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Heat Transfer On MHD Convective Flow Of Heat Generating/Absorbing Second Grade Fluid Through Porous Medium In A Rotating Parallel Plate Channel

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***Abstract:** In this paper, we have considered the heat transfer on the unsteady hydromagnetic convective flow of an incompressible viscous electrically conducting heat generating/ absorbing second grade fluid through porous medium in a rotating parallel plate channel under the influence of uniform transfer magnetic field normal to the channel. The momentum equation for the flow is governed by the Brinkman's model. The analytical solutions for the velocity and temperature distributions are obtained by making use of regular perturbation technique and computationally discussed with reference to flow parameters through the graphs. The skin friction and Nusselt number are also evaluated analytically and computationally discussed with reference to pertinent parameters in detail.*

***Keywords:** heat transfer, second grade fluid, MHD flows, parallel plate channel, porous medium.*

I. INTRODUCTION

The phenomenon of free convection arises in fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. It can be observed in our daily life in atmospheric flow, which is driven by temperature differences. Free convective flow past a vertical plate was studied extensively by Ostrach (1952, 1953) and many other researchers. Channel flows through porous medium have varied applications in the field of chemical engineering, agriculture engineering and petroleum technology. In some applications e.g. in microfluidic and nanofluidic device where the surface to volume ratio is large, the slip behavior is more typical and slip boundary condition is usually used for the velocity field. Tao (1960) reported on combined free and forced convections in channels. Sinha (1969) studied fully developed laminar free convection flow between vertical parallel plates. Soundalgekar (1970) considered hydromagnetic fluctuating flow past an infinite porous plate in slip flow regime. Magnetogasdynamics flow past an infinite porous plate in slip flow regime was investigated by Sastry and Bhadram (1976). Raptis and Peridiks (1985) studied the unsteady free convection flow through a highly porous medium bounded by an infinite porous plate. Singh (1988) investigated natural convection in unsteady Couette motion. Zaturska et al. (1998) reported on the flow of a viscous fluid driven along a channel by suction at porous walls. Barletta (1998) investigated laminar mixed convection with viscous dissipation in a vertical channel. Unsteady MHD convective heat transfer past a semi infinite vertical porous plate with variable suction was presented by Kim (2000). Kamel (2001) discussed unsteady MHD convection through porous medium with combined heat and mass transfer with heat source / sink. Din (2003) reported effect of thermal and mass buoyancy forces on the development of laminar mixed convection between vertical parallel plates with uniform wall heat and mass fluxes. Magnetohydrodynamic mixed convection in a vertical channel was studied by Umavathi and Malashetty (2005). Makinde and Osalusi (2006) considered MHD steady flow in a channel with slip at the permeable boundaries. Unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation was investigated by Sharma and Singh (2008). Zanchini (2008)

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presented mixed convection with variable viscosity in a vertical annulus with uniform wall temperature. Sharma et al. (2009) observed radiation effects on unsteady MHD free convective flow with Hall current and mass transfer through viscous incompressible fluid past a vertical porous plate immersed in porous medium with heat source / sink. Free convective flow of heat generating / absorbing fluid between vertical porous plates with periodic heat input was studied by Jha and Ajibade (2009). Sharma and Mehta (2009) investigated MHD Unsteady slip flow and heat transfer in a channel with slip at the permeable boundaries. Unsteady MHD convective flow within a parallel plate rotating channel with thermal source / sink in a porous medium under slip boundary conditions was studied by Seth et al. (2010). Sharma et al. (2010) presented unsteady MHD free convective flow and heat transfer between heated inclined plates with magnetic field in the presence of radian effects. The effects of slip condition transverse magnetic field and radiative heat transfer to unsteady flow of a conducting thin fluid through a channel was discussed by Hamza et al. (2011). Khem Chand and Sapna (2012) studied hydromagnetic free convective oscillatory Couette flow through a porous vertical channel with periodic wall temperature. Singh (2012) reported solution of MHD oscillatory convection flow through porous medium in a vertical porous channel in slip-flow regime. Effect of volumetric heat generation / absorption on convective heat and mass transfer in porous medium in between two vertical plates was investigated by Sharma and Dadheech (2012). Kesavaiah et al. (2013) observed effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium. Analysis of MHD convective flow along a moving semi-vertical plate with internal heat generation was presented by Sharma and Yadav (2014). Oscillatory MHD free convective flow and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by two infinite vertical parallel porous plates under slip boundary conditions in the presence of heat source is investigated by Sharma et al. (2014).

Recently, Krishna and Swarnalathamma (2016) discussed the peristaltic MHD flow of an incompressible and electrically conducting Williamson fluid in a symmetric planar channel with heat and mass transfer under the effect of inclined magnetic field. Swarnalathamma and Krishna (2016) discussed the theoretical and computational study of peristaltic hemodynamic flow of couple stress fluids through a porous medium under the influence of magnetic field with wall slip condition. Krishna and M.G.Reddy (2016) discussed MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating plate. Krishna and G.S.Reddy (2016) discussed unsteady MHD convective flow of second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source.

Motivated by the above studies, in this paper, we have discussed the heat transfer on the unsteady hydromagnetic convective flow of an incompressible viscous electrically conducting heat generating/absorbing second grade fluid through porous medium in a rotating parallel plate channel under the influence of uniform transfer magnetic field normal to the channel.

II. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady hydromagnetic convective flow of an incompressible viscous electrically conducting heat generating/absorbing second grade fluid through porous medium in a rotating parallel plate channel under the influence of uniform transfer magnetic field H_o normal to the channel. In undisturbed state both the plates and the fluid rotate with the same angular velocity Ω. Since, the plates are widening to infinity along x and y paths, electrically non-conducting and flow is fully developed, so that all the physical quantities except the pressure depend on z and t alone. The physical configuration of the problem is as shown in Fig. 1.

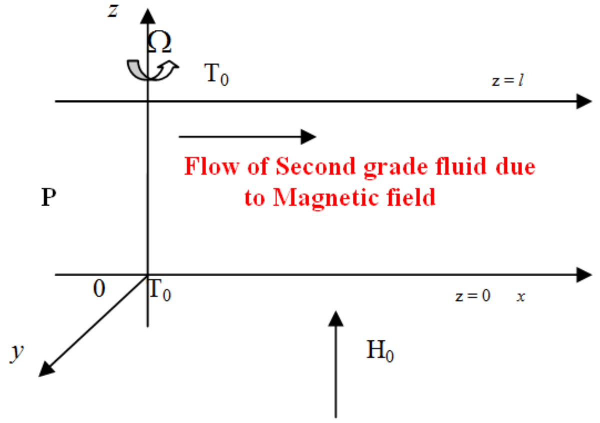

Figure 1: Physical configuration of the Problem

We choose a Cartesian co-ordinate system O(x, y, z) such that the plates are at $z = 0$ and $z = l$ and the z- axis consider with the axis of rotation of the plates. The unsteady hydromagnetic boundary layer equation of motion with respect to a rotating frame moving with angular velocity Ω in the absence of any input electric field are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}-2\Omega v=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2\partial t}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{k}u+g\beta(T-T_0) \qquad (2)$$

$$\frac{\partial v}{\partial t}+2\Omega u=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2\partial t}-\frac{\sigma\beta_0^2}{\rho}v-\frac{\nu}{k}v \qquad (3)$$

$$\frac{\partial T}{\partial t}=\frac{K_1}{\rho C_p}\frac{\partial^2 T}{\partial z^2}-\frac{Q_0}{\rho C_p}(T-T_0) \qquad (4)$$

We have considered oscillatory Hartmann convective flow so pressure p is assumed in the following form

$$p=2Rx\cos(\omega t)+F(y)+G(z) \qquad (5)$$

It is noticed from equations (1), (2), (3), (4) and (5) that pressure p is constant along the axis of rotation i.e., $\frac{\partial p}{\partial z}=G^{'}(z)=0$. The absence of pressure gradient term $\frac{\partial p}{\partial y}=F^{'}(y)$ in equation (2) implies that there is a net cross flow in y-direction. Buoyancy term $g\beta(T-T_0)$ is considered in equation (1) only because free-convection in this problem takes place under gravitational force. Boundary conditions for the fluid velocity are hydrodynamic slip boundary conditions which are given by

$$\mu\frac{du}{dz}=-\beta_1 u \text{ and } \mu\frac{dv}{dz}=-\beta_1 v \quad \text{at} \quad z=0 \qquad (6)$$

$$\mu\frac{du}{dz}=\beta_1 u \text{ and } \mu\frac{dv}{dz}=\beta_1 v \quad \text{at} \quad z=l \qquad (7)$$

Boundary conditions (6-7) for the fluid velocity are well known hydrodynamic slip boundary conditions derived by Beavers and Joseph (1967). Here μ and β_1 are respectively the coefficient of dynamic viscosity and coefficient of sliding friction.

Boundary conditions for the fluid temperature are

$$T=T_0, \quad \text{at} \quad z=0 \qquad (8)$$

$$T=T_0+(T_w-T_0)\cos\omega t \quad \text{at} \quad z=l \qquad (9)$$

here, $T_0<T<T_w$.

Equations (2)and (3), in compact form , become

$$\frac{\partial F}{\partial t}+2i\Omega F=-\frac{1}{\rho}\frac{\partial p}{\partial \xi}+\nu\frac{\partial^2 F}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2\partial t}-\frac{\sigma B_0^2}{\rho}F-\frac{\nu}{k}F+g\beta(T-T_0) \qquad (10)$$

Where, $F=u+iv$.

We introduce non-dimensional variables

$$\xi^*=\frac{\xi}{l},\eta=\frac{z}{l},u^*=\frac{ul}{\nu},v^*=\frac{vl}{\nu},t^*=\frac{t\nu}{l^2},p^*=\frac{l^2 p}{\rho\nu^2},T^*=\frac{(T-T_0)}{(T_w-T_0)}$$

Equations (10) and (4) in non-dimensional form are

$$(1+\alpha i\omega)\frac{d^2F}{d\eta^2}-\left\{M^2+\frac{1}{K_1}+i(2K^2+\omega)\right\}F=R-G_rT \qquad (11)$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial\eta^2}-\phi T, \qquad (12)$$

Boundary conditions (6) and (7), in dimensionless from, are

$$u=-\beta\frac{\partial u}{\partial\eta} \text{ and } v=-\beta\frac{\partial v}{\partial\eta} \quad \text{at} \quad \eta=0 \qquad (13)$$

$$u=\beta\frac{\partial u}{\partial\eta} \text{ and } v=\beta\frac{\partial v}{\partial\eta} \quad \text{at} \quad \eta=1 \qquad (14)$$

$$T=0 \quad \text{at} \quad \eta=0 \qquad (15)$$

$$T=\cos\omega t \quad \text{at} \quad \eta=1 \qquad (16)$$

Where, $K^2=\frac{\Omega l^2}{\nu}$ is the rotation parameter which is reciprocal of Ekman number, $M^2=\frac{\sigma B_0^2 l^2}{\rho\nu}$ magnetic parameter which is square of Hartmann number, $\alpha=\frac{\alpha_1}{\rho l^2}$ the second grade fluid parameter, $\mathrm{Gr}=g\beta\frac{(T_w-T_0)l^3}{\nu^2}$ thermal Grashof number, $\Pr=\frac{\nu\rho C_p}{K_1}$ Prandtl number, $,K_1=\frac{k}{l^2}$ Permeablitity parameter (Darcy parameter), $\phi=\frac{Q_0 l^2}{\rho\nu C_p}$ the heat generation/ absorption coefficient, $\beta=\mu/\beta_1 l$ slip parameter and $\omega^*=\frac{\omega l^2}{\nu}$ frequency parameter respectively.

Boundary conditions (15) and (16), in dimensionless from, are

$$F+\beta\frac{\partial F}{\partial\eta}=0 \text{ at } \eta=0,\ F-\beta\frac{\partial F}{\partial\eta}=0 \text{ at } \eta=1 \qquad (17)$$

It may be noted that the fluid flow past a plate may be induced due to either by motion of the plate or free stream or by heating of the fluid or by both. We have considered oscillatory Hartmann convective flow so fluid flow, in our case, is induced due to applied and oscillatory pressure gradient and by heating of the fluid because lower and upper plates. Therefore, pressure gradient $\frac{\partial p}{\partial\varsigma}$, fluid velocity $F(z,t)$ and fluid temperature $T(z,t)$ are assumed in non-dimensional form, as

$$\frac{\partial p}{\partial\varsigma}=R(e^{i\omega t}+e^{-i\omega t}), \qquad (18)$$

$$F(z,t)=F_1(z)e^{i\omega t}+F_2(z)e^{-i\omega t} \qquad (19)$$

$$\theta(z,t)=T_1(z)e^{i\omega t}+T_2(z)e^{-i\omega t} \qquad (20)$$

Where $R<0$ for favourable pressure

Equations (11) and (12) with the use of (19) and (20) reduce to

$$(1+\alpha i\omega)\frac{d^2F_1}{d\eta^2}-\left\{M^2+\frac{1}{K_1}+i(2K^2+\omega)\right\}F_1=R-\mathrm{Gr}T_1 \qquad (21)$$

$$(1-\alpha i\omega)\frac{d^2F_2}{d\eta^2}-\left\{M^2+\frac{1}{K_1}+i(2K^2-\omega)\right\}F_2=R-\mathrm{Gr}T_2 \qquad (22)$$

$$\frac{d^2T_1}{d\eta^2}-\Pr(\phi+i\omega)T_1=0 \qquad (23)$$

$$\frac{d^2T_2}{d\eta^2}-\Pr(\phi+i\omega)T_2=0 \qquad (24)$$

Boundary conditions (27) and (29) becomes

$$F_1+\beta\frac{\partial F_1}{\partial\eta}=0 \text{ and } F_2+\beta\frac{\partial F_2}{\partial\eta}=0 \text{ at } \eta=0 \qquad (25)$$

$$F_1-\beta\frac{\partial F_1}{\partial\eta}=0 \text{ and } F_2-\beta\frac{\partial F_2}{\partial\eta}=0 \text{ at } \eta=1 \qquad (26)$$

$$T_1=0 \text{ and } T_2=0 \quad \text{at} \quad \eta=0 \qquad (27)$$

$$T_1=1/2 \text{ and } T_2=1/2 \quad \text{at} \quad \eta=1 \qquad (28)$$

Equations (21) to (24) subject to boundary conditions (25) to (28) are solved and the solution for velocity and temperature of the fluid is presented in the following form

$$\theta(\eta,t)=\frac{1}{2}\left[\frac{\sinh m_1\eta}{\sinh m_1}e^{i\omega t}+\frac{\sinh m_3\eta}{\sinh m_3}e^{-i\omega t}\right], \qquad (29)$$

$$F(\eta,t)=\left\{C_1\cosh m_2\eta+C_2\sinh m_2\eta-\frac{R}{m_2^2}-\frac{\mathrm{G_r}\sinh m_1\eta}{2(m_1^2-m_2^2)\sinh m_1}\right\}e^{i\omega t}$$

$$+\left\{C_3\cosh m_4\eta + C_4\sinh m_4\eta - \frac{R}{m_4^2} - \frac{G_r\sinh m_3\eta}{2(m_3^2 - m_4^2)\sinh m_3}\right\}e^{-i\omega t} \quad (30)$$

The non-dimensional skin friction at the lower plate $\eta = 0$ in term of amplitude and phase angle is given by

$$\tau = \left(\frac{dF}{d\eta}\right)_{\eta=0} = \left(\frac{dF_1}{d\eta}\right)_{\eta=0} e^{i\omega t} + \left(\frac{dF_2}{d\eta}\right)_{\eta=0} e^{-i\omega t} \quad (31)$$

The rate of heat transfer co-efficient at the lower plate $\eta = 0$ in term of amplitude and phase angle is given by

$$Nu = \left(\frac{dT}{d\eta}\right)_{\eta=0} = \left(\frac{dT_1}{d\eta}\right)_{\eta=0} e^{i\omega t} + \left(\frac{dT_2}{d\eta}\right)_{\eta=0} e^{-i\omega t} \quad (32)$$

III. RESULTS AND DISCUSSIONS

We discussed the heat transfer on the unsteady hydromagnetic convective flow of an incompressible viscous electrically conducting heat generating/absorbing second grade fluid through porous medium in a rotating parallel plate channel under the influence of uniform transfer magnetic field normal to the channel. The momentum equation for the flow is governed by the Brinkman's model. The analytical solutions for the velocity and temperature distributions are obtained by making use of regular perturbation technique. The closed form solutions for the velocity $F = u + iv$ and temperature θ are obtained making use of perturbation technique. The velocity expression consists of steady state and oscillatory state. It reveals that, the steady part of the velocity field has three layer characters while the oscillatory part of the fluid field exhibits a multi layer character. The Figures (2-4) exhibit the temperature distribution with different variations in the governing parameters P_r and frequency of oscillation ω. The Figures (5-11) shows the effects of non-dimensional parameters M the Hartmann number, K_1 permeability parameter, α the second grade fluid parameter, G_r thermal Grashof number, K^2 rotation parameter and source/sink parameter ϕ.

The numerical values of fluid temperature and are computed from analytical solution mentioned by MATHEMATICA software, are depicted graphically in Figs. 2 to 4 for different values of heat generation coefficient φ (< 0), heat absorption coefficient φ (> 0), Prandtl number P_r and frequency parameter ω taking $\omega t = \pi / 2$. Figure 2 reveals that fluid temperature T increases on increasing φ (< 0) and decreases on increasing φ (> 0) which imply that thermal source tends to increase fluid temperature whereas thermal sink has reverse effect on it. Figure 3 shows that, for both heat generating and absorbing fluids, fluid temperature T increases on increasing Prandtl number P_r. Since Prandtl number P_r is ratio of viscosity to thermal diffusivity. An increase in thermal diffusivity leads to a decrease in Prandtl number. Therefore, thermal diffusion has tendency to reduce fluid temperature for both heat generating/absorbing fluids. It is observed that Prandtl number Pr leads to decrease the temperature uniformly in all layers being the heat source parameter fixed. It is found that the temperature decreases in all layers with increase in the heat source. It is concluded that Prandtl number Pr reduces the temperature in all layers. It is noticed from Fig. 4 that, for both heat generating/absorbing fluids, fluid temperature T decreases in the lower of the channel whereas it decreases, attains a minimum and then increases in magnitude in the upper of the channel on increasing ω which implies that there exists reverse flow of heat in the upper of the channel due to oscillating temperature of plate $\eta = 1$.

To study the effects of magnetic field, rotation, thermal buoyancy force, porosity of medium, oscillations and thermal source/sink on the flow-field numerical values of both primary and secondary fluid velocities, computed from analytical solution are displayed graphically versus channel width variable η for various values of second grade fluid parameter α, magnetic parameter M^2, rotation parameter K^2, Grashof number G_r, permeability parameter K_1, frequency parameter ω, heat generation coefficient φ (< 0) and heat absorption coefficient φ (> 0) in Figs. 5 to 11 taking $\beta = 0.05$, $P_r = 0.71$, $\omega t = \pi / 2$ and $R = -1$.

It is evident from Figs. 5 (a-b) to 6 (a-b) that primary velocity u and secondary velocity v decrease on increasing either second grade fluid parameter α or magnetic parameter M^2 for both heat generating and absorbing fluids which implies that wall slip and magnetic field have tendency to retard fluid flow in the primary and secondary flow directions for both heat generating and absorbing fluids. The application of the transverse magnetic field plays the important role of a resistive type force (Lorentz force) similar to drag force (that acts in the opposite direction of the fluid motion) which tends to resist the flow thereby reducing its velocity. Figures 7 (a-b) show that, for both heat generating and absorbing fluids, primary velocity u decreases whereas secondary velocity v increases with the increase in rotation parameter K^2 which implies that, for both heat generating and absorbing fluids, rotation tends to retard fluid flow in the primary flow direction whereas it has reverse effect on the fluid flow in secondary flow direction. Figures 8 (a-b) to 10 (a-b) reveal that, for both heat generating and absorbing fluids, u and v increase on increasing either G_r or K_1 or ω which implies that, for both heat generating and absorbing fluids, thermal buoyancy force, porosity of medium and oscillations have tendency to accelerate fluid flow in both the primary and secondary flow directions. It is noticed from Figs. 11(a-b) that u and v increase on increasing φ (< 0) and decrease on increasing φ (> 0) which implies that thermal source accelerates fluid flow in both the primary and secondary flow directions whereas thermal sink has reverse effect on it.

It is noted from the table 1 that the magnitudes of both the skin friction components τ_{xz} and τ_{yz} increase with increase in permeability parameter K_1, second grade fluid parameter α and rotation parameter K^2 and where as it reduces with increase in Hartmann number M, thermal Grashof number G_r, Prandtl number P_r. From the table 2 that the magnitude of the Nusselt number Nu increases for the parameters and Prandtl number P_r or time t, and it reduces with the frequency of oscillation ω.

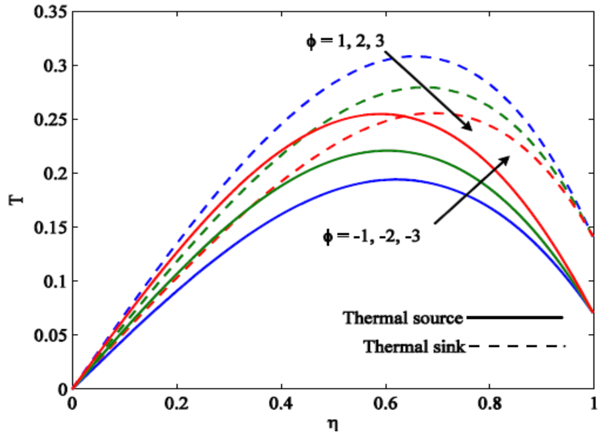

Figure 2: Temperature Profile Pr=0.71 and ω =3

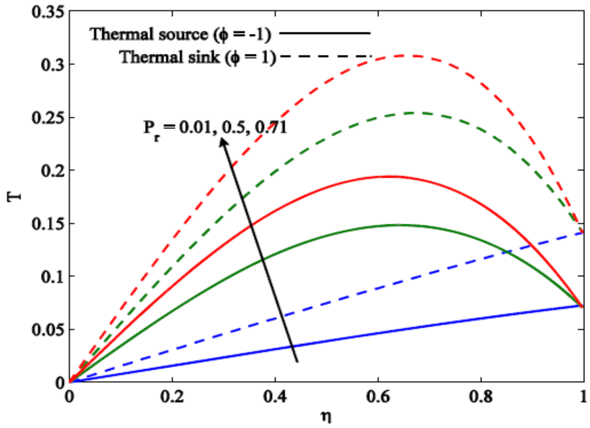

Figure 3: Temperature Profile with ω =3

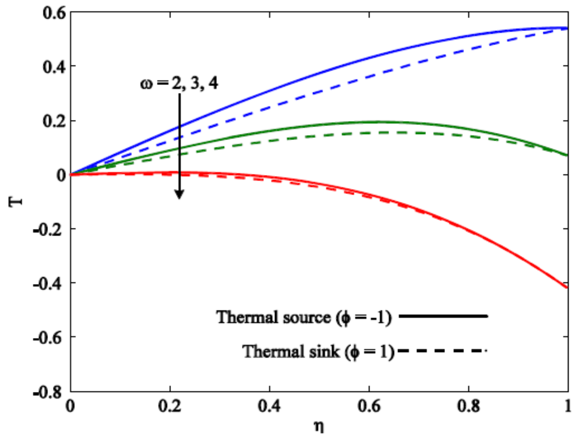

Figure 4: Temperature Profile with Pr = 071

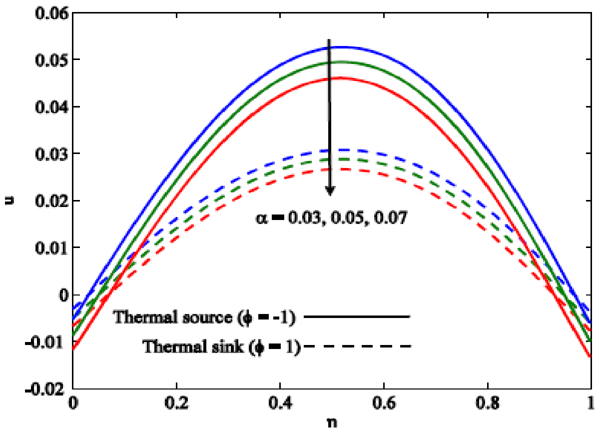

Figure 5(a): The velocity Profile for u against α with
$M^2 = 4, K^2 = 3, G_r = 2, K_1 = 0.2, \omega = 3$

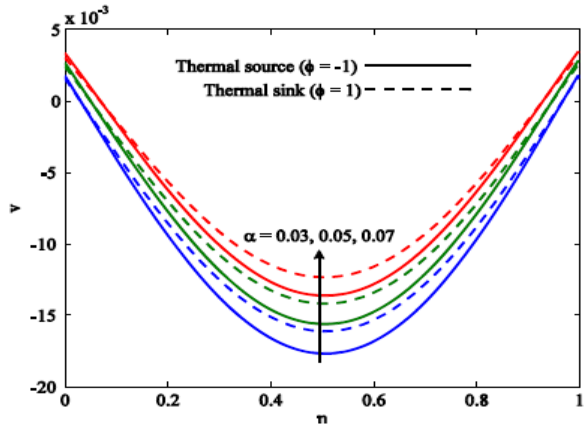

Figure 5(b): The velocity Profile for v against α with
$M^2 = 4, K^2 = 3, G_r = 2, K_1 = 0.2, \omega = 3$

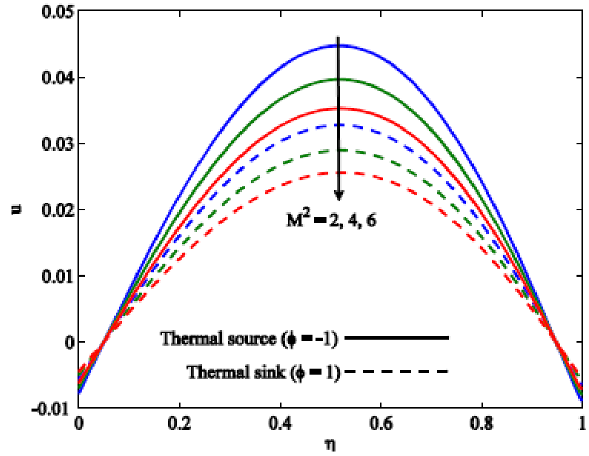

Figure 6(a): The velocity Profile for u against M with
$K^2 = 3, G_r = 2, K_1 = 0.2, \omega = 3, \alpha = 0.05$

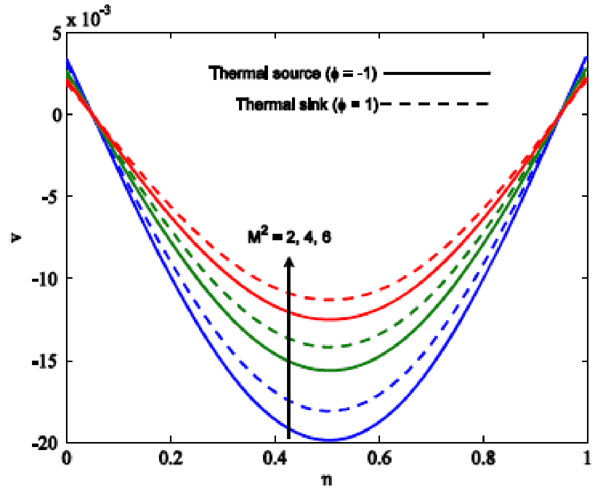

Figure 6(b): The velocity Profile for v against M with
$K^2 = 3, G_r = 2, K_1 = 0.2, \omega = 3, \alpha = 0.05$

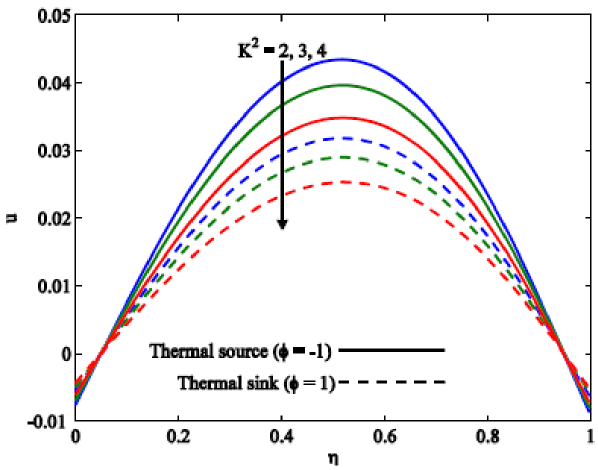

Figure 7 (a): The velocity Profile for u and v against K^2 *with* $M^2 = 4, G_r = 2, K_1 = 0.2, \omega = 3, \alpha = 0.05$

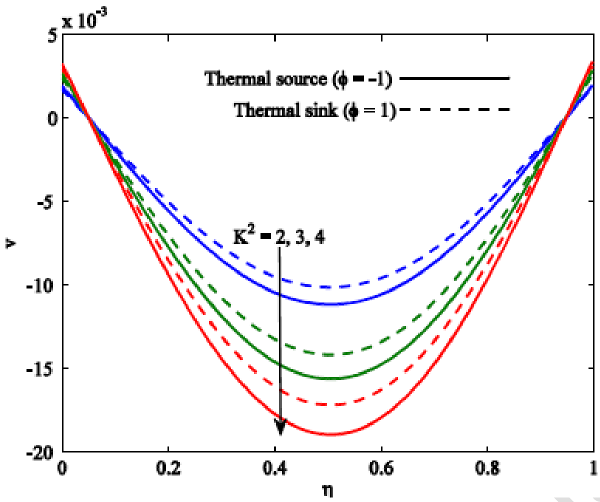

Figure 7 (b): The velocity Profile for u and v against K^2 *with* $M^2 = 4, G_r = 2, K_1 = 0.2, \omega = 3, \alpha = 0.05$

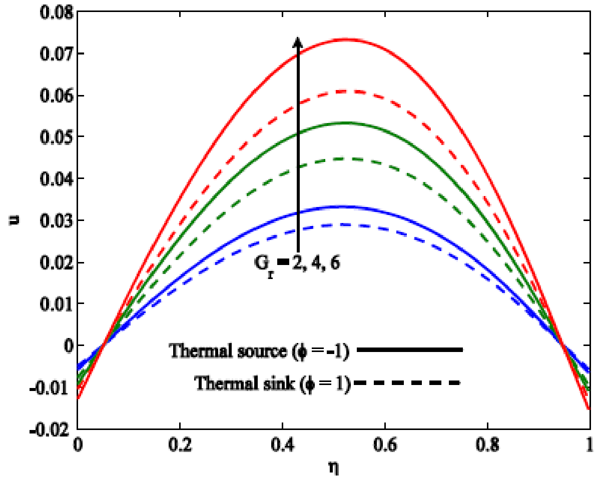

Figure 8 (a): The velocity Profile for u against G_r *with* $M^2 = 4, K^2 = 3, K_1 = 0.2, \omega = 3, \alpha = 0.05$

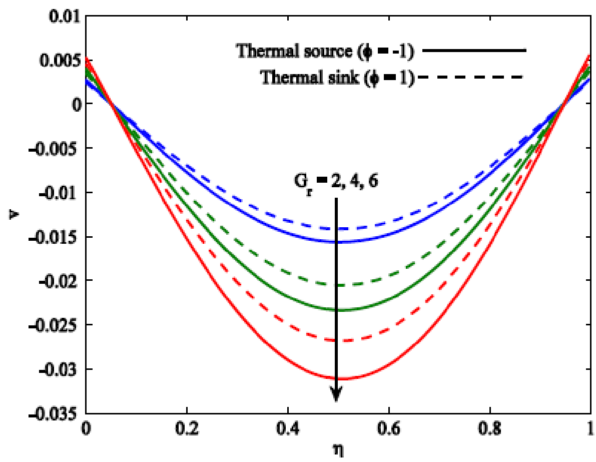

Figure 8 (b): The velocity Profile for v against G_r *with* $M^2 = 4, K^2 = 3, K_1 = 0.2, \omega = 3, \alpha = 0.05$

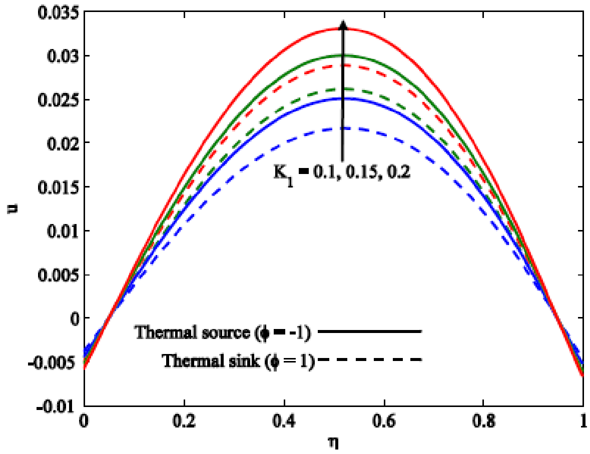

Figure 9 (a): The velocity Profile for u and v against K_1 *with* $M^2 = 4, K^2 = 3, G_r = 2, \omega = 3, \alpha = 0.05$

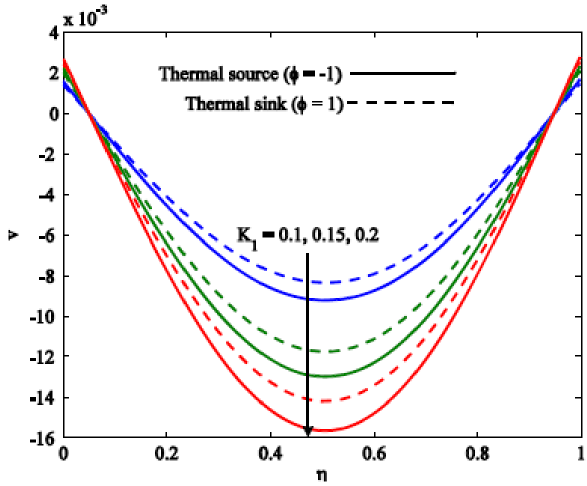

Figure 9 (b): The velocity Profile for u and v against K_1 *with* $M^2 = 4, K^2 = 3, G_r = 2, \omega = 3, \alpha = 0.05$

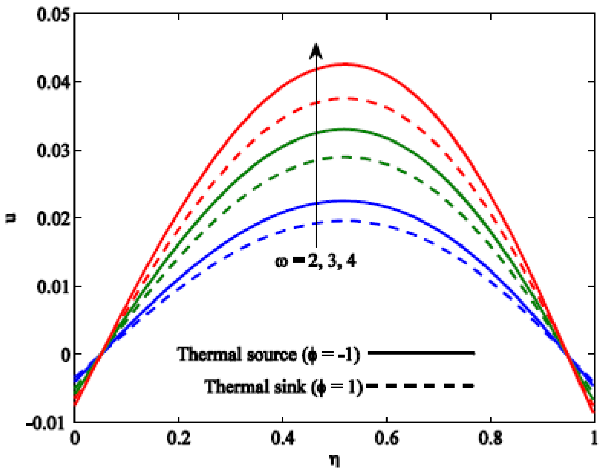

Figure 10(a): The velocity Profile for u against ω with $M^2=4, K^2=3, G_r=2, K_1=0.2, \alpha=0.05$

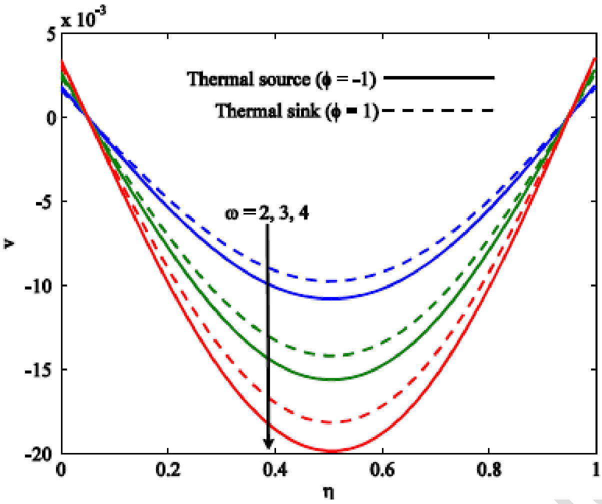

Figure 10(b): The velocity Profile for v against ω with $M^2=4, K^2=3, G_r=2, K_1=0.2, \alpha=0.05$

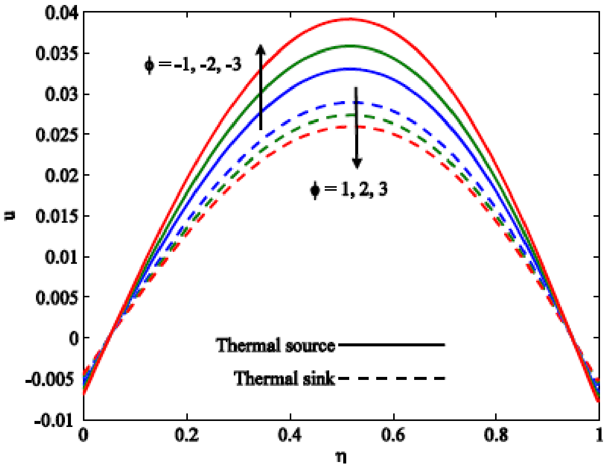

Figure 11: The velocity Profile for u against ϕ with $M^2=4, K^2=3, G_r=2, K_1=0.2, \omega=3, \alpha=0.05$

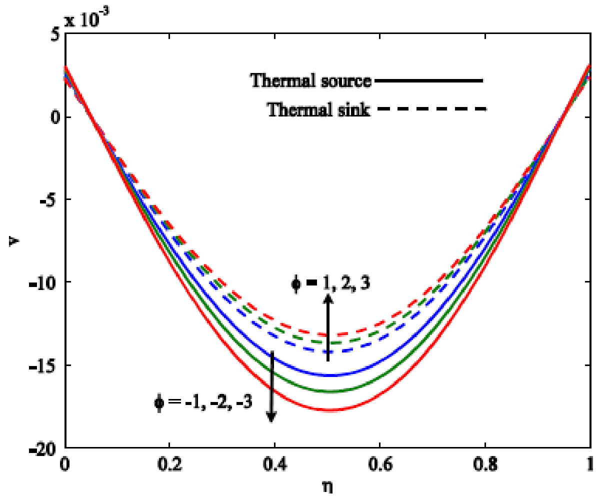

Figure 11 (b): The velocity Profile for v against ϕ with $M^2=4, K^2=3, G_r=2, K_1=0.2, \omega=3, \alpha=0.05$

M^2	K_1	α	K^2	Gr	Pr	τ_{xz}	τ_{yz}
2	0.2	0.05	2	2	0.71	5.455874	-2.685635
4	0.2	0.05	2	2	0.71	5.180014	-2.431979
6	0.2	0.05	2	2	0.71	4.994062	-2.238778
2	0.4	0.05	2	2	0.71	5.630854	-2.798579
2	0.6	0.05	2	2	0.71	5.833692	-2.856412
2	0.2	0.07	2	2	0.71	5.900142	-2.855569
2	0.2	0.1	2	2	0.71	6.874566	-3.552145
2	0.2	0.05	3	2	0.71	5.938664	-2.802852
2	0.2	0.05	4	2	0.71	6.285566	-2.919556
2	0.2	0.05	2	3	0.71	3.606121	-1.635212
2	0.2	0.05	2	4	0.71	2.814612	-1.265465
2	0.2	0.05	2	2	3	4.900744	-2.261745
2	0.2	0.05	2	2	7	4.533414	-2.153403

Table 1: Skin Friction

Pr	ω	t	Nu
0.71	$5\pi/2$	0.2	-1.60653
3	$5\pi/2$	0.2	-4.45861
7	$5\pi/2$	0.2	-8.61827
0.71	$7\pi/2$	0.2	-1.61538
0.71	$9\pi/2$	0.2	-1.61431
0.71	$5\pi/2$	0.4	-1.61854
0.71	$5\pi/2$	0.6	-1.60026

Table 2: Nusselt Number

APPENDIX

$$m_1=\left[\Pr(\phi+i\omega)\right]^{1/2},$$

$$m_2=\left[(1+\alpha i\omega)\left(\frac{1}{K_1}+M^2+i(2K^2+\omega)\right)\right]^{1/2},$$

$$m_3=\left[\Pr(\phi-i\omega)\right]^{1/2},$$

$$m_4=\left[(1-\alpha i\omega)\left(\frac{1}{K_1}+M^2+i(2K^2-\omega)\right)\right]^{1/2},$$

$$m_5 = \left[(1+\beta^2 m_2^2)\sinh m_2 - 2\beta m_2 \cosh m_2 \right]^{-1},$$

$$m_6 = \left[(1+\beta^2 m_4^2)\sinh m_4 - 2\beta m_4 \cosh m_4 \right]^{-1},$$

$$C_1 = -m_5 \left\{ \frac{R}{m_2} \left\{ \beta(1+\cosh m_2) - \frac{1}{m_2}\sinh m_2 \right\} + \frac{\beta m_2 \mathrm{Gr}}{2(m_1^2 - m_2^2)} \left\{ 1 - \frac{\beta m_1}{\sinh m_1} \left(\frac{\sinh m_2}{\alpha m_2} + \cosh m_1 - \cosh m_2 \right) \right\} \right.$$

$$C_2 = m_5 \left\{ \frac{\mathrm{Gr}}{2(m_1^2 - m_2^2)} \left\{ 1 - \frac{\beta m_1}{\sinh m_1} (\cosh m_1 + \cosh m_2 - \beta m_2 \sinh m_2) \right\} + \frac{R}{m_2^2}(1 - \cosh m_2 + \beta m_2 \sinh m_2) \right\}$$

$$C_3 = -m_6 \left\{ \frac{R}{m_4} \left\{ \beta(1+\cosh m_4) - \frac{1}{m_4}\sinh m_4 \right\} + \frac{\beta m_4 \mathrm{Gr}}{2(m_3^2 - m_4^2)} \left\{ 1 - \frac{\beta m_3}{\sinh m_3} \left(\frac{\sinh m_4}{\alpha m_4} + \cosh m_3 - \cosh m_4 \right) \right\} \right\}$$

$$C_4 = m_6 \left\{ \frac{\mathrm{Gr}}{2(m_3^2 - m_4^2)} \left\{ 1 - \frac{\beta m_3}{\sinh m_3} (\cosh m_3 + \cosh m_4 - \beta m_4 \sinh m_4) \right\} + \frac{R}{m_4^2}(1 - \cosh m_4 + \beta m_4 \sinh m_4) \right\}$$

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