87			

Table 11: In-service teachers and Pre-service teachers performance in VHGT levels 1 to 5

The results from the independent sample t-test in table 11 shows that there is statistically significant difference in the scores of ISTs (M =11.55, SD =1.79) and PSTs (M =10.98, SD =1.87), (t (349) = 2.80, p = 0.005 < 0.05). From table 11 the p-value (0.005) is less than the significance level of 0.05. This implies that majority of the ISTs scores fell within the percent range (9.76% - 13.34 %) category, while majority of the PSTs scores fell within the percent range (9.11% - 12.85%) category. Therefore, I reject the null hypothesis that there is no statistically significant difference between ISTs and PSTs. A mean difference of 0.67 was recorded in favor of the ISTs.

VIII. DISCUSSION OF RESULTS

The purpose of this study was to determine Van Hiele's levels of geometric thinking in both ISTs and PSTs. It also sought to find out whether significant differences exist between the ISTs and PSTs in their achievement scores.

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From Table 3, majority of the In-service teachers met the level 3 criterion by obtaining 3 out of 5 correct answers meaning they understood how to develop relationship between properties, and can see hierarchy of properties and shapes. Also, at this level the In-service teachers understood that shapes go together with their properties. This finding correspond with (Halat & Şahin, 2008), (Baffoe & Mereku, 2008), Salifu (2018) and Salifu, et al (2018) who also found majority of students in level 3.

Also, 10 In-service teachers exhibited additional knowledge in working with abstract statements about geometric properties and can make conclusions based on more logic than intuition. They could also proof theorems and had interest in the axiomatic systems themselves and not just the deductions within the systems. They also applied non-Euclidean geometry in solving some of the questions. This indicates reasoning at levels 4 and 5 which also corroborates the studies of Armah et al (2017), (Halat & Şahin, 2008), (Baffoe & Mereku, 2008), Salifu (2018), and Salifu, et al (2018) whose findings also indicate fewer students at levels 4 and 5. However, a few numbering 13 In-service teachers were operating in the lower levels of the Van Hiele model which has serious implication for teaching mathematics especially geometry at the basic level. This is sad because it indicates that those In-service teachers have been struggling to teach geometry concepts of simply shapes and properties of triangles, squares, parallelograms and rhombus. They could not comprehend that if squares are tilted to certain angles they are still squares.

Looking at Table3 again closely, no pre-service teacher attained level 5 meaning that no pre-service teacher could solve questions that demanded deductive axiomatic systems of geometry. They could not also apply non-Euclidean geometry because they lacked the interest in solving questions involving deductions within the system. Also, only 5 pre-service teachers could work problems with abstract statement about geometric properties and draw conclusions based on logic rather than intuition. In simple terms they could identify relationship among properties of geometric objects. This finding is consistent with the studies of Armah et al (2017), Halat & Şahin (2008), (Baffoe & Mereku 2008), Salifu (2018), and Salifu, et al (2018) whose studies revealed few students at levels 4 and 5.

Also, majority of the pre-service teachers were reasoning at informal deduction stage or level 3 because they could classify shapes based on their properties. They could also order shapes base on class inclusion. However, 86 pre-service teachers performance was not encouraging because they attained levels one and two which are below their future students levels, which implies that these pre-service teachers will find it difficult in teaching mathematics topics such as geometry to their basic pupils. These pre-service teachers could not identify square, triangle, and parallelogram when they were presented in different forms by tilting them in different angles. These findings are similar to Salifu (2018), and Salifu, et al (2018) whose studies revealed that some students were reasoning below their future students' level.

Some of the In-service teachers and pre-service teachers encountered difficulties in comprehending properties of rhombus and isosceles triangles because they scored low marks in getting correct answers for diagonals of the rhombus and also establishing the lengths and angles of an isosceles triangle. Further script analysis revealed that In-service teachers and pre-service teachers were confused about the properties of rectangle and that of squares. Some In-service teachers and pre-service teachers understood what all rectangles have that some parallelograms do not have. They perform poorly on these questions.

The performance of the In-service teachers was better than that of the pre-service teachers in the VHGT reasoning levels. A mean difference of 0.67 was recorded in favor of the In-service teachers. Significant difference was only noted in level 3 while the following levels 1, 2, 4 and 5 revealed that there was significant difference among the In-service teachers and pre-service teachers during the Van Hiele's geometry test. This research confirms Halat & Şahin (2008) findings which showed that there was no significant diference between pre-service teachers and in-service Turkish elementary school teachers achievements using the VHGT.

IX. CONCLUSION

For the levels analysis, the In-service teachers representing 9.78 % attained levels 1 and 2 cumulatively, while the pre-service teachers representing 39.45% also reached levels 1 and 2 combined. From the analysis again, the In-service teachers reached level 3 with 82.71% and 58.26 % were recorded for level 3 for the pre-service teachers. Finally, the In-service teachers achieved levels 4 and 5 put together with 7.52 %, while the pre-service teachers with 2.30% attained levels 4 and 5 put together. The In-service teachers did better than the pre-service teachers on Van Hiele's geometry thinking hence there was significant difference between the In-service teachers and pre-service teachers when the overall levels were combined.

On the main achievements scores 25.6% of the In-service teachers scored exactly half mark 12 out of the total 25, while 18.8% of the pre-service teachers scored half the test mark. Also, 28% of the In-service teachers scored above the half mark as against, 19.8% of the pre-service teachers. Significant differences were noted only in level 3 while the following levels 1, 2, 4 and 5 revealed that there were no significant differences among the In-service teachers and pre-service teachers during the Van Hiele's geometry test.

X. RECOMMENDATION

Based on the finding of this study, it is recommended that:

- ✓ PSTs' new 4 year B. Ed Mathematics curriculum that is being designed by Transforming Teacher Education and Learning (T-TEL) and the public universities should capture different models of geometry teaching more especially van Hiele's geometry phase based instruction.
- ✓ Mathematics tutors should be encouraged to use teaching and learning materials in teaching geometry concepts always.

- ✓ Ministry of Education should also encourage mathematics tutors to adopt dynamic geometry systems like GeoGebra, Sketchpad and Cabri in teaching geometry.

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