

Investigation Of Bit Error Rate Performances Of M-Ary Phase Shift Keying In Additive White Gaussian Noise Channel

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Abstract: New technologies are being developed to fulfill the demand for the capacity requirements of users of digital communication systems. This is achieved by developing portable systems which are small, light, of minimal power consumption and, above all, of high spectral efficiency. Phase Shift Key (PSK) modulation utilizes phase that is least susceptible to noise and has been the preferred scheme for efficient higher-data-rate applications. This work presents a comparative study of the major forms of PSK that are widely used in communication, M-ary phase shift key (MPSK) digital modulation schemes in Additive White Gaussian Channel. The criterion for comparison is the ratio of bit energy to noise power spectra density in Additive White Gaussian Noise (AWGN) environment. The modulation techniques studied are: 2-PSK, 4-PSK, 8-PSK and 16-PSK. The relative performances were shown in graphical form. In general, the higher the order forms of modulation the higher the data rates to be carried within a given bandwidth. However, the downside is that they require a better signal to noise ratio before the error rates start to rise. In view of this balance, radio communication systems are able to dynamically choose the form of modulation depending upon the prevailing conditions and requirements.

Keywords: Bandwidth, Bit error rate, Channel, Communication, Digital, Modulation, PSK

I. INTRODUCTION

A. BACKGROUND

In a communication transmitter, the message signal is used to modulate the carrier because, for a number of reasons, it is impossible or undesirable for the message signal to make the journey to the receiver on its own. Feher, K. (1995) reported that Discreet Phase Modulation known as M-ary Phase Shift Keying is among the most frequently used digital modulation techniques. The Binary PSK (BPSK), alters the phase of the carrier sine wave by shifting the phase angle by 180° (or π radians) when the binary signal changes from zero (0) to one (1). In BPSK, a data bit, either a 1 or a 0 is represented as a symbol. A more efficient use of bandwidth is

achieved when each signaling element represents more than one bit. In M-PSK, a symbol is used to deal with $n = \log_2(M)$ bits where $M = 2, 4, 8$ and 16 , which corresponds to $n = 1, 2, 3$ and 4 data bits per symbol. However, increasing the bandwidth efficiency in this way usually increases the bit error rate because of the increases in the number of bits per signal. BPSK ($M = 2$) systems are considered to be the simplest form of M-PSK and adopted by the IEEE 802.11 Committee as the standard modulation in Direct Sequence Spread Spectrum (DS-SS) for Wireless Local Area Network (WLAN). Also, Quadrature Phase Modulations, Quadrature Phase shift key (QPSK) and Offset QPSK (OQPSK) Modems are used in system applications where BPSK is insufficient for available bandwidth. The various modulation techniques used in BPSK also apply to Quadrature systems. Quadrature modulations

are used in microwave radio, satellite applications and adopted for high data transmission for DS-SS for Wireless Local Area Network (WLAN) Feher, K. and Mehdi, H. (1995).

Adachi, F. and Sawahashi, M (1995) investigated the performances of M-ary PSK signal in AWGN channels and reported that M-ary differential (non-coherent) phase shift keying is a bandwidth efficient digital modulation technique, and recently has attracted increased attention in mobile radio where the available radio bandwidth is limited. Coherent detection offers good bit error rate (BER) performance in Additive White Gaussian Noise (AWGN) channels. However, it requires long acquisition times and, in fading environments, exhibits poor BER performance due to fast variations in the received signal phase; thus, differential detection is preferred. In AWGN channels, however, the BER performance of differentially detected M-ary DPSK is inferior to that of coherent detection.

When a signal is transmitted, the signal becomes corrupted by noise (Oladejo, 2009). The contaminations of the signal may take several forms; the noise may be added to the signal, in which case it is called additive noise, or the noise may multiply the signal (multiplicative noise), in which case the effect is called fading. In most practical situations it is adequate to assume that noise is additive (Dunlop and Smith, 1998). The probability density function (PDF) of the sum of many random variables approaches a Gaussian distribution regardless of the distributions of the individual variables (Tanenbaum, 2007). In addition, there is classification of noise by spectral density and it is given colour terminology, with different types named after different colours. The nature of the power spectral density determines the colour of the noise be it white, pink, red, brownian, grey, etc. White noise is a random signal with equal power within a fixed bandwidth at any center frequency (Manhesh, 2007), that is, flat power spectral density. Hence, Additive White Gaussian Noise (AWGN) is a channel model in which the only impairment to communication is a linear addition of white noise with a constant spectral density and a Gaussian distribution of amplitude. This study considers the bit error rate performances of M-PSK modulations in additive white Gaussian noise (AWGN) channel.

B. JUSTIFICATION OF THE STUDY

Bandwidth efficiency describes the ability of a modulation scheme to accommodate data within a limited bandwidth. However, the greater the bandwidth efficiency the greater the bit error rate. The results obtained afford the radio communication systems the opportunity to dynamically choose the form of modulation depending upon the prevailing conditions and requirements.

C. OBJECTIVES OF THE STUDY

The specific objectives of the work are to:

- ✓ obtain the bandwidth requirement for BPSK digital modulation scheme,
- ✓ obtain the bit error rates of 2,4,8,16 – PSK signals in the presence of AWGN to determine the bit error rate performance with least amount of bandwidth and

- ✓ present the bit error rate performances in graphical form.

II. MODEL DESCRIPTION

A. POWER SPECTRAL DENSITY OF A SEQUENCE OF RANDOM PULSES

In digital communication systems, we encounter random pulses. Shanmugam (1979) obtained the power spectra density of sequence of random pulses as follows:

If the Fourier transform of a single pulse $p_1(t)$ is $P_1(f)$ then Parseval's theorem states that the normalized energy of the pulse is

$$E_1 = \int_{-x}^x P_1(f) P_1^*(f) df = \int_{-x}^x |P_1(f)|^2 df$$

The energy in the range df at a frequency f is

$$dE_1 = |P_1(f)|^2 df \quad (2)$$

for a sequence of n successive pulses, and assume that the pulses do not

overlap, the energy in the range df at the frequency f due to the n pulses is

$$dE = dE_1 + dE_2 + \dots + dE_n \\ = \{|P_1(f)|^2 + |P_2(f)|^2 + \dots + |P_n(f)|^2\} df \quad (3)$$

The average value $\overline{|P(f)|^2}$ of the sequence of n pulses is, therefore,

$$\overline{|P(f)|^2} \equiv \frac{1}{n} \{|P_1(f)|^2 + |P_2(f)|^2 + \dots + |P_n(f)|^2\} df \quad (4)$$

so that dE in equation (3) can be written as

$$dE = n \overline{|P(f)|^2} df \quad (5)$$

The average time of separation between pulses is T_b so that n pulses will

occur in a time nT_b . The differential energy, dE contained in the time

interval nT_b from Equation (2.5) is

$$\frac{dE}{nT_b} = \frac{1}{nT_b} n \overline{|P(f)|^2} df \\ = \frac{1}{T_b} \overline{|P(f)|^2} df. \quad (6)$$

Thus, the power spectra density $G(f)$ in the frequency range df is dE/nT_b .

Hence,

$$G(f) = \frac{1}{T_b} \overline{|P(f)|^2} df \quad (7)$$

A modulated signal $s(t)$ can be described in terms of its in-phase and

quadrature components as

$$s(t) = s_i(t) \cos(2\pi f_c t) - s_q(t) \sin(2\pi f_c t) \\ = \text{Re}[s(t) \exp(j2\pi f_c t)], \quad (8)$$

where $\text{Re}[\dots]$ is the real part of the expression inside the square bracket, and

$$s(t) = s_i(t) + js_q(t), \quad (9)$$

$$\exp(j2\pi f_c t) = \cos(2\pi f_c t) + j \sin(2\pi f_c t). \quad (10)$$

The signal $s(t)$ is the complex envelope (the baseband version of the modulated signal) of $s(t)$. The components $s_i(t)$, $s_q(t)$ and $s(t)$ are all low-pass signals. They are uniquely defined in terms of the band-pass signal $s(t)$ and the carrier frequency (f_c) provided that the half-bandwidth of $s(t)$ is less than the carrier frequency. If $s_B(f)$ denotes the power spectra density of complex envelope of the baseband of $s(t)$, then the power spectra density $s_s(f)$ of the band-pass signal

$s(t)$ is a frequency shifted version of $s_B(f)$ except for a scaling factor given by

$$s_s(f) = \frac{1}{4} [s_B(f - f_c) + s_B(f + f_c)].$$

Simon (2001) reported that it is sufficient to evaluate the baseband power spectra density $s_B(f)$ in order to evaluate $s_s(f)$.

A. POWER SPECTRA DENSITY OF BPSK SIGNAL

Equation (7) was used for the analysis of BPSK. The analysis was done to obtain expressions for the power spectral density (PSD). The expressions for PSD were then used to estimate the bandwidth required to transmit r_b , binary transmission rate of signal.

BPSK signal can be described mathematically by

$$s(t) = \begin{cases} s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), & \text{for bit 0} \\ s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), & \text{for bit 1} \end{cases} \quad (12)$$

$$s(t) = m(t) \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_b.$$

where $m(t) = +1$ for bit 1 and -1 for bit 0. Thus, the transmitted signals $s_1(t)$

and $s_2(t)$ can be expressed in terms of $\phi_1(t)$ as

$$s_1(t) = \sqrt{E_b} \phi_1(t), \quad 0 \leq t < T_b \quad (14)$$

and

$$s_2(t) = -\sqrt{E_b} \phi_1(t), \quad 0 \leq t < T_b \quad (15)$$

Thus both transmitted signals have the same energy. The energy per bit, E_b

is given by

$$E_b = A_c^2 T_b / 2, \quad (16)$$

where A_c is the amplitude of the carrier signal.

BPSK signal, $s(t)$ can be expressed in complex envelope form as

$$s(t) = \text{Re}\{g_{BPSK}(t) \exp(j2\pi f_c t)\}, \quad (17)$$

where g_{BPSK} is the complex envelope of the signal given as

$$g_{BPSK}(t) = \sqrt{\frac{2E_b}{T_b}} m(t) e^{j\theta}. \quad (18)$$

The energy spectra density of a Fourier transformable signal $g(t)$ is defined

as the squared magnitude of the signal's Fourier transform. Hence, the

PSD of $g_{BPSK}(t)$ is

$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2E_b \text{sinc}^2(T_b f). \quad (19)$$

The PSD of the BPSK signal can be evaluated by translating the baseband

spectrum to the bandpass using equation (2.11). Hence, the PSD of a BPSK

signal is given by

$$P_{BPSK}(f) = \frac{E_b}{2} \left[\left(\frac{\sin \pi(f-f_c)T_b}{\pi(f-f_c)T_b} \right)^2 + \left(\frac{\sin \pi(-f-f_c)T_b}{\pi(-f-f_c)T_b} \right)^2 \right]. \quad (20)$$

The null-to-null bandwidth is

$$(f_c + R_b) - (f_c - R_b) = 2R_b. \quad (21)$$

V. SIMULATION

Codes were developed in MatLab programming language in conjunction with 'BERTool' (Figure 1), a bit error rate analysis graphical user interface (GUI), to generate BER data for M-PSK; 2-PSK, 4-PSK, 8-PSK, and 16-PSK. Table 1 shows the various Modulation schemes with the detection technique available in BerTool. The simulation results were obtained and presented in graphical form for comparison purpose.

A. CODES ALGORITHM

Generation of random binary data: the conventional format for representing a signal in MATLAB is a vector or matrix. This task used 'randint' function to create a column vector, 'xsym' that listed the successive values of a binary data stream. The length of the binary data stream was set to 10,000;

- ✓ *Modulation:* modulating function was invoked to modulate xsym using the baseband representation;
- ✓ *Addition of White Gaussian Noise:* applying the AWGN function to the modulated signals adds white Gaussian noise to it. An array of ratio of bit energy to noise power spectra density (E_b/N_0) starting from 0 to 10 dB with 0.5 spacing was arbitrarily set.
- ✓ *Demodulation:* applying the demodulate function to the received signal demodulates it.
- ✓ *Generation of BER data:* BERTool iterates over a set of E_b/N_0 values and computes the results.
- ✓ 'for loop' was used to vary the E_b/N_0 value (denoted by E_bN_0 in the code) and simulate the communication system for each value. The inner 'while loop' ensures that the simulation continues to use the given E_b/N_0 value until at least the predefined minimum number of errors (in the code) has occurred.
- ✓ *Computation of bit error rate (BER):* applying 'biterr' function to the original binary vector 'xsym' and to the demodulated binary vector z yielded the number of bit errors and the bit error rate.

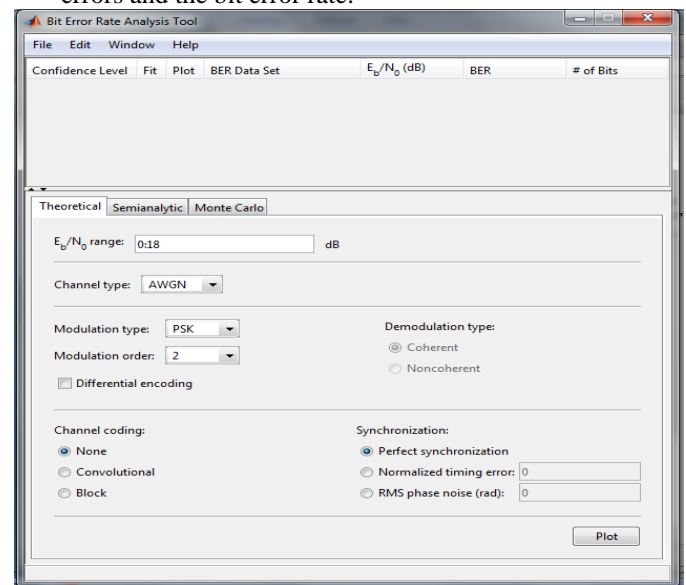


Figure 1: Bit Error Rate Analysis Tool (MatLab R2009)

VI. RESULTS AND DISCUSSION

A. BIT ERROR RATE PERFORMANCES OF 2-PSK, 4-PSK, 8-PSK AND 16-PSK

The results of BER performances of 2-PSK, 4-PSK, 8-PSK and 16-PSK are shown in Figure 2. The bandwidth efficiency of M-PSK modulation scheme increases with increasing in value of M. By using M-PSK modulation, each signaling element represents $n = \log_2(M)$ bits for the same size of bandwidth in this way usually increases the bit error rates, the reason being that distance between adjacent signal reduces with increasing number of bits per signaling; π for BPSK, $\frac{\pi}{2}$ for 4-PSK, $\frac{\pi}{4}$ for 8-PSK and so on This is because the coherent performance of equal energy binary signals in white Gaussian noise depends only on the "distance" between the two signals in the signal space-the larger the distance, the less the probability of error. This is intuitively appealing since the larger the distance, the less the possibility of mistaking one signal for the other. This trend in BER performances of these modulation techniques was actually confirmed. However, a striking result obtained was that the bit error probability of QPSK is identical to BPSK, but twice as much data can be sent in the same bandwidth, $2R_b$ (Equation 21). Thus, when compared to BPSK, QPSK provides twice the bandwidth efficiency with exactly the same energy efficiency while for the higher order, 8-PSK and 16-PSK, the Bit error rate increases with bandwidth efficiency.

Serial number	Modulation	Modulation Order	Available Choices
1	PSK	2, 4, 8, 16	Coherent
2	OQPSK	4	Noncoherent
3	DPSK	4	Coherent or Noncoherent

Table 1: Available Choices of Detection of M-PSK in BerTool (MatLab R2009)

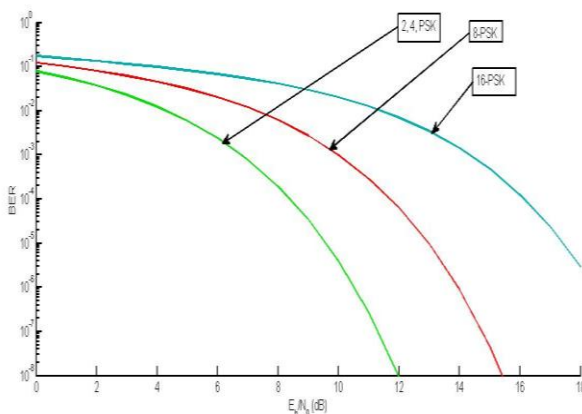


Figure 2: Results of BER Performances of 2-PSK, 4-PSK, 8-PSK and 16-PSK

VII. CONCLUSION AND RECOMMENDATION

This paper considers the performances of M-ary phase shift key in Additive White Gaussian Noise channel. MatLab

programming language in conjunction with BerTool, a bit error rate analysis graphical user interface (GUI) was used to generate BER data for M-ary PSK is developed. The results obtained indicated that by increasing the value of M, The bandwidth efficiency of M-PSK modulation increases while the Error Rate also increases.

The assumption of perfect carrier and bit synchronization was made in order to arrive at the results generated in this research. This is a very significant assumption and is unacceptable for an ideal system, particularly at low to moderate BER. Ideally, the addition of carrier and bit synchronization algorithms should be implemented in the simulator. A dynamic data rate effect can be incorporated in to the modulator instead of a constant because data rate fluctuates in ideal situations. Also, in an ideal communication system, the channel would be subjected to Rayleigh fading in addition to the additive noise resulting in phase shifts, this will have a significant effect on the resulting BER.

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