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Infinitesimal Transformations In A Projective Symmetric Finsler Space

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Abstract: In this paper we find infinitesimal projective and special projective transformations with certain theorems and relations in a projective symmetric Finsler space.

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I. INTRODUCTION

Takano, K. [8] studied the projective motion in a Riemannian space with bi-recurrent curvature. Sinha, R.S [6] discussed the infinitesimal projective transformation in a Finsler space, Yano, K. and Nagano, T. [10] have defined projective conformal transformation in a Riemannian space. Pande, H.D. and Kumar, A [4] have also discussed special infinitesimal projective transformation in a Finsler space and have obtained certain results.

In the present paper studies have been carried out with references to infinitesimal projective and special projective transformations and accordingly certain theorems and relations have been obtained in a projective symmetric Finsler space.

In view of the Berwald's covariant derivative the Lie derivatives of a tensor field $T^i_j(x,\dot{x})$ and the connection parameter $G^i_{jk}(x,\dot{x})$ are given as under:

(1.1) $\pounds_v T^i_j(x,\dot{x}) \underline{\underline{\text{def}}}\, T^i_{j(h)} v^h + \left(\dot{\partial}_n T^i_j\right) v^h_{(s)} \dot{x}^s T^h_j v^i_{(h)} + T^i_h v^h_{(j)}$

And

(1.2) $\pounds_v G^i_{jk}(x,\dot{x}) = v^i_{(j)(k)} H^i_{jkh} v^h + G^i_{sjk} v^s_{(r)} \dot{x}^r$

where $H^i_{jkh}(x,\dot{x})$ is defined by [I-(14.6)].

We also have the following commutation formulae

(1.3) $\dot{\partial}_l\left(\pounds_v T^i_j\right) - \pounds_v\left(\pounds_v T^i_j\right) = 0,$

(1.4) $\pounds_v T^i_{j(k)} - \left(\pounds_v T^i_j\right)_{(k)} = T^h_j \pounds_v G^i_{kh} - T^i_h \pounds_v G^h_{kj} - \left(\partial_h T^i_j\right) \pounds_v G^h_{ks} \dot{x}^s$

And

(1.5) $\left(\pounds_v G^i_{jh}\right)_{(k)} - \left(\pounds_v G^i_{kh}\right)_{(j)} = \pounds_v Q^i_{hjk} + \left(\pounds_v G^r_{kl}\right) G^i_{rjh} \dot{x}^l -$
$- \left(\pounds_v G^r_{jl}\right) \dot{x}^l G^i_{rhk} \dot{x}^l.$

In view of the projective covariant derivative the Lie derivative of $T^i_j(x,\dot{x})$ and the connection parameter $\Pi^i_{jk}(x,\dot{x})$ are given by:

(1.6) $\pounds_v T^i_j(x,\dot{x}) = T^i_{j((r))} v^r + \left(\dot{\partial}_s T^i_j\right) v^s_{((r))} \dot{x}^r - T^r_j v^i_{((r))} + T^i_r v^r_{((j))}$

And

(1.7) $\pounds_v \Pi^i_{mk} = v^i_{((m))((k))} + Q^i_{mkr} v^r + \left(\dot{\partial}_r \Pi^i_{mk}\right) v^r_{((s))} \dot{x}^s$

The connection parameter Π^i_{mk} satisfies the identity

(1.8) $\Pi^i_{rkh} \dot{x}^h = 0.$

Between the operators $\pounds_v$, $\dot{\partial}$ and ((k)), we can obtain the following commutation formulae

(1.9) $\dot{\partial}_l\left(\pounds_v T^i_j\right) - \pounds_v\left(\dot{\partial}_l T^i_j\right) = 0$

(1.10) $\left(\pounds_v T^i_j\right)_{((r))} - \pounds_v T^i_{j((r))} = T^l_j \pounds_v \Pi^i_{rl} - T^i_l \pounds_v \Pi^l_{rj} - \left(\dot{\partial}_l T^i_j\right) \pounds_v \Pi^l_{rm} \dot{x}^m$

And

(1.11) $\left(£_v \Pi^i_{jh}\right)_{((k))} - \left(£_v \Pi^i_{kh}\right)_{((j))} = £_v Q^i_{hjk} + \left(£_v \Pi^r_{kl}\right)\Pi^i_{rjh}\dot{x} - -\left(£_v \Pi^r_{jl}\right)\dot{x}^l \Pi^i_{rhk}$

II. SYMMETRIC FINSLER SPACE AND AFFINE MOTION

A symmetric Finsler space is characterised by the vanishing of the covariant derivative of Berwald's curvature tensor field H^i_{hjk}.

DEFINITION (2.1)

When the extended point transformation

(2.1) $$\xi^{x'} = \xi'^x = f^x\left(\xi^\nu\right)$$

does not change the fundamental function $L(\xi, \dot{\xi})$ of a Finsler space, that is, when we have

(2.2) $$L('x, '\dot{\xi}) = L(\xi, \dot{\xi}),$$

we call this extended point transformation a motion in a Finsler space F_n.

DEFINITION (2.2)

When the following transformations

(2.3) $$\xi'^x = f^r\left(\xi^\nu\right)$$

and

(2.4) $$'\dot{\xi}^x = \left(\partial_\lambda f^x\right)\dot{\xi}^\lambda$$

transform every geodesic into a geodesic and the affine parameter on it into an affine parameter on the deformed geodesic, we call the transformation an affine motion in general affine space of geodesics.

III. NON-AFFINE INFINITESIMAL PROJECTIVE TRANSFORMATION

Let us consider an infinitesimal point transformation

(3.1) $$\bar{x}^i = x^i + v^i(x)dt$$

where $v^i(x)$ determines a non-zero contravariant vector field defined over the domain of the space under consideration and dt is an infinitesimal constant. If the infinitesimal transformation (3.1) transform the system of geodesics of paths into the same system then such a transformation is termed as infinitesimal projective transformation in F_n.

The necessary and sufficient condition in order that an infinitesimal transformation (3.1) be an infinitesimal projective transformation is given by the following equation [3]:

(3.2) $$£_v G^i_{jk} = \bar{G}^i_{jk} - G^i_{jk} = \delta^i_j p_j - g_{jk} g^{il} d_l.$$

here $p_k(x, \bar{x})$ and $d_1(x, \bar{x})$ are vectors and satisfy the following identities

(3.3) (a) $\dot{\partial}_j p = pj$, (b) $p_{hk} = \dot{\partial}_n \dot{\partial}_k p$,

(c) $p_{hk}\dot{x}^h = p_k$, (d) $p_{hk}\dot{x}^h\dot{x}^k = p$,

(e) $\dot{\partial}_j d = dj$ (f) $d_{hk} = \dot{\partial}_h \dot{\partial}_k d$,

(g) $d_{hk}\dot{x}^h = d_k$, (h) $d_{hk}\dot{x}^h\dot{x}^k = d$

With the help of the commutation formula (1.5), the Lie-derivative of H^i_{hjk} is given by

(3.4) $$£_v H^i_{hjk} = \left(£_v G^i_{jh}\right)_{(k)} - \left(£_v G^i_{kh}\right)_{(j)} - \left(£_v G^r_{kl}\right)G^i_{rjh}\dot{x}^l + +\left(£_v G^r_{jl}\right)\dot{x}_l G^i_{rhk}$$

With the help of the equation of Lie-derivative, the equation (3.4) assumes the form

(3.5) $$£_v H^i_{hjk} = \delta^i_j p_{h(k)} - \delta^i_k p_{h(j)} + \delta^i_h p_{j(k)} = \delta^i_h p_{k(j)} - -g_{jh}g^{il}d_{l(k)} + g_{kh}g^{il}d_{l(j)} + +g_{kl}g^{rm}G^i_{rjh}d_m\dot{x}^l - g_{jl}g^{rm}G^i_{rkh}d_m\dot{x}^l.$$

Multiplying (3.5) by $\dot{x}^h\dot{x}^j$ and noting the equation (5.2) and the homogeneity property of $H^i_{hjk}(x, \dot{x})$, we get

(3.6) $$£_v H^i_k = 2\dot{x}^i p_{(k)} - \delta^i_k p_{(j)}\dot{x}^j - \dot{x}^i p_{k(j)}\dot{x}^j - -g_{jh}g^{il}d_{l(k)}\dot{x}^h\dot{x}^j + g_{kh}g^{il}d_{l(j)}\dot{x}^h\dot{x}^j.$$

Contracting (3.6) with respect to the indices i and k we get

(3.7) $$£_v H = -p_{(j)}\dot{x}^j + \frac{1}{n-1}\left\{d_{j)}\dot{x}^j - g_{hk}g^{il}de_{(i)}\dot{x}^h\dot{x}^j\right\}$$

With the help of equation (3.6) and (3.7), we get

(3.8) $$£_v H^i_k - £_v H\delta^i_k = 3\dot{x}^i p_{(x)} - \delta^i_k p_{(j)}\dot{x}^j - \dot{x}^i p_{k(j)}\dot{x}^j + +g_{kh}g^{il}d_{l(j)}\dot{x}^h\dot{x}^j - \frac{1}{n-1}\left\{d_{(k)}\dot{x}^i + +(2-n)g_{jh}g^{il}d_{l(k)}\dot{x}^h\dot{x}^j\right\}$$

Differentiating (3.8) partially with respect to $\dot{x}^r$ and other contracting the resulting equation with respect to the indices i & r, we get,

(3.9) $$\left(£_v\dot{\partial}_r H^r_k - £_v\dot{\partial}_k H\right) = (3n+2)p_{(k)} - (n+3)p_{k(j)} + d_{k(j)}\dot{x}^j + +g_{kh}g^{rl}\dot{x}^h\left\{d_{rl(j)}\dot{x}^j + d_{l(r)}\right\} - \frac{5-n}{n-1}d_{l(k)} +2\dot{x}^h\dot{x}^j\frac{C^l_{sr}}{g_{rs}}\left\{\frac{2-n}{n-1}g_{rh}d_{l(k)} - g_{kh}d_{l(j)}\right\}$$

In view of the equation (3.8) and (3.9), the Lie derivative of $W^i_j(x, \dot{x})$ can be written as:

(3.10) $$£_v W^i_k = \frac{1}{n+1}\left[p_{(k)}\dot{x}^i + 2p_{k(j)}\dot{x}^i\dot{x}^j + \frac{4-n}{n-1}d_{(k)}\dot{x}^i - -\dot{x}^i\left\{d_{k(j)}\dot{x}^j + g_{kh}g^{rl}\dot{x}^h\left(d_{rl(j)}\dot{x}^j + d_{l(r)}\right) + \right.\right.$$

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$$+2\dot{x}^h\dot{x}^j\frac{C^l_{sr}}{g_{rs}}\left(\frac{2-n}{n-1}g_{rh}d_{l(k)}-g_{kh}d_{l(j)}\right)\Big\}\Big]$$

$$-\delta^i_k p_{(j)}\dot{x}^j+g_{kh}g^{il}d_{l(j)}\dot{x}^h\dot{x}^j+\frac{2-n}{n-1}g_{jh}g^{il}d_{l(k)}\dot{x}^h\dot{x}^j.$$

Applying the commutation formula to the projective deviation tensor $W^i_j(x,\dot{x})$, we get

(3.11) $£_v W^i_{j(r)}=\left(£_v W^i_j\right)_{(r)}=W^h_j £_v G^i_{rh}-W^i_h £_v G^h_{jr}-\left(\dot{\partial}_h W^i_j\right)£_v G^h_{rs}\dot{x}^s.$

Substituting the value of $£_v G^i_{jk}$ from (3.2) in (3.11) and using the equations (3.3), we get

(3.12) $£_v W^i_{j(r)}-\left(£_v W^i_j\right)_{(r)}=W^h_j\left(\delta^i_r p_h-g_{rh}g^{il}dl\right)-W^i_r p_j-2W^i_j p_r+$

$$+g^{hl}d_l\left\{W^i_h g_{jr}+\left(\dot{\partial}_h W^i_j\right)g_{rs}\dot{x}^s\right\}-\left(\dot{\partial}_r W^i_j\right)p$$

Contracting (3.12) with respect to the indices i and r and noting we get

(3.13) $£_v W^i_{j(i)}-\left(£_v W^i_j\right)_{(i)}=(n-2)p_h W^h_j-W^h_j d_h+g^{hl}d_l\left\{W^i_h g_{ji}+\right.$

$$\left.+\left(\dot{\partial}_h W^i\right)g_{is}\dot{x}^s\right\}$$

Transvecting (3.12) by $\dot{x}^r$ and noting we get

(3.14) $\left\{£_v W^i_{j(r)}-\left(£_v W^i_j\right)_{(r)}\right\}\dot{x}^r=W^h_j\dot{x}^i p_h-4W^i_j p-W^h_j g_{rh}g^{il}d_l\dot{x}^r+$

$$+g^{hl}d_l\dot{x}^r\left\{W^i_h g_{gr}+\left(\dot{\partial}_h W^i_j\right)g_{rs}\dot{x}^s\right\}$$

At this stage, if we now assume that the Finsler space F_n is symmetric one i.e., $W^i_j(r)=0$? then under this assumption $£_v W^i_{j(r)}=0$ will always hold and therefore equations (3.13) and (3.14) will respectively assume the following form :

(3.15) $\left(£_v W^i_j\right)_{(r)}\dot{x}^r=(2-n)W^h_j p_h+W^h_j d_h-g^{hl}d_l$

$$-g^{hl}d_l\left\{W^i_h g_{jr}+\left(\dot{\partial}_h W^i_j\right)g_{rs}\dot{x}^s\right\}$$

And

(3.16) $\left(£_v W^i_j\right)_{(r)}\dot{x}^r=-W^h_j\dot{x}^i p_h+W^i_j p+W^h_j g_{rh}g^{il}d_l\dot{x}^r-$

$$-g^{hl}d_l\dot{x}^r\left\{W^i_h g_{jr}+\left(\dot{\partial}_h W^i_j\right)g_{rs}\dot{x}^s\right\}$$

With the help of equation (3.15) and (3.16) we eliminate the term $W^h_j p_h$ and the result of elimination gives

(3.17) $M^i_j=\dot{x}^i\left[W^h_j d_h-g^{hl}d_l\left\{W^r_h g_{jr}+\left(\dot{\partial}_h W^r_j\right)g_{rs}\dot{x}^s\right\}\right]$

$$+2(2-n)\left[4W^i_j p+W^h_j g_{rh}g^{il}d_l\dot{x}^r-g^{hl}d_l\dot{x}^r\left\{W^i_h g_{jr}+\right.\right.$$

$$\left.\left.+\left(\dot{\partial}_h W^i_j\right)g_{rs}\dot{x}^s\right\}\right]$$

Where

(3.18) $M^i_j\underline{\underline{\mathrm{def}}}\left(£_v W^i_j\right)_{(r)}\dot{x}^i+(2-n)\left(£_v W^i_j\right)_{(r)}\dot{x}^r.$

If the Finsler F_n admits a projective affine motion, then the equation

(3.19) $£_v=G^i_{jk}=0$

must hold.

Therefore, with the help of equation (3.2) and (3.19), it is clear that the vectors $p(x,\dot{x})$ and $d(x,\dot{x})$ must separately vanish. Thus, we can state

THEOREM (3.1)

If an F_n admits a non-affine infinitesimal projective transformation such that the Berwald's covariant derivatives of W^i_j remains invariant then the equation (3.17) holds.

THEOREM (3.2)

If an F_n admits an affine infinitesimal projective transformation such that the covariant derivative of W^i_j remains invariant then $M^i_j=0$.

THEOREM (3.3)

If a symmetric Finsler space F_n admits a non-affine infinitesimal projective transformation then the equation (3.17) necessarily holds.

THEOREM (3.4)

In a symmetric Finsler space, if an infinitesimal projective transformation is an affine one then the equation $M^i_j=0$ necessarily holds.

IV. INFINITESIMAL SPECIAL PROJECTIVE TRANSFORMATION DEFINITION (4.1)

The necessary and sufficient condition in order that the infinitesimal point transformation (3.1) be an infinitesimal special projective transformation is given by

(4.1) $£_v\Pi^i_{jk}=\overline{\Pi}^i_{jk}-\Pi^i_{jk}=\delta^i_j b_k+\delta^i_j b_k-g_{gk}g^{il}c_l$

Here $b_k(x,\dot{x})$ and $C_1(x,\dot{x})$ are vector which satisfy the following relations

(4.2) (a) $\dot{\partial}_j b=b_j$, (b) $b_{hk}=\dot{\partial}_h\dot{\partial}_k b$,

(c) $b_{hk}\dot{x}^h=b_k$, (d) $b_{hk}\dot{x}^h\dot{x}^k=b$,

(e) $\dot{\partial}_j c=c_j$, (f) $c_{jk}=\dot{\partial}_j\dot{\partial}_k c$,

(g) $c_{hk}\dot{x}^h=c_k$ (h) $c_{nk}\dot{x}^h\dot{x}^k=c.$

With the help of the equation (4.1), (4.2) and the commutation formula (1.11), the Lie-derivative of the projective entity Q^i_{hjk} is given by

(4.3) $£_v Q^i_{hjk}=\delta^i_j b_{h((k))}+\delta^i_h b_{j((k))}-g_{jh}g^{il}c_{l((kk))}-$

$$-g_{jh((k))}g^{il}c_l-g_{jh}g^{il}_{((k))}c_l-\delta^i_k b_{h((j))}-$$

$$-\delta^i_h b_{k((j))}+g_{kh}g^{il}c_{l((j))}+g_{kh((j))}g^{il}c_l+$$

$$+g_{kh}g^{il}_{((j))}c_e-\delta^r_k b\Pi^i_{rjh}+g_{kl}g^{rm}c_m\Pi^i_{rjh}\dot x^i+$$
$$+\delta^r_j b\Pi^i_{rhk}-g_{jl}g^{rm}c_m\dot x^l\Pi^i_{rhk}.$$

Multiplying (4.3) by $\dot x^h\dot x^j$ and using the equation (1.8) and the fact that $Q^i_{hjk}\dot x^h=Q^i_{jk}$, we get

(4.4) $$\pounds_v Q^i_k=2\dot x b_{((k))}-\delta^i_k b_{((j))}\dot x^j+\dot x^h\dot x^j\Big[g_{kh}\left\{g^{il}_{((j))}c_l+g^{il}c_{l((j))}\right\}-g_{gh}\left\{g^{il}_{((k))}c_l+g^{il}c_{l((k))}\right\}-g_{jh((k))}g^{il}c_l\Big]$$

Contracting (4.3) with respect to the indices i and k and then multiplying the resulting equation thus obtaining by $\dot x^h\dot x^j$ and using (1.8), we get

(4.5)
$$\pounds_v Q_{hj}\dot x^h\dot x^j=(1-n)b_{((j))}\dot x^j+c_{((j))}\dot x^j+g^{il}c_l\dot x^h\dot x^j\left\{g_{il((j))}-g_{jh((i))}\right\}-g_{jh}\dot x^h\dot x^j\left\{g^{il}c_{l((i))}+g^{il}_{((i))}c_l\right\}+g_{ih}g^{il}_{((j))}c_l\dot x^h\dot x^j.$$

We now eliminate the term $b_{((j))}\dot x^j$ from the equation (4.4) and (4.5) and get

(4.6) $$M^i_k(x,\dot x)=(1-n)2\dot x^i b_{((k))}-\dot x^i b_{k((j))}\dot x^j+\dot x^h\dot x^j g_{kh}\left\{g^{il}_{((j))}+g^{il}c_{l((j))}\right\}-g_{jh}g^{il}_{((k))}c_l-g_{jh}g^{il}c_{l((k))}-g_{jh((k))}g^{il}c_l+c_{((j))}\dot x^j\delta^i_k+g^{il}c_l\dot x^h\dot x^j\delta^i_k\left\{g_{ih((j))}-g_{jh((i))}\right\}-g_{jh}\dot x^h\dot x^j\delta^i_k\left\{g^{il}c_{l((i))}-g^{il}_{((i))}c_l\right\}+g_{ih}g^{il}_{((j))}c_l\dot x^h\dot x^h\dot x^j\delta^i_k$$

Where

(4.7) $$M^i_k(x,\dot x)\,\underline{\underline{\mathrm{def}}}\,(i-n)\pounds_v Q^i_k+\delta^i_k\pounds_v Q_{hj}\dot x^h\dot x^j$$

Applying the commutation formula (1.9) to the projective derivation tensor $W^i_j(x,\dot x)$ and using the equation (4.2), we get

(4.8) $$(\pounds_v W^i_j)_{((r))}-\pounds_v W^i_{j((r))}=W^l_j\delta^i_r b_l-W^1_j g_{rl}g^{ip}c_p-W^i_r b_j+W^i_l g_{rj}g^{lp}c_p-(\dot\partial_r W^i_j)b-2W^r_j b_r-(\dot\partial_r W^i_j)g_m g^{lp}c_p\dot x^m.$$

Contracting (4.8) with respect to the indices i and r we get

(4.9) $$(\pounds_v W^i_j)_{((i))}-\pounds_v W^i_{j((i))}=(n-2)W^l_j b_l-W^l_j c_l+g^{lp}c_p\left\{W^i_l g_{rj}-(\dot\partial_l W^i_j)g_{im}\dot x^m\right\}.$$

Transvecting (4.8) by $\dot x^r$ we get

(4.10) $$\left\{(\pounds_v W^i_j)_{((r))}-\pounds_v W^i_{j((r))}\right\}\dot x^r=W^l_j b_l\dot x^i-4W^i_j b-W^l_j g_{rl}g^{ip}c_p\dot x^r+W^i_l g_{rj}g^{lp}c_p\dot x^r-(\dot\partial_l W^i_j)g_{rm}g^{lp}c_p\dot x^r\dot x^m$$

Let us now suppose that the infinitesimal special projective transformation (4.1) leaves in variant the projective covariant derivative of the projective deviation tensor i.e.

(4.11) $$\pounds_v W^i_{j((r))}=0.$$

In view of (4.11), equations (4.9) and (4.10) can be alternatively expressed in the following forms

(4.12) $$(\pounds_v W^i_j)_{((j))}=(n-2)W^l_j b_l-W^l_j c_l+g^{lp}c_p\left\{W^i_l g_{rj}-(\dot\partial_l W^i_j)g_{lm}\dot x^m\right\}$$

And

(4.13) $$(\pounds_v W^i_j)_{((r))}\dot x^r=W^l_j b_l\dot x^i-4W^i_j b-W^l_j g_{rl}g^{ip}c_p\dot x^r+W^i_l g_{rj}g^{lp}c_p\dot x^r-(\dot\partial_l W^i_j)g_m g^{ip}c_p\dot x^r\dot x^m$$

respectively.

Eliminating the term $W^l_j b_l$ from equations (4.12) and (4.13), we get

(4.14) $$B^i_j(x,\dot x)=x^i\left[-W^l_j c_l+g^{lp}c_p\left\{W^i_l g_{kj}-(\dot\partial_l W^i_j)g_{im}\dot x^m\right\}\right]-(n-2)\left[-4W^i_j b-W^i_j g_{rl}g^{ip}c_p\dot x^r+W^i_l g_{rj}g^{lp}c_p\dot x^r-(\partial_i W^i_j)g_{rm}g^{lp}c_p\dot x^r\dot x^m\right]$$

Where

(4.15) $$B^i_j(x,\dot x)\,\underline{\underline{\mathrm{def}}}\,(\pounds_v W^i_j)_{((i))}\dot x^i-(n-2)(\pounds_v W^i_j)_{((rr))}\dot x^r.$$

If the Finsler space F_n admits special projective affine motion, then the equation

(4.16) $\pounds_v\Pi^i_{jh}=0$ must hold.

Hence equations (4.1) and (4.16) enables us to infer that the vectors $b(x,\dot x)$ and $c(x,\dot x)$ must separately vanish, Therefore, we can state

THEOREM (4.1)

If an F_n admits a non-affine infinitesimal special projective transformation such that the projective covariant derivative of W^i_j remains invariant then equation (4.15) holds.

THEOREM (4.2)

If an F_n admits an affine infinitesimal special projective transformation such that the covariant derivative of W^i_j remains invariant then B^i_j given in (4.15) should vanish.

In particular, if we now suppose that the Finsler space F_n in the symmetric one i.e. $W^i_{j((r))} = 0$ holds then equation (4.11) always holds. Therefore, we can state

THEOREM (4.3)

If a symmetric Finsler space F_n admits a non-affine infinitesimal special projective transformation then the equation (4.14) holds necessarily.

THEOREM (4.4)

In a symmetric Finsler space F_n if an infinitesimal special projective transformation is an affine one then the equation $B^i_j = 0$ necessarily holds.

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