International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 5 Issue 12, December 2018

ISSN: 2394-4404

Efficiency Stability Of Total Manufacturing Sectors Of Indian States And Union Territories – A Dea Study Based On Free Disposable Hull

E. Ravi

Research Scholar, Department of Statistics, S.V. University, Tirupati, A.P., India

C. Subbarami Reddy

(Rtd) Professor, Department of Statistics, S.V. University, Tirupati, A.P., India

Abstract: *The contribution to GDP by agriculture is found steadily declining in Indian context.. Rapid industrialization is recognized necessary to compensate this decline.*

This study aims at identifying efficient total manufacturing sectors of Indian States and Union Territories. It also enquires the ability of efficient total manufacturing sectors to sustain their efficiency status under input expansion and output contraction, the expansion and contraction being additive. The total manufacturing sectors of Union Territories, on the average, have greater ability to preserve efficiency classification, than those of Indian States. This study made use of data published by Annual Survey of Industries (ASI, 2015). The Mathematical tool that has been implemented to accomplish the purpose is Data Envelopment Analysis based on a Non – Convex Production Possibility set, called Free Disposable Hull (FDH). The input and output thresholds that determine efficiency stability bounds are derived as closed form solutions of 0-1 integer linear programming problem.

Keywords: *Data Envelopment Analysis, Free Disposable Hull, Additive Data Variation, Efficiency Stability.*

I. INTRODUCTION

For efficiency measurement one needs a frontier and a distance function. Frontier is envelopment surface of a production possibility set. Popular are convex and non-convex production possibility sets. The envelopment surface is smooth or piecewise linear, depending upon how the production possibility set is structured. A distance function determines the path along which an inefficient production plan shall traverse to reach the frontier of the production possibility set. Popular are radial and non-radial distance functions.

Radial distance functions were put into practise by Farrell (1957), but provided a theoretical premise by Shephard (1970) whose distance function is inversely related to Farrell's distance function. Farrell's input and output technical efficiency measures were popularized by Charnes, Cooper and Rhodes (1978), and Banker, Charnes and Cooper (1984). Fare et.al (1985) popularised non-radial measures built on Russell's axioms. While the radial measures seek radial input reduction to attain input technical efficiency, radial output expansion to attain output technical efficiency, non-radial measures seek input specific contraction and/or output specific expansion to attain non-radial efficiency. Radial measures specify Pareto-Koopman's efficiency, if and only if, the efficiency score is unity and all input and out slacks vanish. But, for non-radial measures (Russell), unit efficiency score always signals Pareto-Koopman's efficiency.

An important class of distance functions, called directional distance functions were introduced by Chambers et.al (1996). These are flexible distance functions for which input reduction and output expansion take place along a path specified by input and output directional vectors respectively. Efficiency scores based on directional distance functions are sensitive to the directional vectors choosen for input contraction and output expansion (Chung et.al, 1977; Fare et.al, 1985; Fare et.al 2013; Dario and Simar, 2014).

II. RADIUS OF STABILITY

Cooper et.al (2001) formulated a linear programming problem to find the radius of stability, for an efficient decision making unit, of classification, which is always feasible.

$$\delta_{j_0} = Min\,\delta$$

subject to $\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j x_{ij} + s_i^- = x_{ij_0} + \delta,\ i \in M$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j y_{rj} - s_r^+ = y_{rj_0} - \delta,\ r \in S \quad ......\ (2.1)$$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0,\ j \in N$$

$$\delta \geq 0$$

δ_{j_0} is the radius of stability of efficiency classification for DMU_{j_0} that is extremely efficient. DMU_{j_0} retains its efficient status under input expansion from x_{j_0} to $x_{j_0} + \delta_{j_0}$ and output contraction from y_{j_0} to $y_{j_0} - \delta_{j_0}$.

(2.1) was originally formulated by Charnes, Haag, Jaska and Semple (1992), in CCR frame work without input and output slacks.

III. EFFICIENCY CLASSIFICATION STABILITY – ADDITIVE VARIATION OF INPUTS AND OUTPUTS

Seiford and Zhu (1999) proposed a variation of directional distance function to examine efficiency stability under simultaneous variation of inputs and outputs. Their DEA model and the Super Efficiency model introduced by S Ray (2004) are closely related. In terms of arbitrary input and output directional vectors for an efficient test DMU_{j_0}, the directional super efficiency problem may be postulated as follows:

$$\beta_{j_0} = Max\,\beta$$

such that $\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j x_{ij} \leq x_{ij_0} - \beta g_{x_i},\ i \in M \quad ......\ (3.1)$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j y_{rj} \geq y_{rj_0} + \beta g_{y_r},\ r \in S$$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0,\ j \in N,\ j \neq j_0$$

Chung et.al (1997) recommended the input and output vectors of DMU_{j_0} as the directional vectors. Replacing,

g_{x_i} with x_{ij_0}, $i \in M$

g_{y_r} with y_{rj_0}, $r \in S$

in (3.1) the following Super Efficiency problem postulated by S Ray can be obtained

$$\beta_{j_0} = Max\,\beta$$

such that $\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j x_{ij} \leq (1-\beta) x_{ij_0},\ i \in M$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j y_{rj} \geq (1+\beta) y_{rj_0},\ r \in S \quad ......\ (3.2)$$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j = 1$$

$$j \in N,\ j \neq j_0$$

Since β_{j_0} arises to be negative, one can set, $-\beta = \theta$ and (3.2) can be replaced by the following:

$$\theta_{j_0} = Min\,\theta$$

such that $\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j x_{ij} \leq (1+\theta) x_{ij_0},\ i \in M$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j y_{rj} \geq (1-\theta) y_{rj_0},\ r \in S \quad ......\ (3.3)$$

$$\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0,\ j \in N,\ j \neq j_0$$

The Data Envelopment Analysis (DEA) model used by Seiford and Zhu for assessing efficiency stability is (3.3), in which the convexity constraint was excluded. The efficient test DMU_{j_0} remains to be efficient under input expansion from x_{j_0} to $(1+\theta)x_{j_0}$ and output contraction from y_{j_0} to $(1-\theta)y_{j_0}$. But Seiford and Zhu examined efficiency stability varying a subset of inputs and a subset of outputs. Zhu (2001) extended the efficiency stability models of Seiford and Zhu (1998, 1999) under input specific expansion for a subset of inputs and output specific contraction for a subset of outputs. In Seiford and Zhu (1999) frame work (3.3) may be expressed as,

$$\theta_{j_0} = Min\,\theta$$

such that $\sum_{\substack{j\neq j_0\\ j=1}}^{n} \lambda_j x_{ij} \leq (1+\theta) x_{ij_0},\ i \in I \quad ......(3.4)$

$$\sum_{\substack{j=1\\ j\neq j_0}}^{n} \lambda_j x_{ij} \le x_{ij_0},\ i\in I^C$$

$$\sum_{\substack{j=1\\ j\neq j_0}}^{n} \lambda_j y_{rj} \ge (1-\theta)\, y_{rj_0},\ r\in O$$

$$\sum_{\substack{j=1\\ j\neq j_0}}^{n} \lambda_j y_{rj} \ge y_{rj_0},\ r\in O^C$$

$$\sum_{\substack{j=1\\ j\neq j_0}}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0,\ j\in N,\ j\neq j_0$$

IV. FREE DISPOSABLE HULL (FDH)

Banker, Charnes and Cooper (1984) introduced radial distance functions in association with convex production possibility set. BCC production possibility set is based on the postulates: inclusion, free disposability, convexity and minimum extrapolation. The DEA input technical efficiency problem postulated in BCC frame work may be expressed as follows:

$$\lambda_{j_0} = Min\,\lambda$$

that $\sum_{j=1}^{n} \lambda_j x_{ij} \le \lambda x_{ij_0},\ i\in M$ ...... (4.1)

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rj_0},\ r\in S$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0,\ j\in N$$

The DEA output technical efficiency problem postulated by BCC may be expressed as,

$$\theta_{j_0} = Max\,\theta$$

such that $\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ij_0},\ i\in M$ ...... (4.2)

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \theta y_{rj_0},\ r\in S$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0,\ j\in N$$

Deprins et.al (1984) relaxed the assumption of convexity, but maintained the postulates of inclusion, free disposability and minimum extrapolation. For each production plan $(x_j, y_j),\ j\in N$ an orthant is recognized as follows:

$$\Gamma(x_j, y_j) = \{(x,y): x\ge x_j,\ y\le y_j\},\ j\in N$$

The non-convex production possibility set, that Briec et.al (2004) introduced called as Free Disposable Hull (FDH) can be viewed as union of $\Gamma(x_j, y_j),\ j\in N$

$$T_{FDH} = \bigcup_{j=1}^{n} \Gamma(x_j, y_j)$$

Efficiency measurement can be extended from convex to non-convex production possibility set by constraining the intensity parameters of BCC formulation to be bivalent,

i.e $\lambda_j \in \{0,1\},\ \forall j\in N$

On FDH the input/output technical efficiency problems are Integer Linear Programming problems for which the unknown intensity parameters attain one of the two values 0 or 1. Tulkens (1993) proposed a simple enumeration method to evaluate FDH input/output technical efficiencies, instead of solving Integer Linear programming problems. Tulkens' enumeration method can be further refined, involving a subset of efficient production plans in the enumeration process (Burdhan et.al 1996), (3.3), intended to find input and output threasholds to examine efficiency classification preservation by efficient decision making units, can be extended from convex to non-convex (FDH) production possibility set, and the threasholds expressed as 0-1 Integer Linear programming problems can be replaced by closed form mathematical expressions

For any efficient production plan (x_{j_p}, y_{j_p}) for which $\lambda_{j_p} = 1$, under optimum

$$\left.\begin{aligned} (x_{ij_p} - \delta x_{ij_p}) &\le x_{ij_0} + \delta x_{ij_0},\ i\in M \\ (y_{rj_p} + \delta y_{rj_p}) &\ge y_{rj_0} - \delta y_{rj_0},\ r\in S \end{aligned}\right\} \text{...... (4.3)}$$

It is perceived that the efficient test DMU_{j_0} experiences input expansion and output contraction, while its rivals (efficient) strive to reduce inputs and expand outputs.

$$(x_{ij_0} + x_{ij_p})\delta \ge x_{ij_p} - x_{ij_0},\ i\in M$$

$$\delta \ge \left(\frac{x_{ij_p} - x_{ij_0}}{x_{ij_0} + x_{ij_p}}\right),\ i\in M \text{ ..... (4.4)}$$

$$(y_{rj_p} + y_{rj_0})\delta \ge y_{rj_0} - y_{rj_p},\ r\in S$$

$$\delta \ge \left(\frac{y_{rj_0} - y_{rj_p}}{y_{rj_p} + y_{rj_0}}\right),\ r\in S \text{ .... (4.5)}$$

Combining (4.4) and (4.5),

$$\delta \ge \underset{i,r}{Max}\left\{\frac{x_{ij_p} - x_{ij_0}}{x_{ij_0} + x_{ij_p}}, \frac{y_{rj_0} - y_{rj_p}}{y_{rj_p} + y_{rj_0}}\right\}$$

$$\delta^{FDH}\left(j_0\right)=\underset{j_p\in D_0}{Min}\ \underset{i,r}{Max}\left\{\frac{x_{ij_p}-x_{ij_0}}{x_{ij_0}+x_{ij_p}},\frac{y_{rj_0}-y_{rj_p}}{y_{rj_p}+y_{rj_0}}\right\}$$

where

$$D_0=\left\{j_p : j_p\neq j_{0i}\left(x_{j_p},y_{j_p}\right) is\ efficient\ \ production\ \ plan\right\}$$

For all η and T such that

$1\leq\eta\leq 1+\delta^{FDH}\left(j_0\right)$

and $1-\delta_{FDH}\left(j_0\right)\leq T\leq 1$

DMU_{j_0} preserves its efficiency classification. That is, under input expansion from x_{j_0} to $\left(1+\delta^{FDH}\left(j_0\right)\right)x_{j_0}$ and simultaneous output contraction from y_{j_0} to $\left(1-\delta^{FDH}\left(j_0\right)\right)y_{j_0}$ while its (efficient) rivals contract inputs from x_{j_p} to $\left(1-\delta^{FDH}\left(j_0\right)\right)x_{jp}$ and expand outputs from y_{j_p} to $\left(1+\delta^{FDH}\left(j_0\right)\right)y_{j_p}$, DMU_{j_0} remains to be efficient.

The empirical study is to examine classification stability of efficient total manufacturing sectors of Indian States and Union Territories. Annual Survey of Industries (2015) published industrial data related to 28 Indian States and 6 Union Territories. Two DEA inputs and one output are selected for this study

- ✓ Fixed Capital (x_1),
- ✓ Total Persons Engaged in Production (x_2)

DEA output: Net value Added

S. No.	Total Manufacturing Sector of	δ_{FDH}	$1-\delta_{FDH}$	$1+\delta_{FDH}$
1	Maharashtra	0.1298	0.8702	1.1298
2	Gujarat	0.1260	0.8740	1.1260
3	Tamil Nadu	0.2074	0.7926	1.2074
4	Karnataka	0.0775	0.9225	1.0775
5	Haryana	0.0794	0.9206	1.0794
6	Uttar Pradesh	-	--	--
7	Uttarakhand	0.1458	0.8542	1.1458
8	Rajasthan	--	--	--
9	Himachal Pradesh	0.2163	0.7837	1.2163
10	Telangana	---	--	--
11	Andhra Pradesh	---	--	--
12	Madhya Pradesh	---	--	--
13	Jharkhand	0.0683	0.9317	1.0683
14	Punjab	0.1074	0.8926	1.1074
15	West Bengal	--	--	--
16	Chattisgarh	0.0759	0.9241	1.0759
17	Odisha	--	--	--
18	Goa	0.4112	0.5888	1.4112
19	Dadra & Nagar Haveli	0.2188	0.7812	1.2188
20	Kerala	--	--	--
21	Assam	--	--	--
22	Delhi	0.1643	0.8357	1.1643
23	Bihar	--	--	--
24	Daman & Diu	0.0481	0.9519	1.0481
25	Jammu & Kashmir	0.0792	0.9208	1.0792
26	Sikkim	0.6525	0.3475	1.6525
27	Puducherry	--	--	--
28	Chandigarh	0.3152	0.6848	1.3152
29	Meghalaya	--	--	--
30	Tripura	0.1627	0.8373	1.1627
31	Arunachal Pradesh	0.6233	0.3767	1.6233
32	Nagaland	--	--	--
33	Manipur	0.2554	0.7446	1.6233
34	Andaman & N. Island	0.7851	0.2149	1.7851

Table 1

The total manufacturing sector of Andaman and Nicobar Islands, under simultaneous absolute input expansion and output contraction, when its rival DMUs resort to absolute input contraction and output expansion in a simultaneous fashion, has the ability to remain efficient upto a threashold input increase $1.7851\,x_{j_0}$ and output decrease $0.2149\,y_{j_0}$, while its rival DMUs contract inputs upto $0.2149\,x_{j_p}$ and expand outputs upto $1.7851\,y_{j_p}$, where $j_p\in D_0$.

For all $x_j\in\left[x_{j_0},1.6525x_{j_0}\right]$ and $y_j\in\left[0.3475y_{j_0},y_{j_0}\right]$ the total manufacturing sector of Sikkim is efficient, and for its rival DMUs it happens that $x_j\in\left[0.3475x_{j_p},x_{j_p}\right]$ and $y_j\in\left[y_{j_p},1.6525y_{j_p}\right]$, where $j_p\in D_0$.

The TMS of Arunachal Pradesh is efficient and it preserves efficiency classification if its input expansion does not exceed 1.6233 x_{j_0} and output contraction can not go beyond $0.3767\,y_{j_0}$ simultaneously, while for $j_p\in D_0$ input contraction is upto $0.3767\,x_{j_p}$ and output expansion is upto $1.6233\,y_{j_p}$.

For all inputs not exceeding $1.4112\,x_{j_0}$ and outputs not smaller than $0.5888\,y_{j_0}$, for rival DMUs all inputs not smaller than $0.5888\,x_{j_p}$ and outputs not greater than $1.4112\,y_{j_p}$ the total manufacturing sector of Goa preserves its efficiency classification.

To preserve its efficiency classification the total manufacturing sector of Chandigarh should control its input application not to exceed $1.3152\,x_{j_0}$ and output production not to be smaller than $0.6848\,y_{j_0}$, while its rival total manufacturing sectors strive to contract their inputs upto $0.6848\,x_{j_p}$ and expand outputs upto $1.3152\,y_{j_p}$, where $j_p\in D_0$.

The TMS of Manipur under absolute input expansion and output contraction remains to be efficient if its input application does not exceed $1.2554\,x_{j_0}$ and output production does not fall below 0.7446 y_{j_0}, while its rivals states are expected to contract inputs upto 0.7446 x_{j_p} and expand outputs upto 1.3152 y_{j_p}, $j_p \in D_0$.

The total manufacturing sectors of Dadra & Nagar Haveli and Himachal Pradesh appear to exhibit the same ability to preserve efficiency classification while they experience absolute input expansion and output contraction. To retain their efficient status these should control input application not to exceed 1.22 x_{j_0} and output contraction not to fall below 0.78 y_{j_0}, while for all $j_p \in D_0$ DMU_{j_p} are expected to expand their output upto 1.22 y_{j_p} and contract inputs upto 0.78 x_{j_p}.

Tamilnadu is an efficient state in Industrial manufacturing production. For all inputs not exceeding 1.2074 x_{j_0} and outputs not falling below 0.7926 y_{j_0}, while rivals compete to expand outputs upto 1.2074 y_{j_p} and inputs not fall below 0.7926 x_{j_p}, $j_p \in D_0$ the TMS of Tamilnadu remains to be efficient.

For all inputs expansion upto 1.16 x_{j_0} and outputs not falling below 0.84 y_{j_0} and rival DMUs inputs not falling below 0.84 x_{j_p} and outputs not exceeding 1.16 y_{j_0}, the total manufactrung sectors of Delhi and Tripura preserve their efficiency classification .

Uttarakhund has the ability to remain efficient under input expansion upto 1.1458 x_{j_0} and output contraction not falling below 0.8542 y_{j_0} while for every $j_p \in D_0$ input contraction is upto 0.8542 x_{j_p} and output expansion not to exceed 1.1458 y_{j_p}.

The total manufacturing sectors of Maharastra and Gujarat preserve efficiency classification if their input expansion does not exceed 1.13 x_{j_0} and output contraction does not fall below 0.874 y_{j_0} while their rivals are expected to contract inputs upto 0.8542 x_{j_p} and expand outputs not to exceed 1.13 y_{j_p}, $j_p \in D_0$.

The TMS of Punjab has the ability to remain efficient under input expansion upto 1.1074 x_{j_0} and output contraction leading to output not falling below 0.8926 y_{j_0} while its rivals strive to expand outputs upto 1.1074 y_{j_p} and contract inputs not falling below 0.8926 x_{j_p}, where $j_p \in D_0$.

For all outputs falling not below 0.92 y_{j_0} and inputs not exceeding 1.08 x_{j_0} and rival DMUs input contraction is upto 0.92 x_{j_p} and output expansion is upto 1.08 y_{j_p}, $j_p \in D_0$ the total manufacturing sectors of Karnataka, Haryana, Chattisgarh and Jammu & Kashmir remain to be efficient.

Jharkhand is an efficient total manufacturing sector. It can preserve its efficiency classification under input expansion upto 1.6083 x_{j_0} and output contraction upto 0.9317 y_{j_0} while the rival DMUs contract their inputs upto 0.9317 x_{j_p} and expand outputs upto 1.0683 y_{j_p}, $j_p \in D_0$.

The TMS of Daman & Diu remain efficient for all inputs not exceeding 1.0481 x_{j_0} and outputs not falling below 0.9519 y_{j_0}, while the rival DMUs are expected to contract their inputs upto 0.9519 x_{j_p} and expand outputs upto 1.0481 y_{j_p}, $j_p \in D_0$.

V. CONCLUSIONS

- ✓ 21 out of 34 total manufacturing sectors of Indian States and Union territories are efficient.
- ✓ 17 out of 28 total manufacturing sectors of Indian States are efficient (61%)
- ✓ 4 out of 6 total manufacturing sectors of Union territories are efficient (67%)
- ✓ Remarkable ability to preserve their efficiency classification goes with Andaman & Nicobar Islands $(UT, \delta^{FDH} = 0.7851)$, Sikkim $(\delta^{FDH} = 0.6525)$ and Arunachal Pradesh $(\delta^{FDH} = 0.6233)$
- ✓ The ability to remain efficient under input expansion and output contraction of Indian Union territories $(Mean\ \delta^{FDH} = 0.3282)$ is greater than such ability of Indian States $(Mean\ \delta^{FHD} = 0.2167)$
- ✓ The difference of the above stated means is statistically significantly different from zero, at 1 percent level of significance.

REFERENCES

[1] Bardhan I, Bowlin, WF, Cooper WW, Sueyoshi T, (1996), “Models and Measures for efficiency dominance in DEA, Part II: for disposable hull (FDH) and Russell measure (RM) approaches”, Journal of Operations Research Society, Japan, Vol.39, 333-345.

[2] Cazals, Florens and Simar (2002), “Non-Parametric frontier estimation: A robust approach”, Journal of Econometrics, 106, 1-25.

[3] Chung, Fare and Chambers (1997), "The Productivity and Undesirable outputs: A Directional Distance Function Approach", Journal of Environmental Management, vol. 51, 3, 229-240.

[4] Cooper, Seiford, Jone, Thrall and Zhu (2001) “Sensitivity and Stability Analysis in DEA: Some recent developments; journal of productivity Analysis, 15(3), pp. 217-246. Dario and Simar (2014), "Directional Distance and their Robust versions, computational and testing Issues", Technical Report.
[5] Dario and Simar (2014), “Efficiency and Bench marking with directional distances: A data driven approach", working paper.
[6] De Borger, Ferrier and Kirstens (1998), “The choice of technical efficiency measure on the free disposable hull reference technology: A comparison using US banking data”, European Journal of Operational Research, 105, 427-446.
[7] Deprins, D. L. Simar, and H. Tulkens (1984). "Measuring Labor Efficiency in Post Offices," In M. Marchand, P. Pestieau, and H. Tulkens (eds.)", The Performance of Public Enterprises: Concepts and Measurement". Amsterdam: North-Holland, pp. 243-267.
[8] Solow, R.M., (1957), "Technical Change and Aggregate Production Function", 39, 312-320.