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# Numerical Solution Of The Flow Past A Circular Cylinder

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***Abstract:*** ***Numerical solutions have been obtained for steady viscous flow past a circular cylinder at different Reynolds numbers. Polar-region is converted to rectangular region by using a transformation. Governing equations and boundary conditions are approximated by second order derivatives and a new fast accurate algorithm is employed to solve system of simultaneous linear equations. Steady results upto moderately large Reynolds numbers are reported and results obtained compare well with those of previous authors are much accurate.***

***Keywords: Numerical solutions, Reynolds numbers, Algorithm***



## I. INTRODUCTION

The problem of viscous incompressible flow past a circular cylinder has for a long time received much attention, both theoretically and numerically. There are many reasons for the continuing interest in this problem and in attempts to carry numerical calculations to still higher Reynolds numbers. One of these reasons is that it is a good model problem for flows past other bodies of practical importance. Steady solutions for the circular cylinder become experimentally unstable around Re=40. Many difficulties are encountered in attempts to describe analytically the complete flow field. We believe that numerical methods can provide further information on the limiting properties of the steady flow for increasing Reynolds numbers.

Brodetsky [1] suggests a solution for infinite Reynolds number in which vortex sheets bounds an infinite wake region containing stagnant flow. This solution is often referred to as the Helmholtz – Kirchoff free streamline model because of their introduction of vortex sheets. Both an infinite wake and infinite drag is in agreement with experimentally and numerically observed trends for low Reynolds numbers. Batchelor [2] gives however arguments against these features of the limit and suggests that the vorticity inside the wake in the limit need not be zero but can take a constant value on each side of the line of symmetry. Solutions of this kind (if they exist) would allow for a finite wake and therefore no drag on the body.

Numerical results for the steady flow past a rotating cylinder have been obtained by Ingham [3] and he has dealt in detail effects of using different boundary conditions at large distances from cylinder. Numerical studies for different Reynolds numbers past a circular cylinder have been made by Fornberg [4] and effects of different boundary conditions analysed. H.M.Badr et al [5] have obtained steady results up to reynolds number 20 for a flow past a rotating circular cylinder. Tang and Ingham [6] increased the range of Reynolds number to 60 and 100 for a flow past a rotating circular cylinder.

Herein, first the polar-region is converted to rectangular region by using a transformation. Governing equations and boundary conditions are approximated by second order derivatives and the algorithm applied to solve system of simultaneous linear equations. Steady results upto moderately large Reynolds numbers are obtained. Results obtained compare well with those of previous authors.

## II. THE PROBLEM

In terms of stream function $\psi$ and Vorticity $\omega$, satisfying

$u= \partial\Psi/\partial y,\ \ v = -\partial\Psi/\partial x$ ……………….….(1)

$\omega = (\partial v/\partial x) - (\partial u/\partial y)$ …………………………...(2)

The Navier-Stokes equations can be formulated as

$\Delta\Psi + \omega = 0$ ………………………………………..(3)

$$\Delta\omega + \frac{Re}{2}\left\{\frac{\partial\Psi}{\partial x}\frac{\partial\omega}{\partial y} - \frac{\partial\Psi}{\partial y}\frac{\partial\omega}{\partial x}\right\} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$Where..\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

and the Reynolds number Re, based on diameter d, is Re= Ud/γ.

The quantity U is the free stream velocity and γ is the Kinematic coefficient of viscosity. We will from now on assume that U=(1/2)d =1.

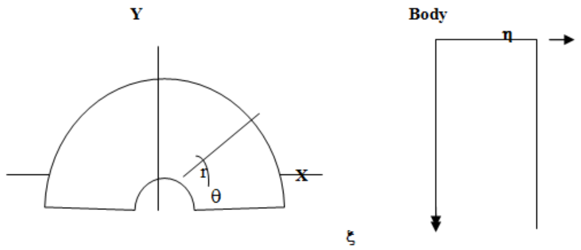


*Figure 1: The x,y and ξ,η Plane*

It is convenient to work both numerically and theoretically with the deviation from uniform flow.

ψ(x,y) = Ψ(x,y)-y ……………………………….(5)

instead of with Ψ.

A polar co-ordinate system can be introduced by the conformal transformation.

ξ+iη = (1/π)Iη(x+iy)……………………………(6)

(The variables ξ and ηare connected to the usual polar co-ordinates r and θ by r $=e^{\pi\xi}$ and θ = πη). The Navier-Stokes equations take in these co-ordinates the form

$\Delta\psi = -\pi^2 r^2\omega$ ……………………………………(7)

$$\Delta\omega = \frac{Re}{2}\left\{\pi r\left(\cos\theta\frac{\partial\omega}{\partial\xi} - \sin\theta\frac{\partial\omega}{\partial\eta}\right) - \frac{\partial\psi}{\partial\xi}\frac{\partial\omega}{\partial\eta} + \frac{\partial\psi}{\partial\eta}\frac{\partial\omega}{\partial\xi}\right\} \ldots\ldots\ldots(8)$$

$$here.....\Delta = \frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}$$

We assume symmetry and consider only the upper half plane. The half plane minus the cylinder gets mapped into the semi-infinite strip 0≤ η ≤1, ξ≥0 (Figure 1)

On the surface of the body, the boundary conditions are

ψ(0,η) = - sin(πη)……………………………..(9)

∂ψ(0,η)/∂ξ = - π sin (πη) ………………………(10)

corresponding to vanishing normal and tangential derivatives of ψ.Symmetry gives ψ=0 and ω=0 as boundary conditions on η=0 and η=1. Numerically, two boundary conditions must also be supplied at some outer limit $r_\infty$ . First boundary condition required for ψ has been taken as normal derivative condition i.e. normal derivative of ψ viz. ∂ψ/∂ξ vanishes for large values of r. A distance r =100 corresponds to ξ= 1.47 and similarly r $=10^6$ to ξ=4.40 . Boundary condition for ω at large value of r is calculated by applying central difference approximation to equation (7), but impose ∂ω/∂ξ=0 on the boundary by equating the values on the last two mesh lines. By using this approximation, problem did not need any information from outer boundary.

## III. DIFFERENCE EQUATIONS

In equations (7) to (8); First and second derivatives are approximated by central difference formulae and following difference equations are obtained respectively.

$$A_1\psi_{i,j-1} + A_2\psi_{i-1,j} + A_3\psi_{i,j} + A_4\psi_{i+1,j}A_5\psi_{i,j+1} = A_6 \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

*where*

$$A_1 = \frac{1}{k^2}$$

$$A_2 = \frac{1}{h^2}$$

$$A_3 = -2\left(\frac{1}{h^2} + \frac{1}{k^2}\right)$$

$$A_4 = \frac{1}{h^2}$$

$$A_5 = \frac{1}{k^2}$$

$$A_6 = -\pi^2 e^{2\pi\xi_{i,j}}.\omega_{i,j}$$

And

$$B_1\omega_{i,j-1} + B_2\omega_{i-1,j} + B_3\omega_{i,j} + B_4\omega_{i+1,j} + B_5\omega_{i,j+1} = B_6 \ldots\ldots\ldots\ldots\ldots(12)$$

*where*

$$B_1 = \frac{1}{k^2} + \frac{Re\,\pi e^{\pi\xi_{i,j}}\cos\theta}{4k} + \frac{Re(\psi_{i+1,j} - \psi_{i-1,j})}{8hk}$$

$$B_2 = \frac{1}{k^2} - \frac{Re\,\pi e^{\pi\xi_{i,j}}\sin\theta}{4h} - \frac{Re(\psi_{i,j+1} - \psi_{i,j-1})}{8hk}$$

$$B_3 = -2\left(\frac{1}{h^2} + \frac{1}{k^2}\right)$$

$$B_4 = \frac{1}{h^2} + \frac{Re\,\pi e^{\pi\xi_{i,j}}\sin\theta}{4h} - \frac{Re(\psi_{i,j+1} - \psi_{i,j-1})}{8hk}$$

$$B_5 = \frac{1}{k^2} - \frac{Re\,\pi e^{\pi\xi_{i,j}}\cos\theta}{4k} - \frac{Re(\psi_{i+1,j} - \psi_{i-1,j})}{8hk}$$

$$B_6 = 0$$

## IV. METHOD OF SOLUTION

Given the complete descretization of equation (7) – (8), following computational process is employed.

- ✓ Assign initial values of ω as zero.
- ✓ Substitute current values of ω in (11) and solve it to obtain values of ψ.
- ✓ Substitute these values of ψ in (12) and solve it to get values of ω.

Repeat steps (b) - (c) iteratively until a desired convergence criterion is satisfied. This convergence criterion is taken up as follows:

$$\text{Max}\left|\omega_k - \omega_{k-1}\right| < 10^{-4}$$

Where $\omega_{k-1}$ and $\omega_k$ are consecutive values of $\omega$.

At each iteration, system of linear equations is solved by a specially designed algorithm [8] which is economical in computer storage and computer time both. It also provides accurate solutions.

## V. RESULTS AND DISCUSSION

To solve the problem, the circular region is transformed into rectangular region so that we can apply algorithm developed for rectangular region [8]. Values of r and θ are converted to ξ and η respectively and limits for ξ and η have been taken from 0 to 1. Mesh-size used in computation is 0.05. Computations have been made for Re= 2, 10 and 20 respectively. Values of stream function ψ for different values of Reynolds number Re at some selected points are reported in table 1 and values of vorticity ω for different values of Re at some selected points are also reported in table 2. The values of (η,ξ) corresponding to (θ,r) are also given along with in table 1 and 2.

Stream function, vorticity curves are drawn corresponding to the result to have clear idea of physical phenomena. Streamlines and vorticity curves for Re=2 are drawn in diagrams 1(a) and 1(b) respectively. The curves drawn compare well with those of previous authors [7]. From diagram 1(a) we observe that streamlines are dense near the cylinder and become a little straight as we move far from the cylinder. Computations have also been made with mesh size 0.025, but results show similar pattern to that of mesh size 0.05. Moreover computer time for mesh size 0.025 is considerably raised. Therefore results for mesh size 0.05 are reported here. In diagram 1(b), vorticity curves for Reynolds number 2 are drawn. The behaviours of vorticity curves in the proximity of cylinder is striking. The vorticities take a turn in the region close to cylinder. In diagram 2(a) and 2(b) , streamlines and vorticities for Re = 10 are drawn. It is apparant from 2(a) that more and more streamlines are generated near the cylinder. From figure 2(b), it is observed that bending of vorticity curves increase with increase in Reynolds number. In diagrams 3(a) and 3(b) streamlines and vorticity curves are depicted for Reynolds number 20. Previous observations can be markedly noted from these diagrams.

All these features compare well with previous authors [9]. As we move for Reynolds number higher, problem of instability arises as compared with another authors (10). Instability arises due to three types of errors.

- ✓ Rounding Errors.
- ✓ Implementation of boundary conditions, especially at r →∞.
- ✓ Turncation errors because of finite differencing of derivatives.

## VI. CONCLUSIONS

From the numerical results obtained, we may conclude as follows:

- ✓ The algorithm developed by [8] when applied to the present problem, works well upto moderately large Reynolds number and after that, problem of instability arises.
- ✓ The computer-time is economic for the results obtained.
- ✓ The pattern of the results obtained viz. Streamlines and vorticity curves are almost identical with those of previous researchers [11].

### A. VALUES OF STREAM FUNCTION AT SOME SELECTED POINTS

| (η,ξ)= → (θ,r)= Re ↓ | (.2,.2) (.6283, 1.874) | (.3,.3) (.9424,2.566) | (.5,.5) (1.5707,4.810) | (.6,.6) (1.8849, 6.586) | (.7,.7) (2.1991,9.017) | (.8,.8) (2.5132, 12.345) |
|---|---|---|---|---|---|---|
| 2 | -0.3266 | -0.2392 | -0.1904 | -0.1442 | -0.1213 | -0.0744 |
| 10 | -0.1704 | -0.2450 | -0.2286 | -0.1851 | -0.1378 | -0.1037 |
| 20 | -0.1427 | -0.1589 | 0.0670 | 0.2556 | 0.4389 | 0.6330 |

*Table 1*

### B. VALUES OF VORTICITY AT SOME SELECTED POINTS

| (η,ξ) = (θ,r) = Re ↓ | (.2,.2) (.6283, 1.874) | (.3,.3) (.9424,2.566) | (.5,.5) (1.5707,4.810) | (.6,.6) (1.8849, 6.586) | (.7,.7) (2.1991, 9.017) | (.8,.8) (2.5132,12.345) |
|---|---|---|---|---|---|---|
| 2 | -0.0039 | -0.0034 | -0.0015 | -0.00028 | -0.00001 | 0.0000 |
| 10 | 0.0010 | -0.0008 | -0.0001 | 0.00001 | 0.0003 | -0.0018 |
| 20 | 0.0003 | -0.0035 | -0.0067 | -0.0111 | -0.0133 | 0.0111 |

*Table 2*



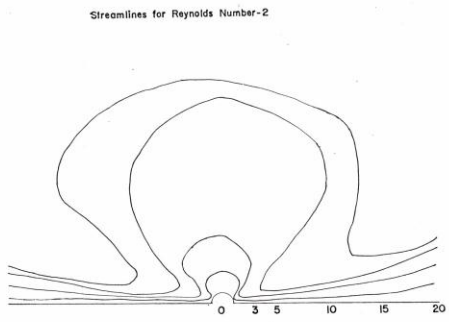


Figure 1(a)

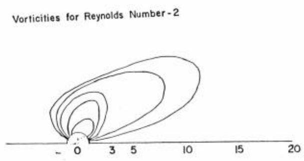


Figure 1(b)

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Streamlines for Reynolds Number - 10

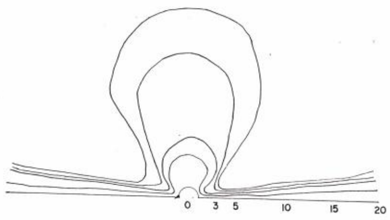


Figure 2(a)

Vorticities for Reynolds Number - 10

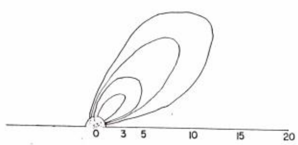


Figure 2(b)

Vorticities for Reynolds Number - 20

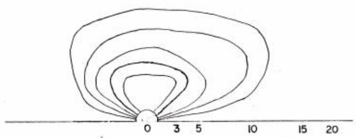


Figure 3(b)

Streamlines for Reynolds Number - 20

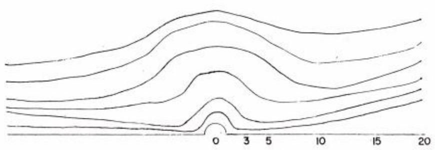


Figure 3(a)

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