

# Critical Speed Analysis Of A Rotating Shaft System

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*Abstract: The most common source of vibration in a rotor is the unbalance amongst all other and it will be always present in the rotor system. The unbalance causes the vibration, acoustic noise, and excessive wear of bearings in a high speed rotating machines when it runs very close to its critical speed. So, it is important to find out the critical speeds of the rotor system during in the designing process. One of the best approach is that the balancing the rotor to reduce its unbalance response. The present works illustrates that to finding the critical speeds of two different rotating shafts by using finite element method. The critical speed has been measured for the various models i.e. (i) a simple rotor system, (ii) A rotor system having disc at the center. A MATLAB rotor FE solver has been used to find out the critical speeds and the results have been verified with the results of ANSYS FE solver. The FE model has been discretized for 15 and 20 elements. A JEFFCOTT rotor model has been studied in detail with steady state response analysis, unbalance analysis and calculation of critical speed of the system. The main contribution of the present work is that the physical characteristics of a rotor system have been studied in detail and the critical speeds for the various types of rotor system have been calculated. The comparative results of MATLAB and ANSYSFE solver shows the good agreement.*

**Keywords:** rotor dynamics, JEFFCOTT ROTOR, unbalance response, critical speed, ANSYS FE Solver.

## I. INTRODUCTION

Induction motors have been used as a power transmitting unit in various industrial applications. There is no such an ideal motor without an unbalance present. The unbalance present in the rotating machinery may cause severe damage to the system and decreasing the performance of the machine while running very close to its unbalance response frequencies. So, it is important to run the rotating machine away from its critical speeds. The unbalance response may reduce by balancing the rotor and providing the damping. Research in rotor dynamics is aimed at improving the understanding of rotor dynamic phenomena and improving the performance of rotating machinery. Nelson [1] analyzed the lateral and torsional motion of rotating rotors is replete with

the application of Newton's and Euler's equation. The first rotor model was proposed by Foppl in 1895 which consisted single disk located on a shaft. In the year 1919, Jeffcott conceived the same model with the behavior including critical speeds, gyroscopic action, Damping and etc. E. Downham [2] in this paper deals with various model experiments designed to establish an accurate method for calculating the critical whirling speeds of complex systems. The critical whirling speeds and natural vibrations of a single shaft flexibly supported and carrying a flexible rotor of appreciable moment of inertia have been investigated and good agreement has been obtained between experimental and calculated results for the rotating system. Mili J. Hota *et al* [2] has analyzed the synchronized critical speed analysis of rotor bearing system. Cylindrical rotor modes are not influenced by gyroscopic

effects and remain at a fairly constant frequency versus rotor speed. J. Zajackowski [4] analyzed the dynamics of the shaft for which the speed is a result of the interaction of the motor and the shaft. It has been noticed that the shaft with one end free to move axially, the energy surface has a minimum below the critical speed and a maximum over the critical speed. As the consequence of the shape of the energy surface, the motion over critical speed is unstable and the speed of the shaft decreases to the critical value. Erik Swanson *et al* [5] gave a practical understanding of terminology and behavior based in visualizing how a shaft vibrates and examining issues that affect vibration. It was shown that cylindrical rotor modes are not influenced by gyroscopic effects and remain at a fairly constant frequency versus rotor speed. Korody Jagannath [6] developed an automated script in ANSYS Mechanical APDL solver to determine the critical speeds high speed turbine. The author has considered the effect of turbine, slip ring and end rings as an external load over the shaft. Naveena *et al*[7] used the FE method to determine the centrifugal rotor frequencies. Forced response analysis has been used to find the peak amplitude at the resonance condition. Deepak Srikrishnanivas [8] analyzed with rotor dynamic study of jet engine using ANSYS. The rotor dynamic characteristics of a RM12 Jet engine have been modelled using ANSYS and its capabilities are evaluated with specialized rotor dynamic's tool, Dyrobes. This helps in understanding, modelling, simulation and post processing techniques for rotor dynamic analysis of Jet engine rotor using ANSYS. Trupti Wani *et al* [9] used the Finite element analysis to overcome problems arising due to damping and give a feasible solution with any complicated geometry having number of components. Modal and Harmonic analysis is carried out to exactly estimate the nature of deflection and stresses in the structure. The analysis results nearness of natural frequency with operational frequency. In recent days, modern computational techniques and experimental findings have been used to solve the real time engineering problems. In this context, the finite element analysis with the help of computations have been utilized to analyze the effect of critical speed on a rotating shaft system. The present paper shows the critical speed measurement of a two different rotor system *ie.*, a rotating shaft with rotor and without rotor. A FE matlab code has been used to find the critical speed calculations and verified with the results of ANSYS FE solver. The results shows good agreement. The effect of varying bearing stiffness on the rotating shaft system has been analyzed.

## II. JEFFCOTT ROTOR MODEL

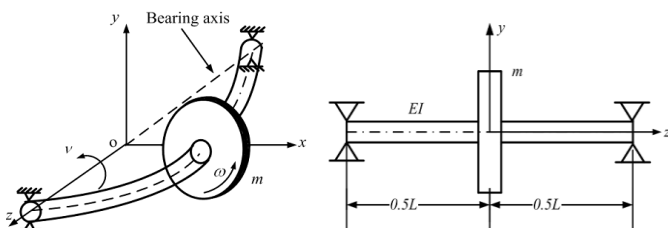


Figure 1: A Jeffcott rotor model Figure 2: A Jeffcott Rotor Model in y-z plane

The Jeffcott Rotor Model consists of a simply supported flexible massless shaft with a rigid thin disc mounted at the mid-span as shown in figure 1.1. The transverse stiffness,  $k$ , of a simply supported shaft is expressed as

$$k = \frac{\text{Load}}{\text{Deflection}} = \frac{48EI}{L^3} \quad (2.1)$$

where  $E$  is the Young's modulus,  $I$  is the second moment of area of the shaft cross-section, and  $L$  is the span of the shaft. Coordinates to define the position of the centre of rotation of the rotor,  $C$ , are given as  $x$  and  $y$ , the location of the unbalance is given by  $\theta$ , which is measured from the  $x$ -axis in the counter clockwise direction. Thus, three geometrical coordinates  $(x, y, \theta)$  are needed to define the position of the Jeffcott rotor (*i.e.* it has three DOFs). The disc is at the mid-span, hence, the tilting of the disc about transverse axes (*i.e.*  $x$  and  $y$ ) are not there.

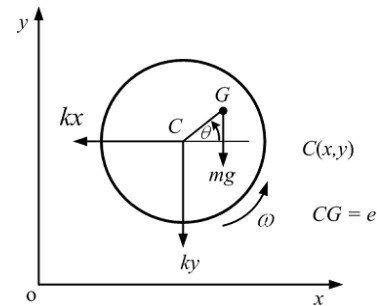


Figure 3: Free body diagram of the disc in the x-y plane

From the above figure, the force and the moment balance in the  $x$ ,  $y$  and  $\theta$  directions can be written as

$$\begin{aligned} \text{Position of center of gravity (G) in X-direction} &= x + e \cos \theta \\ \text{Position of center of gravity (G) in Y-direction} &= y + e \sin \theta \end{aligned}$$

Equating the inertia forces with spring forces and damping forces we have,

$$-kx - c\dot{x} = m \frac{d^2}{dt^2} (x + e \cos \theta) \quad (2.2)$$

$$-ky - c\dot{y} = m \frac{d^2}{dt^2} (y + e \sin \theta) \quad (2.3)$$

Now the weight of the disc is acting at the centre of gravity of the disc.

$$-mg - ky - c\dot{y} = m \frac{d^2}{dt^2} (y + e \sin \theta) \quad (2.4)$$

and the weight ' $mg$ ' also has a moment about the centre of rotation ' $C$ '

$$-mg(e \cos \theta) = I_p \ddot{\theta} \quad (2.5)$$

For the case when  $\theta = \omega t$ , *i.e.* when the disc is rotating at a constant spin speed, the Jeffcott rotor model reduces to two DOF rotor model. Hence the above equations of motion in the  $x$  and  $y$  directions are modified as

$$m\ddot{x} + c\dot{x} + kx = m\omega^2 e \cos \omega t \quad (2.6)$$

$$m\ddot{y} + c\dot{y} + ky = m\omega^2 e \cos \omega t \quad (2.7)$$

In deriving equations of motion, the centrifugal force due to unbalance is not considered as external force. Since the rotor system is symmetric, the rotor system will have two

equal transverse undamped natural frequencies in two orthogonal directions and are given as

$$\omega_n f_{1,2} = \sqrt{\frac{k}{m}} \quad (2.8)$$

### III. RESULTS AND DISCUSSIONS

A simple rotor shaft having a maximum diameter of 75 mm and it is rotating at 5000 rpm has been chosen for the analysis. The same rotating shaft has been used for the analysis with disc of 218.5 mm outer diameter and the thickness is 45.4 mm. The shaft model is discretized for 15 and 20 elements. Figure 4 & 5 shows the shaft model which is discretized for 15 element with and without disc respectively and with a bearing stiffness of  $1e5$ . Figure 6 & 7 shows the shaft model which is discretized for 20 element with and without disc respectively and with a bearing stiffness of  $1e6$ .

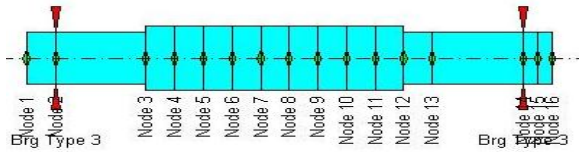


Figure 4: A shaft model having 15 element without disc

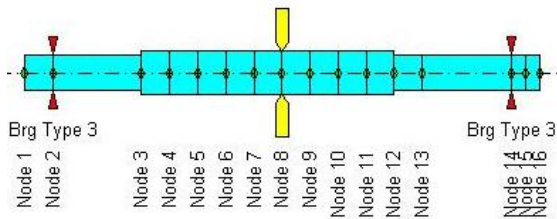


Figure 5: A shaft model having 15 element with disc

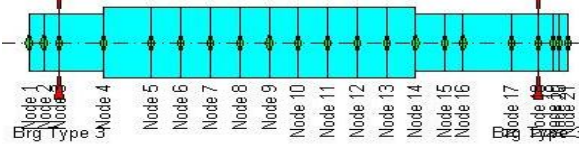


Figure 6: A shaft model having 20 element without disc

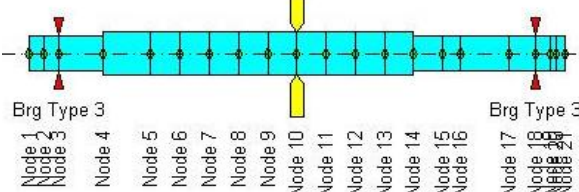


Figure 7: A shaft model having 20 element with disc

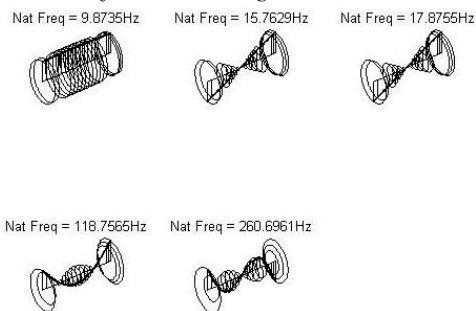


Figure 8: Critical speeds of 15 elements shaft with no disc for  $1e5 \text{ N/m}^2$  Bearing stiffness

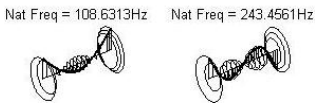
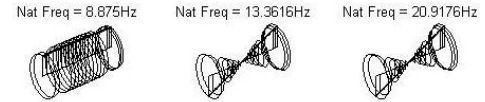


Figure 9: Critical speeds of 15 elements shaft with disc for  $1e5 \text{ N/m}^2$  Bearing stiffness



Figure 10: Critical speeds of 20 elements shaft with no disc for  $1e6 \text{ N/m}^2$  Bearing stiffness



Figure 11: Critical speeds of 20 elements shaft with disc for  $1e6 \text{ N/m}^2$  Bearing stiffness

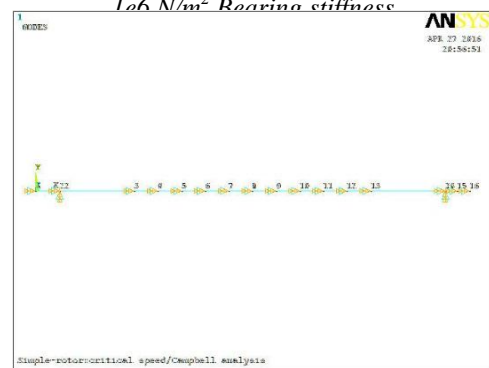


Figure 12: A shaft model having 15 element modelled in ANSYS

Figure 13: A shaft model having 20 element modelled in ANSYS

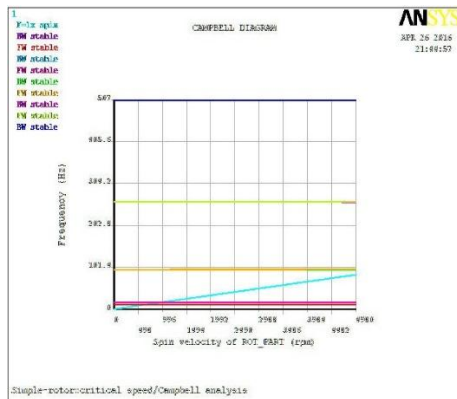


Figure 14: Campbell diagram of 15 element shaft model

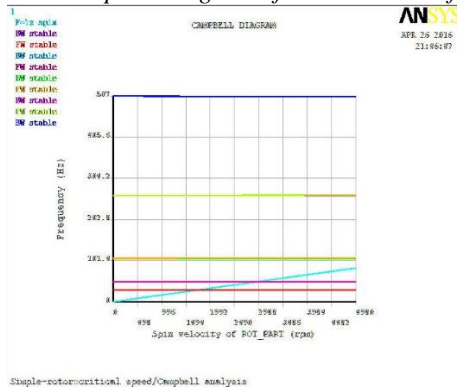


Figure 15: Campbell diagram of 20 element shaft model

SL.NO	Numbers of Elements	Critical Frequency	Ansyz(Hz)	MatLab (Hz)
1	15	1st	8.8756	8.875
2	15	2nd	15.638	13.361

Table 1: Critical frequency comparison of 15 elements shaft with disc (Bearing Stiffness  $1e5 N/m^2$ )

SL.NO	Numbers of Elements	Critical Frequency	Ansyz(Hz)	MatLab (Hz)
1	20	1st	24.026	24.626
2	20	2nd	48.0589	47.0589

Table 2 Critical frequency comparison of 20 elements shaft with disc (Bearing Stiffness  $1e6 N/m^2$ )

From the above frequency tables, Campbell Diagram & comparisons made between Ansys and Matlab it can be found out that by keeping the speed and bearing stiffness constant the critical frequency for a shaft without disc is higher than the shaft with disc. This can be seen from the Fig 8 & 9 for 15 elements and Fig 10 & 11 for 20 elements. From the Campbell diagrams Fig 14 & 15 for a shaft with 15 and 20 elements respectively it is noted that for a shaft with 15 elements the

first and second critical frequency is obtained at 8.8756 Hz & 15.638 Hz in ANSYS software. Likewise, in MATLAB the first and second critical frequency is obtained at 8.875 Hz & 13.361 Hz (Table 1). Now with increase in the number of elements and bearing stiffness to  $1e6$  it is noted that the first and second critical frequency for ANSYS is 24.026 & 48.0589 Hz respectively whereas in MATLAB the first and second critical frequency is obtained at 24.626 and 47.0589.

#### IV. CONCLUSION

Thus the main objective of the paper work to build and to perform rotor dynamics analysis of a rotating shaft of a machine element using ANSYS and MATLAB. It has been shown through simulations and comparisons, the results obtained from Ansys model and Matlab model are in good agreement with each other. The results obtained from analysis may not be entirely concordant with the reality. Extended validations of results are preferable.

Overall, Ansys and Matlab can be used for performing rotor dynamics analysis within the company.

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