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On αrω-LC Continuous Maps In Topological Spaces

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***Abstract:** In this paper, we study some distinct notions of αrω-LC continuous, αrω-LC* continuous, αrω-LC** continuous functions are introduced and we discuss some of their properties.*

***Keywords:** αrω-locally closed sets, αrω-lc-continuous, αrω-lc irresolute, αrω- submaximal space.*

I. INTRODUCTION

Kuratowski and Sierpinski[11] introduced the notion of locally closed sets and locally continuous in topological spaces. According to Bourbaki [6], a subset of a topological space (X, τ) is locally closed in (X, τ) if it is the intersection of an open set and a closed set in (X, τ). Stone[14] has used the term FG for a locally closed subset. Ganster and Reilly[9] have introduced locally closed sets, which are weaker forms of both closed and open sets. After that Balachandran et al [2,3], Gnanambal[10], Arockiarani et al[1], Pusphalatha[12] and Sheik John[13] have introduced α-locally closed, generalized locally closed, semi locally closed, semi generalized locally closed, regular generalized locally closed, strongly locally closed and w- locally closed sets and their continuous maps in topological space respectively. Recently as a generalization of closed sets αrω-closed sets and αrω-continuous maps were introduced and studied by R.S. Wali et[4,5]

II. PRELIMINARIES

Throughout the paper (X,τ), (Y,σ) and (Z,μ) (or simply X,Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), Cl(A), Int(A), αCl(A) and A^c denote the closure of A, the interior of A , the α-closure of A and the compliment of A in X respectively.

We recall the following definitions, which are useful in the sequel.

DEFINITION 2.1

A subset A of topological space (X, τ) is called a

- ✓ locally closed (briefly LC or lc) set [7] if A=U∩F,where U is open and F is closed in X.
- ✓ rw-closed set [13] if Cl(A) ⊆ U whenever A ⊆ U and U is regular semi-open.
- ✓ αrω-closed set [11] if αCl(A) ⊆ U whenever A ⊆ U and U is αrω-open.
- ✓ αg-locally closed set if A=U∩F, where U is αg–open and F is αg–closed in X.
- ✓ α-locally closed set if A=U∩F, where U is α–open and F is α–closed in X.
- ✓ wg-locally closed set if A=U∩F, where U is wg–open and F wg–closed in X.
- ✓ gp-locally closed set if A=U∩F where U is gp–open and F is gp–closed in X.
- ✓ gpr-locally closed set if A=U∩F where U is gpr–open and F gpr-closed in X.
- ✓ g-locally closed set if A=U∩F where U is g-open and F is g-closed in X.
- ✓ rwg-locally closed set if A=U∩F where U is rwg-open and F is rwg-closed in X.
- ✓ gspr-locally closed set if A=U∩F where U is gspr-open and F is gspr-closed in X.

- ✓ ωα-locally closed set if A=U∩F where U is ωα-open and F is ωα-closed in X.
- ✓ αgr-locally closed set if A=U∩F where U is αgr-open and F αgr-closed in X.
- ✓ gs- locally closed set if A=U∩F where U is gs-open and F is gs-closed in X.
- ✓ w-lc set if A=U∩F where U is w-open and F is w-closed in X.
- ✓ gprw-lc set if A=U∩F where U is gprw-open and F is gprw-closed in X.
- ✓ rw-lc set if A=U∩F where U is rw -open and F is rw -closed in X.
- ✓ rgα-lc set if A=U∩F where U is rgα-open and F is rgα-closed in X.
- ✓ αrω-LC set if A=U∩F where U is αrω-open and F is αrω-closed in X.
- ✓ αrω-LC* set if A=U∩F where U is αrω-open and F is closed in X.
- ✓ αrω-LC** set if A=U∩F where U is open and F is αrω-closed in X.

DEFINITION 2.2

A topological space (X, τ) is said to be a

(i) Sub maximal space [7] if every dense subset of (X, τ) is open in (X, τ).
(ii) Door space [8] if every subset of (X, τ) is either open or closed in (X, τ).
(iii) $T_{\alpha r\omega}$-space[4] if every αrω-closed set is closed

DEFINITION 2.3

A map f: (X, τ) → (Y, σ) is called

(i) LC-continuous [9](resp. α-continuous [12], αg-LC-continuous [10]) if $f^{-1}(G)$ is locally closed (resp. α-locally closed , αg-locally closed) set.
(ii) LC-irresolute [9] if $f^{-1}(G)$ is locally closed set in (X,τ) for locally closed set G of (Y,σ).

III. αrω-LC CONTINUOUS FUNCTIONS

In this section, we define αrω-LC continuous maps which is lies between LC-continuous and αgLC-continuous functions and study their relations with existing ones. We also define αrω-LC* continuous maps, αrω-LC** continuous maps.

DEFINITION 4.1

A function f:(X,τ)→(Y,σ) is called αrω-LC continuous (resp. αrω-LC* continuous, αrω-LC** continuous) function if $f^{-1}(G)$∈αrω-LC(X,τ) (resp. $f^{-1}(G)$∈αrω-LC*(X,τ), $f^{-1}(G)$∈αrω-LC**(X,τ)) for each open set G of (Y,σ).

THEOREM 4.2

If f:(X,τ)→(Y,σ) is LC continuous then f is αrω-LC continuous (resp. αrω-LC* continuous and αrω-LC** continuous).

PROOF: Let G be open set in Y. Since f is LC continuous then $f^{-1}(G)$ is locally closed set in X. Every locally closed set is αrω-locally closed set. Therefore $f^{-1}(G)$ is αrω-locally closed set in X. Hence f is αrω-LC continuous.

similarly other proof.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 4.3

Let X = Y = {a, b, c}, τ = {ϕ, {a}, X} and σ ={ϕ, {b}, {c},{b, c},Y}. Then the identity map f:(X,τ)→(Y,σ), f is αrω-LC-continuous, αrωLC*-continuous and αrω-LC**-continuous but not LC-continuous, since for the open set A = b}∈(Y, σ), f^{-1}({b})= {b} ∈ LC(X, τ).

THEOREM 4.4

If f:(X,τ)→(Y,σ) αLC continuous function then αrω-LC continuous.

PROOF: Let G be open set in Y. Since f is α-LC continuous then $f^{-1}(G)$ is α-locally closed set in X. Every α-locally closed set is αrω-locally closed set. Therefore $f^{-1}(G)$ is αrω-locally closed set in X. Hence f is αrω-LC continuous.

Following example shows that converse need not be true.

EXAMPLE 4.5

Let X = {a, b, c, d} = Y, σ = {Y, ϕ, {a},{b},{a, b},{a, b, c}} and τ = {X, ϕ, {a}, { c,d}, {a, c, d}}. Then the identity map f:(X,τ)→(Y,σ) is αrωLC-continuous but not α-lc-continuous, since for the open set {a,b,c} in (Y,σ), f^{-1}({a,b,c}) = {a,b,c} is not αlc- set in (X, τ).

THEOREM 4.6

If f:(X,τ)→(Y,σ) αrω-LC continuous function then αg-LC continuous.

PROOF: Let G be open set in Y. Since f is αrω-LC continuous then $f^{-1}(G)$ is αrω-locally closed set in X. Every αrω-locally closed set is αg-locally closed set. Therefore $f^{-1}(G)$ is αg-locally closed set in X. Hence f is αg-LC continuous.

Following example shows that converse need not be true.

EXAMPLE 4.7

Let X = {a, b, c} = Y, τ = {X, ϕ, {a},{b,c}} and σ = {Y, ϕ, {b}, {c}, {b, c}. Then the identity map f: (X, τ)→(Y,σ) is αg-LC-continuous but not αrω-lc-continuous, since for the open set {b} in (Y,σ), f^{-1}({b})={b} is not αrω-lc-set in (X, τ).

THEOREM 4.8

If f: (X, τ)→(Y, σ) is αrω-LC*-continuous (resp αrω-LC**-continuous) then f is αrω-LC -continuous.

PROOF: Let U be open in Y and f: (X, τ) → (Y, σ) is αrω-LC*-continuous or αrω-LC**-continuous. f^{-1}(U) αrω-LC* set (resp. αrω-LC** set) in X s By Every αrω-LC* set (resp. αrω-LC** set) is αrω-LC set. Therefore f is αrωLC - continuous.

The converse of the above theorem need not be true as seen from the following examples.

EXAMPLE 4.9

Let X = {a, b, c} = Y, τ = {ϕ, {a}, X} and σ = {ϕ, {a},{b}, {a,b}, Y}. Let f: (X, τ) → (Y, σ) be the identity map. Then f is αrω-LC-continuous but not αrωLC*-continuous and not αrωLC**-continuous. For the open set A = {a,b} ∈ (Y, σ), f^{-1}({a, b}) = {a,b} ∉ αrωLC*(X, τ) and {a, b} ∉ αrωLC**(X, τ).

REMARK 4.10

Composition of two αrω-LC-continuous (resp. αrω-LC*-continuous, αrω-LC**-continuous) maps need not be αrω-LC-continuous (resp. αrω-LC*-continuous, αrω-LC**-continuous) as seen from the following example.

EXAMPLE 4.11

Let X = Y = Z = {a, b, c}, τ = {ϕ,{a},{b, c}, X} and σ = {ϕ ,{a},Y}, μ = {ϕ,{b},{c},{b,c}, Z}. Define a map f:(X, τ)→(Y,σ) and g:(Y,σ)→(Z,μ) are the identity map. Then both f and g are αrω-LC -continuous (αrω-LC*-continuous, αrω-LC**-continuous) but the composition gof:(X, τ)→(Z, μ) is not αrω-LC-continuous (resp. αrωLC*-continuous, αrω-LC**-continuous), since for the open set A = {b} in (Z, μ), $(gof)^{-1}$({b}) = f^{-1}(g^{-1}{b}) = f^{-1}{b} = {b}∉αrω-LC(X, τ) (resp. {b}∉ αrω-LC*(X, τ), {b} ∉ αrω-LC**(X, τ)).

THEOREM 4.12

If f: (X, τ) → (Y,σ) be αrω-LC-continuous (resp. αrω-LC*-continuous, αrω-LC**-continuous) maps and g: (Y,σ) → (Z,μ) be continuous then gof:(X, τ)→(Z, μ) is αrω-LC-continuous (resp. αrω-LC*-continuous, αrω-LC**-continuous) maps.

PROOF: Let G be open set in Z , $(gof)^{-1}$(G)= f^{-1}(g^{-1}(G)) is αrω-LC closed set since g: (Y,σ) → (Z,μ) be continuous, g^{-1}(G) be open set in Y and also since f: (X,τ)→(Y,σ) be αrω-LC-continuous (resp. αrω-LC*-continuous, αrω-LC**-continuous) maps, f^{-1}(g^{-1}(G))∈αrω-LC(X,τ)(resp. f^{-1}(g^{-1}(G))∈αrωLC*(X,τ) and f^{-1}(g^{-1}(G))∈αrωLC**(X,τ)). Therefore gof:(X, τ)→(Z,μ) is αrω-LC-continuous (resp. αrω-LC*-continuous, αrω-LC**-continuous) maps.

IV. αrω-LC IRRESOLUTE FUNCTIONS

In this section, we define αrω-LC irresolute maps, αrω-LC* irresolute maps, αrω-LC** irresolute maps and study some of their properties.

DEFINITION 5.1

A function f:(X,τ)→(Y,σ) is called αrω-LC irresolute (resp. αrω-LC* irresolute, αrω-LC** irresolute) function if f^{-1}(G)∈αrω-LC(X,τ) (resp. f^{-1}(G)∈ αrω-LC*(X, τ), f^{-1}(G) ∈ αrω-LC**(X, τ)) for each G∈αrω-LC(Y, σ) (resp. G∈αrω-LC*(Y, σ) , G∈αrω-LC**(Y, σ)).

THEOREM 5.2

If f:(X,τ)→(Y,σ) is αrω-irresolute then f is αrω-LC irresolute.

Proof: Let f is αrω-irresolute and V∈αrω-LC(Y,σ) . Then V=U∩G for some αrω-open set U and some αrω-closed set G in (Y,σ). we have f^{-1}(V)= f^{-1}(U∩G)= f^{-1}(U)∩ f^{-1}(G) ,where f^{-1}(U) is αrω-open and f^{-1}(G) is αrω-closed set in (X, τ), since f is αrω-irresolute. This shows that f^{-1}(V) is αrω-locally closed set in X. Hence f is αrω-LC irresolute.

THEOREM 5.3

Let f:(X,τ)→(Y,σ) be function

(i) If f is αrω-LC-irresolute then f is αrω-LC continuous.
(ii) If f is αrω-LC* irresolute then f is αrω-LC* continuous
(iii) If f is αrω-LC** irresolute) then f is αrω-LC** continuous

PROOF: (i) Let G be open set in Y and also G be αrω-locally closed set in Y, Since f is αrω-LC-irresolute then f^{-1}(G) is αrω-locally closed set in X. Hence f is αrω-LC continuous.

Similarly (ii) and (iii)

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 5.4

Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b,c}, X} and σ ={ϕ, {a}, Y}. Then the identity map f: (X, τ) → (Y, σ), f is αrω-LC-continuous, αrω-LC*-continuous and αrω-LC**-continuous but not αrω-LC-irresolute (resp. αrω-LC* irresolute, αrω-LC** irresolute), since for the open set A={b}∈αrω-LC(Y,σ), f^{-1}({b})={b}∉αrω-LC(X,τ)(resp. f^{-1}({b})∉αrω-LC*(X,τ) and f^{-1}({b})∉αrω-LC**(X,τ)) .

THEOREM 5.5

Any map defined on a door space is αrω-LC continuous (resp αrω-LC irresolute).

Proof: Let f: (X,τ)→ (Y,σ) be a map, where (X, τ) be a door-space and (Y,σ) be any topological space. Let A ∈ σ (resp, A ∈αrω-LC(Y,σ)). Then by the assumption on (X,τ), f^{-1}(A) is either open or closed. In both cases f^{-1}(A)∈αrω-LC(X,τ) and therefore f is αrω-LC continuous (resp. αrω-LC irresolute).

THEOREM 5.6

Let f: (X, τ) → (Y,σ) and g: (Y,σ) → (Z,μ) be any two functions.

(i) If f is αrω-LC-irresolute and g is is αrω-LC-continuous then gof :(X, τ)→ (Z, μ) is αrω-LC-continuous.
(ii) If f is αrω-LC*-irresolute and g is is αrω-LC*-continuous then gof :(X, τ)→ (Z, μ) is αrω-LC*-continuous.
(iii) If f is αrω-LC**-irresolute and g is is αrω-LC**-continuous then gof :(X, τ)→ (Z, μ) is αrω-LC**-continuous.

PROOF: (i) Let U∈(Z,μ), since g is is αrω-LC-continuous, $g^{-1}(U)$∈αrω-LC(Y,σ). Then $f^{-1}(g^{-1}(U))$∈αrω-LC(X,τ) since f is αrω-LC-irresolute. So $f^{-1}(g^{-1}(U))=(gof)^{-1}(U)$∈αrω-LC(X,τ). Hence gof is αrω-LC-continuous.

(ii) and (iii) are similar to (i).

THEOREM 5.7

Let f: (X, τ) → (Y,σ) and g: (Y,σ) → (Z,μ) be any two functions

(i) If f is αrω-LC-irresolute and g is is LC-continuous then gof :(X, τ)→(Z,μ) is αrω-LC-continuous.
(ii) If f is αrω-LC-irresolute and g is is αrω-continuous then gof :(X, τ)→(Z,μ) is αrω-LC-continuous.

PROOF:

(i) (i) Let U∈(Z,μ), since g is is LC-continuous, $g^{-1}(U)$∈LC(Y,σ) and $g^{-1}(U)$ ∈ αrω-LC(Y,σ). Then $f^{-1}(g^{-1}(U))$∈αrω-LC(X,τ) since f is αrω-LC-irresolute. So $f^{-1}(g^{-1}(U))=(gof)^{-1}(U)$∈ αrω-LC(X,τ). Hence gof is αrω-LC-continuous.
(ii) Let U∈(Z,μ), since g is is αrω-continuous, $g^{-1}(U)$∈αrω-O(Y,σ), every αrω-open set is αrω-lc closed set and $g^{-1}(U)$∈αrω-LC(Y,σ). Then $f^{-1}(g^{-1}(U))$∈αrω-LC(X,τ) since f is αrω-LC-irresolute. So $f^{-1}(g^{-1}(U))=(gof)^{-1}(U)$∈αrω-LC(X,τ). Hence gof is αrω-LC-continuous.

THEOREM 5.8

Let f: (X, τ) → (Y,σ) and g: (Y,σ) → (Z,μ) be any two functions.

(i) If f and g are αrω-LC-irresolute then gof :(X, τ)→ (Z, μ) is αrω-LC- irresolute.
(ii) If f and g are αrω-LC*-irresolute then gof :(X, τ)→ (Z, μ) is αrω-LC*- irresolute.
(iii) If f and g are αrω-LC**-irresolute then gof :(X, τ)→ (Z, μ) is αrω-LC**- irresolute.

PROOF: (i) Let U∈αrω-LC(Z,μ), since g is is αrω-irresolute s, $g^{-1}(U)$∈αrω-LC(Y,σ). Then $f^{-1}(g^{-1}(U))$∈αrω-LC(X,τ) since f is αrω-LC-irresolute. So $f^{-1}(g^{-1}(U))=(gof)^{-1}(U)$∈ αrω-LC(X, τ). Hence gof is αrω-LC-irresolute.

(ii) and (iii) are similar to (i).

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