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Invariant Submanifold Of $\tilde{\psi}(5,3)$ Structure Manifold

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Abstract: ***In this paper, we have studied various properties of a $\tilde{\psi}(5,3)$ structure manifold and its invariant submanifold. Under two different assumptions, the nature of induced structure ψ, has also been discussed.***

Keywords: Invariant submanifold, Nijenhuis tensor, projection operators and complementary distributions.

I. INTRODUCTION

Let V^m be a C^∞ m-dimensional Riemannian manifold imbedded in a C^∞ n-dimensional Riemannian manifold M^n, where $m<n$. The imbedding being denoted by

$$f: V^m \longrightarrow M^n$$

Let B be the mapping induced by *f* i.e. $B=df$

$$df: T(V) \longrightarrow T(M)$$

Let $T(V,M)$ be the set of all vectors tangent to the submanifold $f(V)$. It is well known that

$$B: T(V) \longrightarrow T(V,M)$$

Is an isomorphism. The set of all vectors normal to $f(V)$ forms a vector bundle over $f(V)$, which we shall denote by $N(V,M)$. We call $N(V,M)$ the normal bundle of V^m. The vector bundle induced by f from $N(V,M)$ is denoted by $N(V)$. We denote by $C: N(V) \longrightarrow N(V,M)$ the natural isomorphism and by $\eta_s^r(V)$ the space of all C^∞ tensor fields of type (r,s) associated with N (V). Thus $\zeta_0^0(V)=\eta_0^0(V)$ is the space of all C^∞ functions defined on V^m while an element of $\eta_0^1(V)$ is a C^∞ vector field normal to V^m and an element of $\zeta_0^1(V)$ is a C^∞ vector field tangential to V^m.

Let $\bar{X}$ and $\bar{Y}$ be vector fields defined along $f(V)$ and $\tilde{X},\tilde{Y}$ be the local extensions of $\bar{X}$ and $\bar{Y}$ respectively. Then $[\tilde{X},\tilde{Y}]$ is a vector field tangential to M^n and its restriction $[\tilde{X},\tilde{Y}]/f(V)$ to $f(V)$ is determined independently of the choice of these local extension $\tilde{X}$ and $\tilde{Y}$. Thus $[\bar{X},\bar{Y}]$ is defined as

(1.1) $$[\bar{X},\bar{Y}]=[\tilde{X},\tilde{Y}]/f(V)$$

Since B is an isomorphism

(1.2) $$[BX,BY]=B[X,Y] \text{ for all } X,Y\in\zeta_0^1(V)$$

Let $\bar{G}$ be the Riemannain metric tensor of M^n, we define g and g^* on V^m and $N(V)$ respectively as

(1.3) $$g(X_1,X_2)=\tilde{G}(BX_1,BX_2)f,$$ and

(1.4) $$g^*(N_1,N_2)=\tilde{G}(CN_1,CN_2)$$

For all $X_1,X_2\in\zeta_0^1(V)$ and $N_1,N_2\in\eta_0^1(V)$

It can be verified that g and g^* are the induced metrics on V^m and $N(V)$ respectively.

Let $\tilde{\nabla}$ be the Riemannian connection determined by $\tilde{G}$ in M^n, then $\tilde{\nabla}$ induces a connection ∇ in $f(V)$ defined by

(1.5) $\nabla_{\bar{X}}\bar{Y} = \tilde{\nabla}_{\tilde{X}}\tilde{Y} / f(V)$

where $\bar{X}$ and $\bar{Y}$ are arbitrary C^{∞} vector fields defined along $f(V)$ and tangential to $f(V)$.

Let us suppose that M^n is a C^{∞} $\tilde{\psi}$ $(5,3)$ structure manifold with structure tensor $\tilde{\psi}$ of type (1,1) satisfying

(1.6) $\tilde{\psi}^5 + \tilde{\psi}^3 = 0$

Let $\tilde{L}$ and $\tilde{M}$ be the complementary distributions corresponding to the projection operators

(1.7) $\tilde{l} = \tilde{\psi}^4, \qquad \tilde{m} = I - \tilde{\psi}^4$

where I denotes the identity operator.

From (1.6) and (1.7), we have

(1.8) (a) $\tilde{l} + \tilde{m} = I$ (b) $\tilde{l}^2 = \tilde{l}$

(c) $\tilde{m}^2 = \tilde{m}$ (d) $\tilde{l}\,\tilde{m} = \tilde{m}\,\tilde{l} = 0$

Let D_l and D_m be the subspaces inherited by complementary projection operators l and m respectively.

We define

$$D_l = \{X \in T_p(V) : lX = X, mX = 0\}$$

$$D_m = \{X \in T_p(V) : mX = X, lX = 0\}$$

Thus $T_p(V) = D_l + D_m$

Also $Ker\, l = \{X : lX = 0\} = D_m$

$Ker\, m = \{X : mX = 0\} = D_l$

at each point p of $f(V)$.

II. INVARIANT SUBMANIFOLD OF $\tilde{\psi}$ $(5,3)$ STRUCTURE MANIFOLD

We call V^m to be invariant submanifold of M^n if the tangent space $T^p(f(V))$ of $f(V)$ is invariant by the linear mapping $\tilde{\psi}$ at each point *p* of $f(V)$. Thus

(2.1) $\tilde{\psi}BX = B\psi X$, for all $X \in \zeta_0^1(V)$,, and ψ being a (1,1) tensor field in V^m.

THEOREM (2.1): Let $\tilde{N}$ and *N* be the Nijenhuis tensors determined by $\tilde{\psi}$ and ψ in M^n and V^m respectively, then

(2.2) $\tilde{N}(BX, BY) = BN(X,Y)$, for all $X, Y \in \zeta_0^1(V)$

PROOF: We have, by using (1.2) and (2.1)

(2.3) $\tilde{N}(BX, BY) = [\tilde{\psi}BX, \tilde{\psi}BY] + \tilde{\psi}^2[BX, BY] - \tilde{\psi}[\tilde{\psi}BX, BY] - \tilde{\psi}[BX, \tilde{\psi}BY]$

$= [B\psi X, B\psi Y] + \tilde{\psi}^2 B[X,Y] - \tilde{\psi}[B\psi X, BY] - \tilde{\psi}[BX, B\psi Y]$

$= B[\psi X, \psi Y] + B\psi^2[X,Y] - \tilde{\psi}B[\psi X, Y] - \tilde{\psi}B[X, \psi Y]$

$= B\{[\psi X, \psi Y] + \psi^2[X,Y] - \psi[\psi X, Y] - \psi[X, \psi Y]\}$

$= BN(X,Y)$

III. DISTRIBUTION $\tilde{M}$ NEVER BEING TANGENTIAL TO $f(V)$

THEOREM (3.1) if the distribution $\tilde{M}$ is never tangential to $f(V)$, then

(3.1) $\tilde{m}(BX) = 0$ for all $X \in \zeta_0^1(V)$

and the induced structure ψ on V^m satisfies

(3.2) $\psi^4 = I$

PROOF : if possible $\tilde{m}(BX) \neq 0$. From (2.1) We get

(3.3) $\tilde{\psi}^4 BX = B\psi^4 X$; from (1.7) and (3.3)

$\tilde{m}(BX) = (I - \tilde{\psi}^4)BX$

$= BX - B\psi^4 X$

(3.4) $\tilde{m}(BX) = B(X - \psi^4 X)$

This relation shows that $\tilde{m}(BX)$ is tangential to $f(V)$ which contradicts the hypothesis. Thus $\tilde{m}(BX) = 0$. Using this result in (3.4) and remembering that *B* is an isomorphism, We get

(3.5) $\psi^4 = I$,

THEOREM (3.2) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.6) $\underset{\tilde{m}}{\tilde{N}}(BX, BY) = 0$

PROOF: We have

(3.7) $\underset{\tilde{m}}{\tilde{N}}(BX, BY) = [\tilde{m}BX, \tilde{m}BY] + \tilde{m}^2[BX, BY] - \tilde{m}[\tilde{m}BX, BY] - \tilde{m}[BX, \tilde{m}BY]$

Using (1.2), (1.8) (c) and (3.1), we get (3.6).

THEOREM (3.3) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.8) $\underset{\tilde{l}}{\tilde{N}}(BX, BY) = 0$

PROOF: We have

(3.9) $\underset{\tilde{l}}{\tilde{N}}(BX, BY) = [\tilde{l}BX, \tilde{l}BY] + \tilde{l}^2[BX, BY] - \tilde{l}[\tilde{l}BX, BY] - \tilde{l}[BX, \tilde{l}BY]$

Using (1.2), (1.8) (a), (b) and (3.1) in (3.9); we get (3.8)

THEOREN (3.4) Let $\tilde{M}$ be never tangential to $f(V)$. Define

(3.10) $\tilde{H}(\tilde{X}, \tilde{Y}) = \tilde{N}(\tilde{X}, \tilde{Y}) - \tilde{N}(\tilde{m}\tilde{X}, \tilde{Y}) - \tilde{N}(\tilde{X}, \tilde{m}\tilde{Y})$

$$+\tilde{N}\left(\tilde{m}\tilde{X},\tilde{m}\tilde{Y}\right)$$

For all $\tilde{X},\tilde{Y}\in\zeta_0^1(M)$, then

(3.11) $\tilde{H}\left(BX,BY\right)=BN\left(X,Y\right)$

PROOF: Using $\tilde{X}=BX$, $\tilde{Y}=BY$ and (2.2), (3.1) in (3.10) We get (3.11).

IV. DISTRIBUTION $\tilde{M}$ ALWAYS BEING TANGENTIAL TO $f(V)$

THEOREM (4.1) Let $\tilde{M}$ be always tangential to $f(V)$, then

(4.1) (a) $\tilde{m}\left(BX\right)=Bm\,X$ (b) $\tilde{l}\left(BX\right)=Bl\,X$

PROOF: from (3.4), We get (4.1) (a). Also

(4.2) $l=\psi^4$

$lX=\psi^4\,X$

(4.3) $BlX=B\psi^4\,X$

Using (2.1) in (4.3)

(4.4) $BlX=\tilde{\psi}^4\,BX=\tilde{l}\left(BX\right),$

which is (4.1) (b).

THEOREM (4.2) Let $\tilde{M}$ be always tangential to $f(V)$, then l and m satisfy

(4.5) (a) $l+m=I$ (b) $lm=ml=0$ (c) $l^2=l$ (d) $m^2=m$.

PROOF: Using (1.8) and (4.1) We get the results.

THEOREM (4.3) If $\tilde{M}$ is always tangential to $f(V)$, then

(4.6) $\psi^5+\psi^3=0$

PROOF: From (2.1)

(4.7) $\tilde{\psi}^5\,BX=B\psi^5\,X$

Using (1.6) in (4.7)

$-\tilde{\psi}^3\,BX=B\psi^5\,X$

$-B\psi^3X=B\psi^5\,X$

Or $\psi^5+\psi^3=0$ which is (4.6)

THEOREM (4.4): If $\tilde{M}$ Is always tangential to $f(V)$ then as in (3.10)

(4.8) $\tilde{H}\left(BX,BY\right)=BH\left(X,Y\right)$

PROOF: from (3.10) we get

(4.9) $\tilde{H}\left(BX,BY\right)=\tilde{N}\left(BX,BY\right)-\tilde{N}\left(\tilde{m}BX,BY\right)$
$-\tilde{N}\left(BX,\tilde{m}BY\right)+\tilde{N}\left(\tilde{m}BX,\tilde{m}BY\right)$

Using (4.1) (a) and (2.2) in (4.9) we get (4.8).

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