

# Linear Uncertain Discrete Time Delay System: A Survey

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*Abstract: In several field of engineering and mathematics especially in the field of control theory, problems formulated in terms of Linear Matrix Inequality (LMI) and solved by Semidefinite Programming (SDP) techniques became more and more common in last decades. Such methods are applied for the analysis of system stability, synthesis of stabilizing robust controllers for uncertain systems and synthesis of optimal control modules. The large number of applications of SDP has led to an intense research and development of software for solving the optimization problems. However, these solvers typically take the problem description in a very compact format, making immediate use of the solvers time-consuming and error prone. Stability analysis is very important criteria which is not only important but also mandatory for the system to perform under unusual situation. In this paper various researches that has been done on this field of engineering has been presented and discussed. The stability condition can be solved by dividing the situation in two parts i.e. parsing the conditions and solving the conditions. For these two task various tools has been used in recent publications. This literature survey will provide clear insight of handing the inequality problem in control system with its benefits and drawbacks.*

*Keywords: LMI (Linear Matrix Inequality), SDP (Semidefinite Programming), PTVMSFC (periodically time-varying memory state-feedback controller).*

## I. INTRODUCTION

Linear Matrix Inequalities (LMIs) and LMI techniques have emerged as powerful design tools in areas ranging from control engineering to system identification and structural design. Three factors make LMI techniques appealing. a) A variety of design specifications and constraints can be expressed as LMIs. b) Once formulated in terms of LMIs, a problem can be solved exactly by efficient convex optimization algorithms (the “LMI solvers”). c) While most problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework. This makes LMI-based design a valuable alternative to classical “analytical” methods.

## II. LINEAR MATRIX INEQUALITIES

A linear matrix inequality (LMI) is any constraint of the form:

$$A(x) := A_0 + x_1 A_1 + \dots + x_N A_N < 0 \quad (1)$$

Where

$x = (x_1, \dots, x_N)$  is a vector of unknown scalars (the decision or optimization variables).  $A_0, \dots, A_N$  are given symmetric matrices. And  $< 0$  stands for “negative definite”, i.e. the largest eigenvalue of  $A(x)$  is negative. The LMI presented in equation 1, is a convex constraint on  $x$  since  $A(y) < 0$  and  $A(z) < 0$  imply that  $A(\frac{y+z}{2}) < 0$ . As a result, its solution set, called the feasible set, is a convex subset of  $\mathbb{R}^N$  and finding a solution ‘ $x$ ’ to above equation, if any, is a convex optimization problem. Convexity has an important consequence: even though (1) has no analytical solution in general, it can be solved numerically with guarantees of

finding a solution when one exists. There are three Generic LMI Problems namely:

- ✓ Feasibility problem
- ✓ Linear objective minimization problem
- ✓ Generalized eigenvalue minimization problem

Many control problems and design specifications have LMI formulations [9]. This is especially true for Lyapunov-based analysis and design, but also for optimal LQG control,  $H_\infty$  control, covariance control, etc. Further applications of LMIs arise in estimation, identification, optimal design, structural design [6, 7], matrix scaling problems, and so on. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

Accompanying the growth of the usage of LMI conditions, a large number of solvers based on interior point methods were developed, as well as interfaces for parsing the LMIs, most of them free and easily accessible. Thanks to such remarkable advance in the computational tools to define, manipulate and solve LMIs, in many cases one can say that if a problem can be cast as a set of LMIs, then it can be considered as solved (Boyd et al., 1994). Unfortunately, this is not completely true for large scale systems, since LMI solvers are limited to a few thousands of variables and LMI rows, but progresses are being made.

### III. TOOLBOX

Usually, LMIs are solved in two steps: first, an interface for parsing the conditions is used, for example the YALMIP parser (Löfberg, 2004) or the LMI Control Toolbox from Matlab (Gahinet et al., 1995); and an LMI solver is then applied to find a solution (if any), for example SeDuMi (Sturm, 1999) or SDPT3 (Toh et al., 1999). Some auxiliary toolboxes may also be used in addition to the parser and the solver, for example the SOSTOOLS (Prajna et al., 2004), which is used to transform a sum of squares problem into a SDP formulation, and Gloptipoly (Henrion and Lasserre, 2003), used to handle optimization problems over polynomials.

### IV. METHODOLOGY

To handle the stability of the system for LMI (Linear Matrix Inequalities) various methods has been proposed by the researchers over the years. A summary of the proposed methods has been presented in this section.

Kalman et al. in 1960, presented the classical filtering and prediction problem and re-examined using the Bode- Shannon representation of random processes and the “state transition” method of analysis of dynamic systems. New results were:

- ✓ The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite memory filters.
- ✓ A nonlinear difference (or differential) equation is derived for the covariance matrix of the optimal estimation error. From the solution of this equation the coefficients of the

difference (or differential) equation of the optimal linear filter are obtained without further calculations.

- ✓ The filtering problem has shown to be the dual of the noise-free regulator problem.

The new methods developed here were applied to two well-known problems, confirming and extending earlier results. The problems included as a special case the problems of filtering, prediction, and data smoothing mentioned earlier. It also includes the problem of reconstructing all the state variables of a linear dynamic system from noisy observations of some of the state variables. The discussion is largely self-contained and proceeds from first principles; basic concepts of the theory of random processes. The author formulated and solved the Wiener problem from the “state” point of view. On the one hand, this leads to a very general treatment including cases which cause difficulties when attacked by other methods. On the other hand, the Wiener problem has been shown to be closely connected with other problems in the theory of control. [1960]

Lee et al. in 2015, addresses the problem of static output feedback (SOF) stabilization for discrete-time LTI systems. The author approaches this problem using the periodically time-varying memory state-feedback controller (PTVMSFC) design scheme. A bilinear matrix inequality (BMI) condition which uses a pre-designed PTVMSFC is developed to design the periodically time-varying memory SOF controller (PTVMSOFC). The BMI condition can be solved by using BMI solvers. Alternatively, the authors comments that the method can apply two-steps and iterative linear matrix inequality algorithms that alternate between the PTVMSFC and PTVMSOFC designs. The results presented by the author shows that at the price of a higher computational cost, the proposed method offers improvement over the previous approaches except for the full-order DOF design. The number of parameters of the controller is  $mpN(N + 1)/2$  for the PTVMSOFC while  $n^2 + np + mn + mp$  for the full-order DOF controller. Thus, in many cases, the number of parameters for the PTVMSOFC is smaller than that of the DOF controller. This means that the online computational cost of the PTVMSOFC can be lower. For this reason, the PTVMSOFC can be a useful alternative to the DOF controller in some cases. The comparison results between the two-steps algorithm and the ILMI algorithm suggest that some improvement can be achieved by adopting the ILMI method. The result presented by the author has been shown below.

Methods	$N_{stable}$	Time (s)
Cone complementarity linearization algorithm in El Ghaoui et al. (1997) (discrete-time version)	509	34.35
Discrete P-problem in Crusius and Trofino (1999)	248	0.10
Discrete W-problem in Crusius and Trofino (1999)	243	0.10
Algorithm A in Rosinová et al. (2003) with $(R, Q) = (0.01I_m, I_n)$	306	0.04
Two-steps LMI approach of Theorem 3.1 in Mehdi et al. (2004) with constraint $-G - G^T < 0$	427	0.18
Two-steps LMI approach of Theorem 3.1 in Mehdi et al. (2004) with $F1 = F3$	427	0.18

$= 0$		
Lemma 3 in Dong and Yang (2007) with $T = [C^T (CC^T)^{-1} C_1]$ (method in de Oliveira et al., 2002)	287	0.10
Theorems 3.1 and 3.3 in Bara and Boutayeb (2005) with $T = [C^T (CC^T)^{-1} C_1]$	222	0.14
Algorithm 1 in Shu et al. (2010)	309	9.90
PENBMI (Kočvara & Stingl, 2005)	484	0.08
Full-order DOF design (discrete-time version of Scherer et al. (1997))	1000	0.10

Table 1: Number Of Stabilizable Systems,  $N_{stable}$ , And The Average Computational Time

Jia You et al. in 2013, investigates the  $H_\infty$  filtering problem for a class of discrete-time systems with time-varying delay. A model transformation is first applied by employing a two-term approximation for delayed state variables, which has a smaller approximation error than the other one-term approach. By using Scaled Small Gain Theorem and Lyapunov–Krasovskii functional approach, sufficient conditions are provided for the stability of the filtering error system with a prescribed  $H_\infty$  performance level. In addition, sufficient conditions for the existence of the  $H_\infty$  filters are established. The existence of time-delay is often a source of poor performance and even instability [14], whereas it is commonly encountered in practical dynamic systems due to various reasons. Over the past few decades, time delay systems have drawn considerable interest and a large number of results have been reported in the literatures, such as stability analysis [15], stabilization problem [16,17], and especially filtering problem with constant [18,19] and time-varying [20–22] delays. For the time varying delay case, it has been proved that delay independent methods [23–25] are more conservative than delay-dependent methods [8], especially for small time-delays [26]. Thus, researchers have focused on designing the time-dependent  $H_\infty$  filters and the main concern is to reduced the conservatism of these conditions [27, 28]. By Lyapunov method, a large number of stability results have been established [8, 29–31]. On the other hand, an input–output (IO) approach has been introduced for constant delays in [32], and then extended to the time-varying delays [33]. This approach, that analyzes the stability of the original systems by means of interconnection of two subsystems, could give results with much less conservatism. The key point of this approach is to find a proper approximation for the time-varying delay, such that the approximation error is as small as possible. The author we investigate the  $H_\infty$  filtering problem for discrete-time systems with time-varying delay. The main contribution of this paper was in employing an input–output approach based on the Scaled Small Gain Theorem to design a full-order filter such that the filtering error system is asymptotically stable and preserves a guaranteed  $H_\infty$  performance. First, a model transformation is applied to the original system. In order to ensure the approximation error to be as small as possible, a two-term approximation constructed by the lower and upper delay bounds is performed for the time-varying delay. By using a Lyapunov–Krasovskii approach, a new sufficient condition for the existence of the  $H_\infty$  filter is established. Then, the corresponding  $H_\infty$  filter

design technique, which is much less conservative than the previously known ones, is proposed.

Lacerda et al. in 2013, presents new robust linear matrix inequality (LMI) conditions for robust  $H_\infty$  full order filter design of discrete-time linear systems affected by time-invariant uncertainty and a time-varying state delay. Thanks to the use of a larger number of slack variables, the proposed robust LMI conditions contain and generalize other results from the literature. LMI relaxations based on homogeneous polynomial matrices of arbitrary degree are used to determine the state space realization of the full order filter that can also be implemented with delayed state terms whenever the time delay is available in real time. As another contribution, an iterative LMI-based procedure involving the decision variables was proposed to improve the  $H_\infty$  filter performance. Numerical experiments illustrate the better performance of the proposed filter when compared to other approaches available in the literature. The authors investigate the problem of robust  $H_\infty$  full order filter design for discrete-time linear systems affected by time-invariant polytopic uncertainty and a time-varying state delay. The main contribution is to provide new delay dependent robust linear matrix inequality (LMI) conditions for the filter design. The appropriate choice of a Lyapunov–Krasovskii function with parameter-dependent matrices to be determined and the Jensen’s inequality [38] provide stability conditions that depend on the minimum and maximum values of the time-varying delay (i.e., delay-dependent conditions). Additionally, the Finsler’s Lemma [39] is employed to derive design conditions in an augmented space with additional slack variables. Thanks to the parameter-dependent matrices and the extra slack variables, the proposed delay-dependent robust LMI conditions contain and generalize other results from the literature. Differently from [35], that also uses homogeneous polynomials of arbitrary degrees in the Lyapunov-Krasovskii function and proposes conditions that depend on the interval where the time-varying delay lies, this paper uses a larger number of slack variables, Jensen’s inequality and Finsler’s Lemma to obtain more general conditions. Compared to [36], that proposes a technique based on the partition of the delay interval and also uses Jensen’s inequality, the conditions in this paper allow a larger number of slack variables to be considered as homogeneous polynomials of arbitrary and independent degrees (while in [36] only affine parameter-dependent matrices are used) without any delay partition scheme. LMI relaxations based on homogeneous polynomial matrices of arbitrary and independent degree are used to determine the matrices of the state space realization of the full order filter. Whenever available in real time (measured or estimated), the time-varying delay can also be used as additional information in the filter state space implementation. As another contribution, an iterative LMI-based procedure involving the decision variables is proposed to improve the  $H_\infty$  filter performance. The authors in this paper presented new delay-dependent robust LMI conditions for the design of full order robust  $H_\infty$  filters for discrete-time uncertain polytopic linear systems with time-varying delays. A state space filter implementation that can be constructed with state delayed terms, a larger number of slack variables, an iterative procedure and LMI relaxations based on homogeneous polynomials of arbitrary degree

provided less conservative results when compared to other existing techniques.

## V. EXAMPLE

Consider the discrete-time LTI system described by

$$\begin{cases} x(K+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \quad (1)$$

where  $k \in \mathbb{N}$ ;  $x(k) \in \mathbb{R}^n$  is the state;  $u(k) \in \mathbb{R}^m$  is the control input;  $y(k) \in \mathbb{R}^p$  is the measured output;  $\Sigma := (A, B, C) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{n \times m} \times \mathbb{R}^{p \times n}$  is a tuple of constant matrices. Inspired by the recently developed PTVMSFC, we suggest the PTVMSOFC (or N-PTVMSOFC) of the following form:

$$u(k) = \sum_{i=0}^{\lceil k \rceil_N} F_{SOF}^{(\lceil k \rceil_N, i)} y(k-i) \quad (2)$$

where  $N \in \mathbb{N}_+$  is the period of the controller and  $F_{SOF}^{(\lceil k \rceil_N, i)} \in \mathbb{R}^{m \times p}$ ,  $(\lceil k \rceil_N, i) \in Z_{[0, N-1]} \times Z_{[0, N-1]}$  are the SOF gains to be designed. In the case  $N = 1$ , this is the classical SOF controller. Substituting (2) into (1), the N-periodic control system (closed-loop system) can be written as

$$x(k+1) = Ax(k) + B \sum_{i=0}^{\lceil k \rceil_N} F_{SOF}^{(\lceil k \rceil_N, i)} Cx(k-i) \quad (3)$$

The problem addressed here seek to the N-PTVMSOFC (2) such that the N-periodic control system (3) is asymptotically stable.

## VI. CONCLUSION

The paper presented above describes parameter-dependent LMI conditions for the design of full-order robust  $H_2$  and  $H_\infty$  memory filters for discrete-time LTI systems with multiple state delay and polytopic uncertainties. Numerical examples borrowed from the literature showed that the proposed conditions can be less conservative than others available in the literature for increasing quantity of used memory in the structure of the filter, illustrating the efficacy of filter structures with past states and past output measures. Moreover, the methodologies proved to be an interesting alternative to deal with systems with multiple constant delays, unknown or time-varying delays belonging to intervals, or no delays. Generally, the increase in the memory size of the filter demands a higher computational effort during the design phase (performed off-line), but for the practical implementation of the filter, the necessary memory to store the past information can be easily handled through the digital platforms available nowadays. As future works, the authors are concerned in applying the methodology to discrete-time fuzzy and stochastic systems as well as in extending the method to deal with sampled data filter design with memory.

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