International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 4 Issue 4, April 2017

ISSN: 2394-4404

A Profit Maximization Model For Yoghurt Production: Linear Programming Approach (A Case Study Of Rhan Tito Dairies, Makurdi – Benue State Of Nigeria)

James Friday Emmanuel
Department of Mathematics and Statistics,
Federal University Wukari, Wukari, Taraba State

Shewa Gladys A.
Department of Mathematical Sciences,
Taraba State University, Jalingo, Taraba State

Ameh Douglas Ekwule
Department of Mathematics/Statistics/Computer Science,
University of Agriculture, Makurdi, Benue State

***Abstract:** The primary purpose of establishing most firms is to derive profit on a daily, weekly or monthly basis. Thus, profit maximization is the main aim of organizations. This work is on the use and application of linear programming to obtain a maximum profit per day from the production of four varieties of yoghurt for Rhan-Tito Dairies, Makurdi metropolis – Nigeria. The linear programming model was solved using the simplex algorithm embedded in the TORA software. The result shows that 6 units of Probiotic, 22 units of Regular, 6 units of Furaghurt would yield a maximum profit of ₦178,686.27 per day. The Model identified that the labour used in the production was in excess of 10 units, which is recommended to be channelled to other departments for efficiency. Significant quantities of 90% of the raw materials in stock were unused. The management of Rhan Tito Dairies is advised to curtail expenses on these materials.*

***Keywords:** Linear programming, Optimization, Objective function, Constraints, Simplex Algorithm.*

I. INTRODUCTION

Often times, organization's managers are faced with the challenge of making decisions among several alternatives in relation to limited available resources – human labour (man), materials, mechanical resources (machines) and money. These limited resources pose a challenge as management would not achieve as much as it has targeted. It is incumbent on the management of the companies to decide on which resources would be allocated to obtain the optimum results in terms of profit.

A number of methods of optimization are available to help solve the aforementioned problem. One of the simplest methods used in obtaining optimal solution with several alternative solutions is the linear programming. Linear Programming is a method of allocating resources in an optimal way, as one of the most widely used Operation Research tools which has been successfully used as a decision making aid in almost all industries including financial and service organizations (Reeb and Leavengood , 1998). It is also a powerful quantitative tool used by operations managers and other managers to obtain optimal solutions to problems that involve restrictions or limitations, such as budget and available materials, labour and machine (Stevenson, 2005).

Linear programming model can be formulated and optimal solutions obtained to determine the best course of action in terms of profit within the constraints that exist. The model usually comprises the objective function and a set of constraints. This research is aimed at maximizing the objective function which is the daily profit from four products of yoghurt for the popular Rhan Tito Dairies in Makurdi, subject to raw materials and labour. The scope of the reply is to apply linear programming model to the products. The data obtained is based on quantity of raw materials, the maximum stock available and the profit gained per product. The profits constitute the objective function whereas the raw materials available in stock are used as constraints. The data is primary data collected by direct interview method.

The problem uses simplex algorithm invented by George Dantzig (1963). It is a standard technique for solving a linear programming problem since the 1940s. Simplex method passes from vertex to vertex on the boundary of the feasible polyhedron repeatedly increasing the objective function until either an optimal solution is found, or it is established that no solution exists. A linear programming problem as used in this work must satisfy major requirements viz: (Gupta and Hira, 1992)

- ✓ There must be a well-defined objective function which is either maximized or minimized and which can be expressed as a linear function of decision variables.
- ✓ It must have the presence of restrictions or constraints that limit the degree to which we can pursue our objective
- ✓ The decision variables should be interrelated and non-negative. This non-negativity condition shows that linear programming deals with real life situations for which negative quantities are generally illogical.
- ✓ There must be alternative course of action.

II. MATERIALS/DATA

The data used in this work was collected through the direct interview method as it is preferred to the questionnaire method since the researchers had a direct access to the management of Rhan Tito Dairies. The data covers:

- ✓ the varieties of products produced by the company
- ✓ the quantities of each product produced
- ✓ raw materials used
  - sugar
  - powdered milk
  - preservatives
  - treated water
  - yeast plus
  - millet
  - ginger
  - clove
  - sweetener
- ✓ the maximum stock of raw materials
- ✓ Daily profit from sales of each unit product

Raw Materials	**Products (yoghurt)**				**Maximum stock available**
	Probiotic	**Regular**	**Fura**	**Diet**	
Sugar (grams)	50	50	50	0	3578
Powdered milk (grams)	35	54	54	20	4500
Preservatives (ml)	25	20	20	10	5200
Treated water (litres)	56	50	50	5	5000
Yeast plus (grams)	20	0	0	20	4500
Millet (grams)	0	20	25	20	5235
Ginger (grams)	0	0	85	50	500
Clove (grams)	0	0	12	25	850
Sweetener (ml)	0	25	0	20	550
Labour (units)	15	15	15	15	500
Profit(₦)	2500	6500	3750	5000	

Table 1: Quantity of Raw materials, labour and daily profits per product

III. THE METHOD: SIMPLEX ALGORITHM

The simplex method uses a four step process (based on the Gauss Jordan method for solving a system of linear equations) to go from one tableau or vertex to the next. The steps are highlighted thus:

- ✓ Select the pivot column(the column with the "most negative: element in the objective function row)
- ✓ Select the pivot row (the row with the smallest non-negative result when the last element in the row is divided by the corresponding element in the pivot column.
- ✓ Calculate new values for the pivot row (divide every number in the row by the pivot number).
- ✓ Use row operations to make all numbers in the pivot column equal to zero except for the pivot number

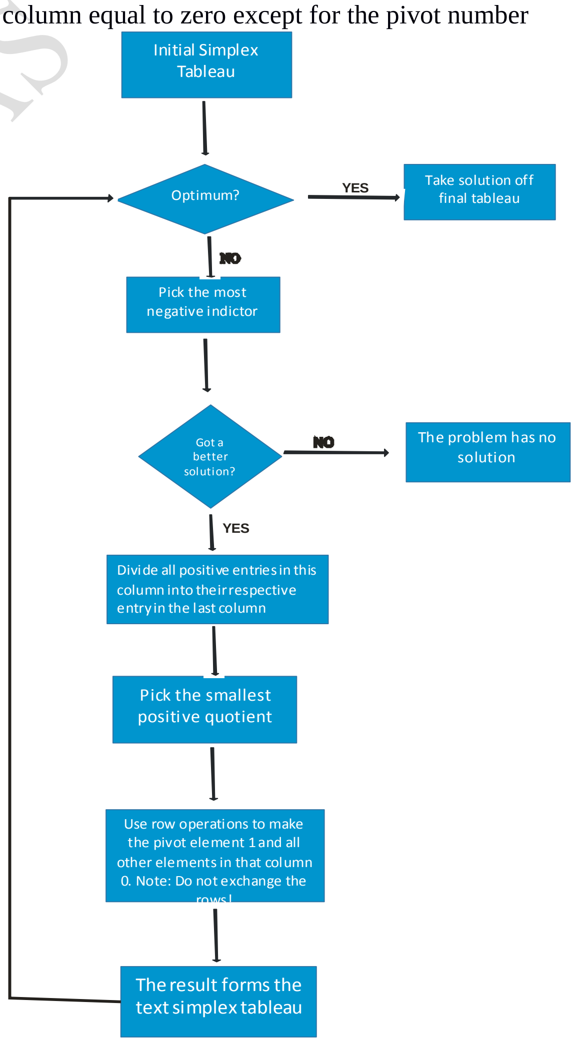

Figure 1: Simplex Algorithm Flowchart

IV. MODEL FORMULATION

Objective function can generally be written as:

$$Z = C_1X_1 + C_2X_2 + C_3X_3 + \cdots + C_nX_n$$

Where n=the number of decision variables (that is, the products) = 4

C_n = objective coefficient

X_1 = unit of probiotic Yoghurt

X_2 = unit of Regular Yoghurt

X_3 = unit of Furaghurt

X_4 = unit of Dietghurt

Hence the objective function becomes:

$$Z = 2500X_1 + 6500X_2 + 3750X_3 + 5000X_4$$

The constraint matrix:

Sugar:

$$50X_1 + 50X_2 + 50X_3 + 0X_4 \leq 3758$$

Powdered milk:

$$35X_1 + 54X_2 + 54X_3 + 20X_4 \leq 4500$$

Preservatives:

$$25X_1 + 20X_2 + 20X_3 + 10X_4 \leq 5200$$

Treated water:

$$56X_1 + 50X_2 + 50X_3 + 5X_4 \leq 5000$$

Yeast plus:

$$20X_1 + 0X_2 + 0X_3 + 20X_4 \leq 4500$$

Millet:

$$0X_1 + 20X_2 + 25X_3 + 20X_4 \leq 5235$$

Ginger:

$$0X_1 + 0X_2 + 85X_3 + 50X_4 \leq 500$$

Clove:

$$0X_1 + 0X_2 + 12X_3 + 25X_4 \leq 850$$

Sweetener:

$$0X_1 + 25X_2 + 0X_3 + 20X_4 \leq 550$$

Labour:

$$15X_1 + 15X_2 + 15X_3 + 15X_4 \leq 500$$

$X_1, X_2, X_3, X_4 \geq 0$ (This is the non-negativity condition)

To solve a Linear Programming problem using the simplex algorithm, it is important to first of all standardize the inequalities. This is to eliminate the inequalities by adding new variables which represent the amount by which the solution avoids violating the constraints. These variables are called slack or surplus variables.

To obtain a values with a high degree of precision, the TORA software is not out of place in solving such a problem as this.

Maximize: $Z = 2500X_1 + 6500X_2 + 3750X_3 + 5000X_4$

Subject to:

$$50X_1 + 50X_2 + 50X_3 + 0X_4 + S_1 = 3578$$
$$35X_1 + 54X_2 + 54X_3 + 20X_4 + S_2 = 4500$$
$$25X_1 + 20X_2 + 20X_3 + 10X_4 + S_3 = 5200$$
$$56X_1 + 50X_2 + 50X_3 + 5X_4 + S_4 = 5000$$
$$20X_1 + 0X_2 + 0X_3 + 20X_4 + S_5 = 4500$$
$$0X_1 + 0X_2 + 25X_3 + 20X_4 + S_6 = 1235$$
$$0X_1 + 0X_2 + 85X_3 + 50X_4 + S_7 = 500$$
$$0X_1 + 0X_2 + 12X_3 + 25X_4 + S_8 = 850$$
$$0X_1 + 25X_2 + 0X_3 + 20X_4 + S_9 = 550$$
$$15X_1 + 15X_2 + 15X_3 + 15X_4 + S_{10} = 500$$

For: $X_1, X_2, X_3, X_4 \geq 0,\ S_1, S_2, S_3, S_4, \ldots S_{10} \geq 0$

Where S_1, S_2, S_3, S_4, ...S_{10} are slack variables

V. RESULTS

Figure 2: Original Data Entry

Figure 3: Final Iteration/Solution

The result is extracted from the Linear Programming output summary. The maximum profit is =.

The results show that for optimum production and to obtain a maximum profit of ₦178,686.27 per day, approximately 6 units (5.45) of X_1 (Probiotic Yoghurt), 22 units of X_2 (Regular Yoghurt) , and approximately 6 units (5.88) of X_3 (Furaghurt) should be produced.

The objective function is as shown below:

$$Z_{max} = C^TX = C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4$$

$= 2500(5.45) + 6500(22) + 3750(5.88)$

$=$ ₦178,675

Using the raw material constraints, the needed raw material for optimum production are:

Sugar:

$$50(6) + 50(22) + 50(6) = 1700g$$

Powdered milk:

$$35(6) + 54(22) + 54(6) = 1722g$$

Preservatives:

$$25(6) + 20(22) + 20(6) = 710ml$$

Treated Water:

$$56(6) + 50(22) + 50(6) = 1736litres$$

Yeast Plus:

$$20(6) = 120g$$

Millet:

$$25(6) = 150g$$

Ginger:

$85(6) = 510g$

Clove:

$12(6) = 72g$

Sweetener:

$25(22) = 550ml$

Labour:

$15(6) + 15(22) + 15(6) = 510$ units

VI. RECOMMENDATION

From the computation, it is noticed that at optimum production levels, the amount of stock of sweetener was exhausted. However, remnant of sugar, powdered milk, preservatives, treated water, millet, ginger and clove are observed and so should be curtailed.

Furthermore, the units of labour in the production is in excess by 10. These should be utilized in other departments of the company to achieve optimality and avoid wastage.

This work embarked upon was to use the meagre resources to maximize profits and find the best production plan for the case study. By using the simplex method, the efficacious solution has been:

- ✓ A realizable maximum profit of ₦178,686.27/day
- ✓ Determination of a commensurate level of optimal production of the current range of products. That is, 6 units of probiotic Yoghurt; 22 units of Regular Yoghurt and 6 units of Furaghurt
- ✓ Determination of maximum amount of resources being used for the range of optimality.

The simplex method is therefore recommended to Rhan Tito Dairies, Makurdi and other production companies to be applied in their production processes, so as to obtain an optimum production and determine among the limited resources, the optimal quantities of raw materials that would generate the maximum profit and avoid wastage.

REFERENCES

[1] Balogun O.S; Jolayemi E.T; Akingbade T.J and Muazu H.G (2012): *Use of Linear Programming for optimal production in a production line in Coca-Cola Bottling Company Ilorin*, International Journal of Engineering Research and Application, 2(5), pp 2004 – 2007.

[2] Dantziq G.B (1963): *Linear Programming and Extensions*, Princeton, NJ Princeton University Press

[3] Prem Kumav Gupta and Hira D.S (1992): *Operations Research*, S.Chand and Company pvt Ltd, RAM NAGAR, New Delhi

[4] Reebs J and Leavengood S (1998): *Using the Graphical Method to solve Linear programs*, Performance Excellence in the wood products industry, Oregon State University Extension Service.

[5] Stevenson W.J (2005): *Operations Management*, McGraw Hill, New York