International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 6 Issue 2, February 2019

ISSN: 2394-4404

Application Of Split - Deflection Method In Buckling Analysis Of CCSS And CCCS Thin Rectangular Isotropic Plates Under Vibration

Ibearugbulem, Owus M.

Ezeh, John C.

Civil Engineering Department, Federal University Of Technology, Owerri, Nigeria

Nwachukwu, Uchechukwu C.

Civil Engineering Technology Department, Auchi Polytechnic, Auchi, Nigeria

***Abstract:** This paper presents an exact approach to buckling analysis of thin rectangular plates under vibration. Total potential energy functional for a thin rectangular plate subjected to both vibration and buckling loads was formulated from principles of theory of elasticity using split-deflection approach. General variation was applied on the total potential energy with respect to deflection function to obtain the fourth order governing equation of equilibrium of forces acting on the plate. This fourth order differential equation was solved, and the function (deflection) that satisfied it was obtained with unknown coefficients. This deflection function is of trigonometric family. This was followed by the application of direct variation of the total potential energy functional with respect to coefficient of deflection to obtain the formula for critical buckling load for plate. Numerical analyses for plate with one edge simply supported and the other three edges clamped (CCCS), and a plate with two adjacent edges clamped and the other two edges simply supported (CCSS) were performed. The non-dimensional critical buckling load results from the present work under no vibration for various aspect ratios were compared with the ones from the work of Ibearugbulem. The maximum percentage differences between the results of the present and past for CCSS is 0.036. For CCCS plates the maximum recorded percentage difference is 0.024. These percentage differences show close agreements. Also, the non – dimensional critical buckling load for rectangular CCSS and CCCS plates under vibration at various aspect ratios were determined. Hence, the present trigonometric shape functions and the equation for non dimensional buckling load under vibration developed are reliable and are recommended for use in classical analysis.*

***Keywords:** Split-deflection, total potential energy functional, direct variation, trigonometric Function.*

I. INTRODUCTION

Classical plate theory (CPT) buckling analysis is dominated by energy methods such as Ritz, Galerkin, Raleigh, Raleigh-Rit, minimum potential energy, work-error, etc. (Ugural, 1999, Ventsel and Krauthammer, 2001 and Ibearugbulem et al., 2014). Most of these energy methods applied single orthogonal deflection function. This make it difficult for deflection function to be separated into two components. Most previous research works on CPT analysis on rectangular plates as seen from the literature rely on this single orthogonal deflection function (Hutchinson, 1992, Jianqiao, 1994, Ugural, 1999, Ventsel and Krauthmmer, 2001, Wang et al., 2002, Taylor and Govindjee, 2004, Szilard, 2004, Jiu et al., 2007, Erdem et al., 2007, Ezeh et al., 2013, Ibearugbulem, 2014). Ibearugbulem et al. (2016) tried to ease this difficulty of using single orthogonal function by separating it into two independent distinct functions (w= $w_x \times w_y$) but was based on assumptions. Based on some previous works on free-vibration analyses of thin rectangular plate that were clamped on all edges using numerical approaches (Lee, 2004, Werfalli and Karaid, 2005 and Misra, 2012) and energy variational methods (Lalet al., 2009, Shu et al., 2007), one could say that works on areas of vibration are complex and difficult to analyze. Several works on the buckling analysis of plate had been done in the past. Ibearugbulem et al.(2016) derived an equation for critical buckling load for rectangular plates using the Work – error and Split – deflection methods given in Equation (1) as:

$$N_x = \frac{\frac{D}{a^2}[K_x + \frac{2}{P^2}K_{xy} + \frac{K_y}{P^4}]}{K_{Nx}} \qquad 1$$

In evolving the split-deflection method, Ibearugbulem et al.(2016) assumed that the general deflection, w is split into w_x and w_y. That is:

$$w = w_x . w_y \qquad 2a$$

$$w = Ah_x . h_y \qquad 2b$$

Where w_x and w_y are x and y directional components of the deflection respectively, h_x and h_y are shape functions in x and y direction respectively, and A is deflection coefficient.

These plates (CCSS and CCCS plates) are subjected to lateral loads on both sides at x – axis as shown in Figures 1 & 2.

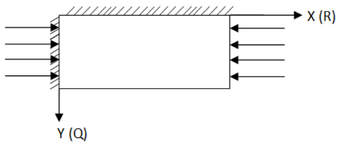

Figure 1: CCSS plate under lateral loads

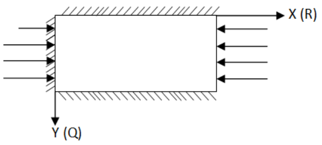

Figure 2: CCCS plate under lateral loads

The boundary conditions for these plates at x – axis are:

$$w(R=0) = \frac{dw(R=0)}{dR} = 0$$

$$w(R=1) = \frac{d^2w(R=1)}{dR^2} = 0$$

The main reason for this paper is to provide easy and less stressful method of plate analysis, using it to develop an equation for critical buckling load under vibration and trigonometric shape functions of a thin rectangular plate of particular boundary condition.

II. TOTAL POTENTIAL ENERGY

The strain energy, U is defined as:

$$U = \frac{1}{2}\int_x \int_y [\int_{-t/2}^{t/2} (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy})dz]dxdy \qquad 3$$

Whre σ_{xx} and σ_{yy}, are normal stresses along x and y directions, ε_{xx} and ε_{xx} are normal strains along x and y directions, and τ_{xy} and γ_{xy} are the shear stress and strain within the x-y plane respectively.

But

$$\varepsilon_{xx} = \frac{du}{dx} = -Z\frac{d^2w}{dx^2} \qquad 4a$$

$$\varepsilon_{yy} = \frac{dv}{dy} = -Z\frac{d^2w}{dy^2} \qquad 4b$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} - 2Z\frac{d^2w}{dxdy} \qquad 4c$$

$$\sigma_{xx} = \frac{E}{1-\mu^2}[\varepsilon_{xx} + \mu\varepsilon_{yy}] = \frac{-EZ}{1-\mu}\left[\frac{d^2w}{dx^2} + \mu\frac{d^2w}{dy^2}\right] \qquad 5a$$

$$\sigma_{yy} = \frac{E}{1-\mu^2}[\mu\varepsilon_{xx} + \varepsilon_{yy}] = \frac{-EZ}{1-\mu}\left[\mu\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2}\right] \qquad 5b$$

$$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy} = \frac{-EZ(1-\mu)d^2w}{(1-\mu^2)dxdy} \qquad 5c$$

Where E is the Young's modulus of elasticity and u is the Poisson's ratio of the plate.

The external work under buckling and vibration is given as:

$$V = \frac{1}{2}\int_x \int_y \left[N_x(\frac{dw}{dx})^2 + m\theta^2 w^2\right] \mathrm{dxdy} \qquad 6$$

Substituting Equations 2a, 4a, 4b, 4c, 5a, 5b and 5c into Equation 3 gives strain energy - deflection relationship as:

$$U = \frac{D}{2}\int_x \int_y \left[\left(\frac{d^2w_x}{dx^2}\right)^2 w_y^2 + 2\left(\frac{dw_x}{dx}.\frac{dw_y}{dy}\right)^2 + \left(\frac{d^2w_y}{dy^2}\right)^2 w_x^2\right] dxdy \qquad 7a$$

Similarly, substituting Equations 2b, 4a, 4b, 4c, 5a, 5b and 5c into Equation 3 gives strain energy - deflection relationship as:

$$U = \frac{A^2D}{2}\int_x \int_y \left[\left(\frac{d^2h_x}{dx^2}\right)^2 h_y^2 + 2\left(\frac{dh_x}{dx}.\frac{dh_y}{dy}\right)^2 + \left(\frac{d^2h_y}{dy^2}\right)^2 h_x^2\right] dxdy \qquad 7b$$

Where t is the thickness of the plate and D if the flexural rigidity of the plate defined as:

$$D = \frac{Et^3}{24(1-\mu^2)} \qquad 8$$

Substituting Equation 2 into Equation 6 gives:

$$V = \frac{1}{2}\int_x \int_y \left[N_x\left(\frac{dw_x}{dx}\right)^2 h_y^2 + m\theta^2 w_x^2 w_y^2\right] dxdy \qquad 9a$$

Similarly, substituting Equation 2b into Equation 6 gives:

$$V = \frac{A^2}{2}\int_x \int_y \left[N_x\left(\frac{dh_x}{dx}\right)^2 h_y^2 + m\theta^2 h_x^2 h_y^2\right] dxdy \qquad 9b$$

Adding Equation 7a and Equation 9a algebraically gives the total potential energy functional as:

$$\pi = \frac{D}{2}\int_x \int_y \left[\left(\frac{d^2w_x}{dx^2}\right)^2 w_y^2 + 2\left(\frac{dw_x}{dx}.\frac{dw_y}{dy}\right)^2 + \left(\frac{d^2w_y}{dy^2}\right)^2 w_x^2 - \frac{N_x}{D}\left(\frac{dw_x}{dx}\right)^2 w_y^2 - \frac{m\theta^2}{D}w_x^2 w_y^2\right] dxdy \qquad 10a$$

In the same way, algebraic summation of Equation 7b and Equation 9b gives:

$$\pi = \frac{A^2D}{2}\int_x \int_y \left[\left(\frac{d^2h_x}{dx^2}\right)^2 h_y^2 + 2\left(\frac{dh_x}{dx}.\frac{dh_y}{dy}\right)^2 + \left(\frac{d^2h_y}{dy^2}\right)^2 h_x^2 - \frac{N_x}{D}\left(\frac{dh_x}{dx}\right)^2 h_y^2 - \frac{m\theta^2}{D}h_x^2 h_y^2\right] dxdy \qquad 10b$$

Equation 10a and Equation 10b can be written in terms on non dimensional coordinates R and Q as:

$$\pi = \frac{D}{2a^4}\int_0^1 \int_0^1 \left[\left(\frac{d^2w_x}{dR^2}\right)^2 w_y^2 + \frac{2}{p^2}\left(\frac{dw_x}{dR}.\frac{dw_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2w_y}{dQ^2}\right)^2 w_x^2 - \frac{N_x a^2}{D}\left(\frac{dw_x}{dR}\right)^2 w_y^2 - \frac{m\theta^2 a^4}{D}w_x^2 w_y^2\right] abdRdQ \qquad 11a$$

$$\pi = \frac{A^2D}{2a^4}\int_0^1 \int_0^1 \left[\left(\frac{d^2h_x}{dR^2}\right)^2 h_y^2 + \frac{2}{p^2}\left(\frac{dh_x}{dR}.\frac{dh_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2h_y}{dQ^2}\right)^2 h_x^2 - \frac{N_x a^2}{D}\left(\frac{dh_x}{dR}\right)^2 h_y^2 - \frac{m\theta^2 a^4}{D}h_x^2 h_y^2\right] abdRdQ \qquad 11b$$

Where the non dimensional coordinates R and Q are the ratios of dimensional coordinates x and y to the lengths of the plate along x and y directions. That is R is ratio of x to a (that is R = x/a) while Q is the ratio of y to b (that is Q = y/b). Aspect ratio, p is defined as the ratio of b to a (p = b/a).

GENERAL VARIATION OF THE TOTAL POTENTIAL ENERGY

Minimization of the total potential energy functional with respect to deflection function gives the governing differential equation of forces in equilibrium for the plate. Equation 11a can be modified as:

$$\pi = \pi_x + \pi_y = \frac{D}{2a^4}\int_0^1\int_0^1\left[\left(\frac{d^2w_x}{dR^2}\right)^2 w_y{}^2 + \frac{2}{p^2}\left(\frac{dw_x}{dR}\cdot\frac{dw_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2w_y}{dQ^2}\right)^2 w_x{}^2 - \frac{N_x a^2}{D}\left(\frac{dw_x}{dR}\right)^2 w_y{}^2 - \frac{m\theta^2 a^4}{D} w_x{}^2 w_y{}^2 (n_x + n_y)\right] abdRdQ \qquad 11c$$

Where:

$$\pi_x = \frac{D}{2a^4}\int_0^1\int_0^1\left[\left(\frac{d^2w_x}{dR^2}\right)^2 w_y{}^2 - \frac{N_x a^2}{D}\left(\frac{dw_x}{dR}\right)^2 w_y{}^2 - \frac{m\theta^2 a^4}{D} w_x{}^2 w_y{}^2 . n_x\right] abdRdQ \qquad 12a$$

$$\pi_y = \frac{D}{2a^4}\int_0^1\int_0^1\left[\frac{2}{p^2}\left(\frac{dw_x}{dR}\cdot\frac{dw_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2w_y}{dQ^2}\right)^2 w_x{}^2 - \frac{m\theta^2 a^4}{D} w_x{}^2 w_y{}^2 . n_y\right] abdRdQ \qquad 12b$$

$$n_x + n_y = 1 \qquad 13$$

Thus minimizing Equation 12a with respect to w_x gives:

$$\frac{d\pi_x}{dw_x} = \frac{D}{2a^4}\int_0^1\int_0^1\left[2\frac{d^4w_x}{dR^4} w_y{}^2 + 2\frac{N_x a^2}{D}\frac{d^2w_x}{dR^2} w_y{}^2 - 2\frac{m\theta^2 a^4}{D} w_x w_y{}^2 . n_x\right] abdRdQ = 0 \text{. That is:}$$

$$\int_0^1\left[\frac{d^4w_x}{dR^4} + \frac{N_x a^2}{D}\frac{d^2w_x}{dR^2} - \frac{m\theta^2 a^4}{D} w_x . n_x\right] dR . \int_0^1 w_y{}^2\, dQ = 0 \qquad 14$$

Carrying out the integration of Equation 14 with respect to Q gives:

$$\int_0^1\left[\frac{d^4w_x}{dR^4} + \frac{N_x a^2}{D}\frac{d^2w_x}{dR^2} - \frac{m\theta^2 a^4}{D} w_x . n_x\right] dR . c_1 = 0 \text{ . That is:}$$

$$\int_0^1\left[\frac{d^4w_x}{dR^4} + \frac{N_x a^2}{D}\frac{d^2w_x}{dR^2} - \frac{m\theta^2 a^4}{D} w_x . n_x\right] dR = 0 \qquad 15$$

Where c_1 is a constant.

Similarly, minimizing Equation 12b with respect to w_y gives:

$$\frac{d\pi_y}{dw_y} = \frac{D}{2a^4}\int_0^1\int_0^1\left[\frac{4}{p^2}\cdot\frac{d^2w_y}{dQ^2}\left(\frac{dw_x}{dR}\right)^2 + \frac{2}{p^4}\frac{d^4w_y}{dQ^4} w_x{}^2 - 2\frac{m\theta^2 a^4}{D} w_x{}^2 . w_y n_y\right] abdRdQ = 0\text{. That is:}$$

$$\frac{2}{p^2}\cdot\int_0^1\left(\frac{dw_x}{dR}\right)^2 dR . \int_0^1 \frac{d^2w_y}{dQ^2} dQ + \frac{1}{p^4}\int_0^1 w_x{}^2 dR . \int_0^1 \frac{d^4w_y}{dQ^4} dQ - \frac{m\theta^2 a^4}{D} n_y \int_0^1 w_x{}^2 dR . \int_0^1 w_y\, dQ = 0 \qquad 16$$

Carrying out the integration of Equation 16 with respect to R gives:

$$\frac{2}{p^2}.c_2.\int_0^1 \frac{d^2w_y}{dQ^2} dQ + \frac{1}{p^4}.c_3.\int_0^1 \frac{d^4w_y}{dQ^4} dQ - \frac{m\lambda^2 a^4}{D} n_y . c_4 . \int_0^1 w_y\, dQ = 0 \text{ . That is:}$$

$$\int_0^1\left[\frac{2}{p^2}.c_2.\frac{d^2w_y}{dQ^2} + \frac{1}{p^4}.c_3.\frac{d^4w_y}{dQ^4} - \frac{m\lambda^2 a^4}{D} n_y . c_4 . w_y\right] dQ = 0 \qquad 17$$

Where c_2, c_3 and c_4 are constants.

For cases of pure buckling (that is in the absence of inertia force), Equation 15 and Equation 17 become:

$$\int_0^1\left[\frac{d^4w_x}{dR^4} + \frac{N_x a^2}{D}.\frac{d^2w_x}{dR^2}\right] dR = 0 \qquad 18$$

$$\int_0^1\left[\frac{2c_2}{p^2}.\frac{d^2w_y}{dQ^2} + \frac{c_3}{p^4}.\frac{d^4w_y}{dQ^4}\right] dQ = 0 \qquad 19$$

The ready solutions for the integrands of Equation 18 and Equation 19 are:

$$w_x = d_0 + d_1 R + d_2 e^{ig_1 R} + d_3 e^{-ig_1 R} \qquad 20$$

$$w_y = d_4 + d_5 Q + d_6 e^{g_2 Q} + d_7 e^{-ig_2 Q} \qquad 21$$

Where d_0, d_1, d_2, d_3, d_4, d_5, d_6 and d_7 are integration constants, and

$$g_1 = \sqrt{\frac{N_x a^2}{D}} \quad and\ g_2 = \sqrt{\frac{2c_2 p^2}{c_3}} \qquad 22$$

Transforming Equation 20 in trigonometric form gives:

$$w_x = d_0 + d_1 R + (d_2 + d_3)\cos g_1 R + (id_2 - id_3)\sin g_1 R \text{ . That is:}$$

$$w_x = a_0 + a_1 R + a_2 \cos g_1 R + a_3 \sin g_1 R \qquad 23$$

Where $a_0 = d_0$, $a_1 = d_1$, $a_2 = d_2 + d_3$ and $a_3 = id_2 - id_3$

Similarly, transforming Equation 21 in trigonometric form gives:

$$w_y = d_4 + d_5 Q + (d_6 + d_7)\cos g_2 Q + (id_6 - id_7)\sin g_2 Q \text{ . That is:}$$

$$w_y = b_0 + b_1 Q + b_2 \cos g_2 Q + b_3 \sin g_2 Q \qquad 24$$

Where $b_0 = d_4$, $b_1 = d_5$, $b_2 = d_6 + d_7$ and $b_3 = id_6 - id_7$

Substituting Equation 23 and Equation 24 into Equation 2a gives:

$$w = (a_0 + a_1 R + a_2 \cos g_1 R + a_3 \sin g_1 R).(b_0 + b_1 Q + b_2 \cos g_2 Q + b_3 \sin g_2 Q) \qquad 25$$

DIRECT VARIATION OF THE TOTAL POTENTIAL ENERGY

Formula for analysis is usually obtained after minimization of the total potential energy functional with respect the coefficient of deflection. Hence, minimizing Equation 11b with respect to deflection coefficient gives:

$$\frac{d\pi}{dA} = \frac{AD}{a^4}\int_0^1\int_0^1\left[\left(\frac{d^2h_x}{dR^2}\right)^2 h_y{}^2 + \frac{2}{p^2}\left(\frac{dh_x}{dR}\cdot\frac{dh_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2h_y}{dQ^2}\right)^2 h_x{}^2 - \frac{N_x a^2}{D}\left(\frac{dh_x}{dR}\right)^2 h_y{}^2 - \frac{m\theta^2 a^4}{D} h_x{}^2 h_y{}^2\right] abdRdQ = 0 \text{ . That is:}$$

$$\frac{d\Pi}{dA} = [k_{xR}.k_{xQ}] + \frac{2}{p^2}[k_{xyR}.k_{xyQ}] + \frac{1}{p^4}[k_{yR}.k_{yQ}] - \frac{N_x.a^2}{D}[k_{xyR}.k_{xQ}] - \frac{m\theta^2 a^4}{D}[k_{yR}.k_{xQ}] = 0 \qquad 26$$

Where:

$$k_{xR} = \int_0^1\left(\frac{d^2h_x}{dR^2}\right)^2 dR; \quad k_{xQ} = \int_0^1 h_y{}^2\, dQ; \quad k_{xyR} = \int_0^1\left(\frac{dh_x}{dR}\right)^2 dR$$

$$k_{xyQ} = \int_0^1\left(\frac{dh_y}{dQ}\right)^2 dQ; \quad k_{yR} = \int_0^1 h_x{}^2\, dR; \quad k_{yQ} = \int_0^1\left(\frac{d^2h_y}{dQ^2}\right)^2 dQ$$

Rearranging Equation 26 gives:

$$\frac{N_x.a^2}{D} = \frac{k_T - \frac{m\theta^2 a^4}{D} k_{yR}.k_{xQ}}{k_{xyR}.k_{xQ}} \qquad 27$$

Where:

$$k_T = k_{xR}.k_{xQ} + \frac{2}{p^2}k_{xyR}.k_{xyQ} + \frac{1}{p^4}k_{yR}.k_{yQ}$$

Under free – vibration only, the numerator of Equation 27 is zero. The vibration frequency at free vibration is natural frequency, λ.Thus:

$$k_T - \frac{m\lambda^2 a^4}{D} k_{yR}.k_{xQ} = 0 \qquad 28$$

Rearranging Equation 28 gives:

$$\frac{m\lambda^2 a^4}{D} = \frac{k_T}{k_{yR}.k_{xQ}} \qquad 29$$

The frequency of vibration θ is alway a fraction of the natural frequency. Its range is $0 \le \theta \le \lambda$. That is:

$$\theta = n.\lambda \qquad 30$$

Where n = resonating frequency ratio

$$0 \le n \le 1 \qquad 31$$

Substituting Equation 30 into Equation 29 gives:

$$\frac{m\theta^2 a^4}{D} = n^2 . \frac{k_T}{k_{yR} \, . \, k_{xQ}} \qquad 32$$

Substituting Equation 32 into Equation 27 gives:

$$\frac{N_x . a^2}{D} = \frac{k_T - \left(n^2 . \frac{k_T}{k_{yR} \, . \, k_{xQ}}\right) k_{yR} \, . \, k_{xQ}}{k_{xyR} \, . k_{xQ}} \; . \; That\ is:$$

$$\frac{N_x . a^2}{D} = \frac{k_T}{k_{xyR} \, . k_{xQ}} [1 - n^2] \qquad 33$$

This Equation 33 is the formula for calculating the non-dimensional critical buckling load for a rectangular plate under vibration.

III. NUMERICAL ANALYSES

Analyze a classical rectangular thin rectangular isotropic plate with:

- ✓ Three edges clamped and one edge simply supported (CCCS) using trigonometric function for both w_x and w_y.
- ✓ Two adjacent edges clamped and the other two edges simply supported (CCSS) using trigonometric function for both w_x and w_y.

FOR CCCS PLATE

After satisfying the boundary condition for cccs plate the deflection components obtained are:

$$w_x = A_x(g_1 - g_1 R - g_1 \cos g_1 R + \sin g_1 R) \qquad 34a$$

$$w_y = A_y(1 - \cos 2\pi Q) \qquad 34b$$

Where $g_1 = 4.49340946$

From Equation34a and Equation 34b the shape functions are:

$$h_x = g_1 - g_1 R - g_1 \cos g_1 R + \sin g_1 R \qquad 35a$$

$$h_y = (1 - \cos 2\pi Q) \qquad 35b$$

FOR CCCS PLATE

After satisfying the boundary condition for cccs plate the deflection components obtained are:

$$w_x = A_x(g_1 - g_1 R - g_1 \cos g_1 R + \sin g_1 R) \qquad 36a$$

$$w_y = A_y(g_2 - g_2 Q - g_2 \cos g_2 Q + \sin g_2 Q) \qquad 36b$$

Where $g_1 = 4.49340946$ and $g_2 = 4.49340946$

From Equations36a and Equation 36b the shape functions are:

$$h_x = g_1 - g_1 R - g_1 \cos g_1 R + \sin g_1 R \qquad 37a$$

$$h_y = g_2 - g_2 Q - g_2 \cos g_2 Q + \sin g_2 Q \qquad 37b$$

With these components of shape functions the stiffness coefficient are calculated and tabulated on Table 1

CCCS	k_{xR}	k_{xQ}	k_{xyR}	k_{xyQ}	k_{yR}	k_{yQ}
	4376.113774	1.5	230.4800172	$2\pi^2$	19.42379403	$8\pi^4$
	$k_{xR}.k_{xQ}$	$k_{xyR}\,.k_{xyQ}$	$k_{yR}\,.k_{yQ}$	$k_{xyR}\,.k_{xQ}$		
	6564.170661	4549.493184	15136.43297	345.7200258		
CCSS	k_{xR}	k_{xQ}	k_{xyR}	k_{xyQ}	k_{yR}	k_{yQ}
	4376.113774	19.42379403	230.4800172	230.4800172	19.42379403	4376.113774
	$k_{xR}.k_{xQ}$	$k_{xyR}\,.k_{xyQ}$	$k_{yR}\,.k_{yQ}$	$k_{xyR}\,.k_{xQ}$		
	85000.7326	53121.03833	85000.7326	4476.796382		

Table 1: Stiffness coefficients for the two plates

$$For\ cccs\ plate: \quad k_T = 6564.170661\left(1 + \frac{1.386159}{\propto^2} + \frac{2.305917038}{\propto^4}\right) \qquad 38$$

$$\frac{k_T}{k_{xyR} \, .k_{xQ}} = 18.98695526\left(1 + \frac{1.386159}{\propto^2} + \frac{2.305917038}{\propto^4}\right) \qquad 39$$

$$For\ ccss\ plate: \quad k_T = 85000.7326\left(1 + \frac{1.249896}{\propto^2} + \frac{1}{\propto^4}\right) \qquad 40$$

$$\frac{k_T}{k_{xyR} \, .k_{xQ}} = 18.98695526\left(1 + \frac{1.249896}{\propto^2} + \frac{1}{\propto^4}\right) \qquad 41$$

IV. RESULTS AND DISCUSSION

The split deflection total potential energy functional for thin rectangular plate loaded simultaneously with in-plane and inertia loads was formulated as shown on Equation 10a, Equation 10b, Equation 11a and Equation 11b. The equations are so unique such that it can easily be seperated (uncoupled). This fit was evident when general variation was applied on it. The governing equations obtained after general variation with respect to w_x and w_y were shown on Equation 14 and Equation 16. These equations were reduced to easily solvable equations as shown on Equation 18 and Equation 19. Upon solving Equation 18 and Equation 19, the trigonemetric expressions for w_x and w_y were obtained. See Equation 23 and Equation 24 for expressions for wx and wy. Equation 25 is the general orthogonal equation of deflection of rectangular plate under buckling load.

After direct variation was applied on the total potential energy functional, the formula for calculating the buckling load when the plate is acted upon by in-plane and inertia loads. This formula is shown on Equation 33. It is an easy to use formula. Two cases of plates of different boundary conditions were analysed. After satisfying the boundary conditions for cccs and ccss plates their unique shape functions were determined as shown on Equation 35a, Equation 35b, Equation 37a and Equation 37b. The formulas for calculating the buckling loads for cccs and ccss plate under both in-plane and inertia loads are obtained by substituting Equation 39 and Equation 41 into Equation 33. They are vrespectively given as:

$$\frac{N_x . a^2}{D} = 18.98695526\left(1 + \frac{1.386159}{\propto^2} + \frac{2.305917038}{\propto^4}\right)[1 - n^2] \quad for\ cccs\ plate$$

$$\frac{N_x . a^2}{D} = 18.98695526\left(1 + \frac{1.249896}{\propto^2} + \frac{1}{\propto^4}\right)[1 - n^2] \quad for\ ccss\ plates$$

International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 6 Issue 2, February 2019

ISSN: 2394-4404

It is noticed that at zero vibration (n = 0), the critical buckling equations obtained herein are the same with those from earlier studies. However, the equations from the present study differ from the onesobtained by previous study when inerial load is applied ($n \neq 0$). This difference accounts for the effect of vibration on buckling of rectangular plates.

The non-dimensional critical buckling load results of this present work when inertia load was absent (n = 0) for aspect ratios ($1 \leq P \leq 2$) where compared with the results from the work of Ibearugbulem et al.(2014).These comparisons were presented on Table 2 and Table 3. The percentage differences between the results of the present and past for CCSS and CCCS plates ranges from 0.000 to 0.036 and 0.018 to 0.024 respectively. This showsthat the differences are insignificant.

(n = 0)

Aspect ratio (P = b/a)	Ibearugbulem et al. (2014) N_P	Present study N_E	Percentage difference $\frac{\lvert N_P - N_E \rvert}{N_E} \times \frac{100}{1}$
1.0	89.354	89.333	0.024
1.1	71.131	71.115	0.022
1.2	59.060	59.047	0.022
1.3	50.735	50.724	0.022
1.4	44.795	44.785	0.022
1.5	40.433	40.424	0.022
1.6	37.148	37.140	0.022
1.7	34.620	34.613	0.020
1.8	32.637	32.631	0.018
1.9	31.056	31.050	0.019
2.0	29.777	29.771	0.020

Table 2: Non - dimensional critical buckling loads for CCCS Rectangular plates under uniform unilateral stress

(n= 0)

Aspect ratio (P = b/a)	Ibearugbulem et al. (2014) N_P	Present study N_E	Percentage difference $\frac{\lvert N_P - N_E \rvert}{N_E} \times \frac{100}{1}$
1.0	64.736	64.737	0.002
1.1	54.133	54.134	0.002
1.2	46.916	46.917	0.002
1.3	41.806	41.806	0.000
1.4	38.066	38.067	0.003
1.5	35.253	35.253	0.000
1.6	33.074	33.086	0.036
1.7	31.382	31.382	0.000
1.8	30.018	30.018	0.000
1.9	28.910	28.910	0.000
2.0	27.997	27.997	0.000

Table 3: Non - dimensional critical buckling loads for CCSS Rectangular plates under uniform unilateral stress

This paper presents the non-dimensional critical buckling load results of CCSS and CCCS plates when inertia loads were applied ($n \neq 0$) for aspect ratios ($1 \leq p \leq 2$) on Table 4 and table 5 show that as aspect ratio ($P = {}^{b}/_{a}$) increases from 1.0 to 2.0 at each resonating frequency ratio / inertia load (n) ranging from 0 to 0.9, non –dimensional critical buckling load $\left(\frac{N_x a^2}{D}\right)$decreases, thereby causing the plate to get more slender. Furthermore, as resonating frequency ratio (n) increases from 0 to 0.9 at each aspect ratio (P = b/a) ranging from 1.0 to 2.0 , non-dimensional critical buckling load also decreases, thereby weakening the strength of the plate and hence requires less effort to cause it to buckle. At resonating frequency ratio (n = 1) at all aspect ratios, non-dimensional critical buckling load equals zero, $[(\frac{N_x a^2}{D}) = 0]$.At this stage, the plate buckles without buckling load.

	Non – dimensional critical buckling load ($\overline{N_x}$)										
	Resonating frequency ratio (n)										
Aspect ratio (P=b/a)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	61.706	61.089	59.237	56.152	51.833	46.279	39.492	31.470	22.214	11.724	0
1.1	51.568	51.053	49.506	46.927	43.317	38.676	33.004	26.300	18.565	9.798	0
1.2	44.624	44.178	42.839	40.608	37.484	33.468	28.559	22.758	16.065	8.479	0
1.3	39.677	39.280	38.090	36.106	33.329	29.758	25.393	20.235	14.284	7.539	0
1.4	36.037	35.677	34.596	32.794	30.271	27.028	23.064	18.379	12.973	6.911	0
1.5	33.285	32.952	31.954	30.289	27.959	24.964	21.302	16.975	11.983	6.324	0
1.6	31.154	30.843	29.908	28.350	26.170	23.366	19.939	15.889	11.216	5.919	0
1.7	29.472	29.177	28.293	26.819	24.756	22.104	18.862	15.031	10.610	5.600	0
1.8	28.120	27.839	26.995	25.589	23.621	21.090	17.997	14.341	10.123	5.343	0
1.9	27.018	26.748	25.937	24.586	22.695	20.263	17.291	13.779	9.726	5.133	0
2.0	26.107	25.846	25.062	23.757	21.930	19.580	16.708	13.314	9.398	4.960	0

Table 4: Non – dimensional Critical Buckling load for Rectangular CCSS Plate under Vibration

	Non – dimensional critical buckling load ($\overline{N_x}$)										
	Resonating frequency ratio (n)										
Aspect ratio (P= b/a)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	89.088	88.197	85.525	81.070	74.834	66.816	57.016	45.435	32.072	16.927	0
1.1	70.642	69.936	67.816	64.284	59.339	52.982	45.211	36.027	25.431	13.422	0
1.2	58.378	57.794	56.043	53.124	49.038	43.784	37.362	29.773	21.016	11.092	0
1.3	49.890	49.391	47.874	45.400	41.907	37.417	31.929	25.444	17.960	9.479	0
1.4	43.812	43.374	42.059	39.869	36.802	32.859	28.040	22.344	15.772	8.324	0
1.5	39.333	38.939	37.759	35.793	33.039	29.499	25.173	20.060	14.160	7.473	0
1.6	35.948	35.589	34.511	32.713	30.197	26.961	23.007	18.334	12.941	6.830	0
1.7	33.336	33.003	32.002	30.336	28.002	25.002	21.335	17.001	12.001	6.334	0
1.8	31.281	30.968	30.030	28.466	26.276	23.461	20.020	15.953	11.261	5.943	0
1.9	29.637	29.341	28.452	26.970	24.895	22.228	18.968	15.115	10.669	5.631	0
2.0	28.303	28.020	27.171	25.756	23.775	21.227	18.114	14.435	10.189	5.378	0

Table 5: Non – dimensional Critical Buckling load for Rectangular CCCS Plate under Vibration

V. CONCLUSION

The conclusion is drawn that with the close agreement of the results obtained from past and present works at n = 0, it follows that the results of non – dimensional critical buckling load for n – values and their corresponding aspect ratios presented in Table 4 and Table 5 (for which there are no other existing results to compare with in literature) are also correct.

Hence, the present equations developed are reliable and are recommended for use in classical analysis. For future studies, it is recommended that buckling analysis of a thin rectangular plate under vibration outside these aspect ratios should be carried out using this present method.

REFERENCES

[1] Erdem , C. lmrak and Ismail Gerdemeli (2007). The problem of isotropic rectangular plate with four clamped edges. S-adhan-a. Vol. 32, part 3, pp. 181 – 186.

[2] Ezeh, J. C., Ibearugbulem, O. M., Njoku, K.O., and Ettu, L.O. (2013). Dynamic Analysis of Isotropic SSSS plate using Taylor Series Shape Function in Galerkin's Functional. International Journal of Emerging Technology and Advanced Engineering, 3 (5) : 372 – 375.

[3] Hutchinson, J.R. (1992). On the bending of rectangular plates with two opposite edges simply supported . J. Appl .Mech. Trans. ASME 59:679 – 681.

[4] Ibearugbulem, O.M. (2014). Using the product of two mutually perpendicular truncated polynomial series as shape function for rectangular plate analysis. International Journal of Emerging Technologies and Engineering (IJETE). ISSN: 2348 – 8050, ICRTIET – 2014. Conference proceeding, 30th – 31st August 2014, 1- 4

[5] Ibearugbulem, O.M., Ezeh, J.C. and Ettu, L.O. (2014). Energy Methods in Theory of Rectangular plates (Use of Polynomial Shape Functions), Liu House of Excellence Venture – Imo State, Nigeria.

[6] Ibearugbulem, O. M., Ibearugbulem, C.N., HabibMomoh and Asomugha, U.C. (2016). Buckling Analysis of Rectangular Plate by Split – Deflection Method. International Journal of Scientific and Research Publications, Vol. 3, Issue 5.

[7] Jiu, Hiu Wu, A.Q. Liu, and H. L. Chen (2007). Exact Solutions for Free – Vibration Analysis of Rectangular Plates. Journal of Applied Mechanics. Vol. 74: PP 1247 - 1251

[8] Lal, R., Kumar, Y. & Gupta, U.S. (2009) Transverse Vibrations of Non – Homogenous Rectangular Plates of Uniform Thickness using Boundary Characteristics Orthogonal Polynomials. Int .J. of Appl .Maths and Mech., 6 (14), 93 -109.

[9] Lee, S. J. (2004). Free- Vibration Analysis of plates by using a Four- Note Finite Element Formulated with Assumed Natural Transverse Shear Strain. Journal of Sound and Vibration, 278, 657 – 684.

[10] Misra, R.K.(2012). Static and Dynamic Analysis of Rectangular Plate using Multiquadric Radial Basic Function. International Journal of Management, I .T and Engineering, 2 (8), 166 – 178

[11] Szilard, R. (2004).Theories and Applications of Plate Analysis. New Jersey: John Wiley & sons Inc.

[12] Shu, C., Wu, W.X., Ding, H. &Wang C.M. (2007). Free vibration Analysis of Plates using Least – Square – Based Finite Difference Method. Comput. Methods Appl. Mech. Engrg., 196, 1330 – 1343.

[13] Taylor, R.L. and Govindjee, S.(2004). Solution of Clamped Rectangular Plate Problems, Communications in Numerical Methods in Engineering, 20 : PP. 757 – 765.

[14] Ugural, A.C.(1999). Stresses in Plates and Shells, 2nd ed. Singapore: McGraw- hill

[15] Ventsel, E. and T. Krauthammer (2001). Thin Plates and Shells: Theory, Analysis and Applications. New York: Marcel Dekker.

[16] Wang, C.M., Y.C. Wang and J.N. Reddy (2002). Problems and remedy for the Ritz method in determining stress resultant of corner supported rectangular plates . Comput.Struct. 80:145 – 154.

[17] Werfalll, N.M. &Karaid, A.A. (2005). Free – Vibration Analysis of Rectangular Plates using Galerkin – Based Finite Element Method. International of Mechanical Engineering, 2(2), 59- 67.

[18] Ye, Jianqiao (1994). Large deflection of imperfect plates by iterative BE-FE method. Journal of Engineering Mechanics, Vol. 120, N0.3 (March).