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Radiation Effects On An Oscillatory Flow Of A Viscous Fluid In A Circular Tube

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Abstract: In this problem, we studied the effects of radiation on oscillatory flow of viscous incompressible fluid in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. The effects of various emerging parameters on the velocity field and temperature field studied in detail with the help of graphs.

Keywords: Radiation on oscillatory flow, Cylindrical tubes, parameters on the velocity field and temperature field.

I. INTRODUCTION

The study of oscillatory flow of a viscous fluid in cylindrical tubes has received the attention of many researchers as they play a significant role in understanding the important physiological problem, namely the blood flow in arteriosclerotic blood vessel. Womersley (1955) have investigated the oscillating flow of thin walled elastic tube. Unsteady and oscillatory flow of viscous fluids in locally constricted, rigid, axisymmetric tubes at low Reynolds number has been studied by Ramachandra Rao and Devanathan (1973), Hall(1974) and Schneck and Ostrach (1975). Haldar (1987) have considered the oscillatory flow of a blood through an artery with a mild constriction. Several other workers, Misra and Singh (1987), Ogulu and Alabraba (1992), Tay and Ogulu (1997) and Elshahed (2003), to mention but a few, have in one way or the other modeled and studied the flow of blood through a rigid tube under the influence of pulsatile pressure gradient.

In view of these, we studied the effects of radiation on oscillatory flow of viscous incompressible fluid in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. The effects of various emerging parameters on the velocity field and temperature field studied in detail with the help of graphs.

II. MATHEMATICAL FORMULATION

We consider an oscillatory flow of a Newtonian fluid in a heated uniform cylindrical tube of constant radius R. The wall of the tube is maintained at a temperature T_w. We choose the cylindrical coordinates (r, θ, z) such that $r = 0$ is the axis of symmetry. The flow is considered as axially symmetric and fully developed. The geometry of the flow is shown in Fig. 1.

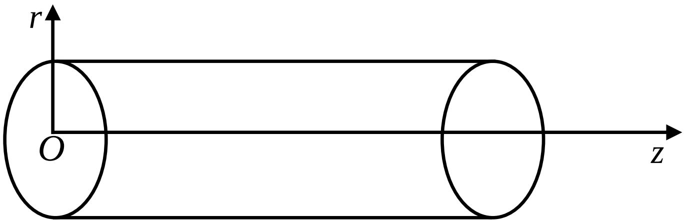

Figure 1: The physical model of the problem

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The equations governing the flow are given by

$$\rho\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\rho g\beta\left(T-T_\infty\right) \quad (1)$$

$$\rho c_p\frac{\partial T}{\partial t}=k_0\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)-\frac{\partial q}{\partial r} \quad (2)$$

where ρ is the fluid density, μ is the fluid viscosity, p is the pressure, w is the velocity component in z-direction, g is the acceleration due to gravity, σ is electrical oconductivity, β coefficient of thermal expansion, T is the temperature, k_0 is the thermal conductivity and c_p is the specific heat at constant pressure. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y}=4\alpha^2\left(T_\infty-T\right) \quad (3)$$

here α is the mean radiation absorption coefficient.

The appropriate boundary conditions are

$$w=0,\quad T=T_w \quad \text{at} \quad r=R$$

$$\frac{\partial w}{\partial r}=0,\quad T=T_\infty \quad \text{at} \quad r=0 \quad (4)$$

Introducing the following non-dimensional variables

$$\bar{r}=\frac{r}{R},\ \bar{z}=\frac{z}{R},\ \bar{t}=\frac{w_0 t}{R},\ \alpha^2=\frac{\rho R^2}{\mu},\ \lambda=\frac{R}{w_0},$$

$$\bar{w}=\frac{w}{w_0},\ \bar{p}=\frac{p-p_w}{\mu},\ \theta=\frac{T-T_\infty}{T_w-T_\infty}$$

$$\Pr=\frac{\mu c_p}{k_0},\quad \mathrm{Re}=\frac{\rho w_0 R}{\mu},$$

into the Eqs. (1) and (2), we get (after dropping bars)

$$\mathrm{Re}\frac{\partial w}{\partial t}=-\lambda\frac{dp}{dz}+\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right]+\frac{Gr}{\mathrm{Re}}\theta \quad (5)$$

$$\Pr\mathrm{Re}\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+N^2\theta \quad (6)$$

where $\Pr$ is the Prandtl number, is the Darcy number and Re is the Reynolds number.

The corresponding non-dimensional boundary conditions are

$$w=0,\quad \theta=1 \quad \text{at} \quad r=1$$

$$\frac{\partial w}{\partial r}=0,\quad \frac{\partial\theta}{\partial r}=0 \quad \text{at} \quad r=0 \quad (7)$$

III. SOLUTION

It is fairly unanimous that, the pumping action of the heart results in a pulsatile blood flow so that we can represent the pressure gradient (pressure in the left ventricle) as

$$-\frac{dp}{dz}=p_0 e^{iwt} \quad (8)$$

and flow variables expresses as

$$\theta\left(y,t\right)=\theta_0\left(r\right)e^{iwt} \quad (9)$$

$$w\left(y,t\right)=w_0\left(r\right)e^{iwt} \quad (10)$$

Substituting Eqs. (8) - (10) into Eqs. (5) and (6) and solving the resultant equations subject to the boundary conditions in (7), we obtain

$$\theta_0=\frac{I_0\left(\Omega r\right)}{I_0\left(\Omega\right)} \quad (11)$$

$$w_0=\frac{Gr}{\mathrm{Re}}\frac{1}{\left(\beta_1^2+\Omega^2\right)}\left[\frac{I_0\left(\beta_1 r\right)}{I_0\left(\beta_1\right)}-\frac{I_0\left(\Omega r\right)}{I_0\left(\Omega\right)}\right]+\frac{\lambda p_0}{\beta_1^2}\left[1-\frac{I_0\left(\beta_1 r\right)}{I_0\left(\beta_1\right)}\right] \quad (12)$$

Here $\Omega^2=i\omega\Pr\mathrm{Re}-N^2$, $\beta_1^2=\left(i\omega\mathrm{Re}\right)$ and $I_0\left(x\right)$ is the modified Bessel function of first kind of order zero.

Thus the temperature distribution and the axial velocity are given by

$$\theta=\frac{I_0\left(\Omega r\right)}{I_0\left(\Omega\right)}e^{i\omega t} \quad (13)$$

$$w=\left(\frac{Gr}{\mathrm{Re}}\frac{1}{\left(\beta_1^2+\Omega^2\right)}\left[\frac{I_0\left(\beta_1 r\right)}{I_0\left(\beta_1\right)}-\frac{I_0\left(\Omega r\right)}{I_0\left(\Omega\right)}\right]+\frac{\lambda p_0}{\beta_1^2}\left[1-\frac{I_0\left(\beta_1 r\right)}{I_0\left(\beta_1\right)}\right]\right)e^{i\omega t} \quad (14)$$

IV. DISCUSSION OF THE RESULTS

Fig.2 depicts the effects of Prandtl number Pr on w for, $N=1,\ p=1,\ \omega=1,\ \lambda=0.1, Gr=1, \mathrm{Re}=1$ and $t=0.1$. It is noted that, the axial velocity w increases on increasing Prandtl number Pr. effects of radiation parameter N on w for $p=1,\ \omega=1, Gr=1,\ \lambda=0.1,\ Pr=0.7, \mathrm{Re}=1$ and $t=0.1$ is presented in Fig. 3. It is observed that, the axial velocity w increases with increasing N.

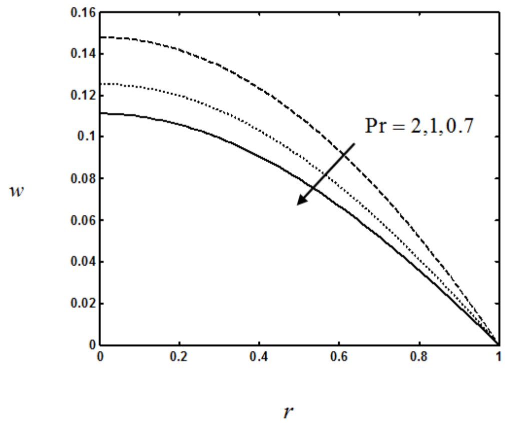

Figure 2: Effects of Prandtl number $\Pr$ on w for $p_0=1$, $\omega=1,\lambda=0.1,\ Gr=1,\ N=1,\ \mathrm{Re}=1$ and $t=0.1$

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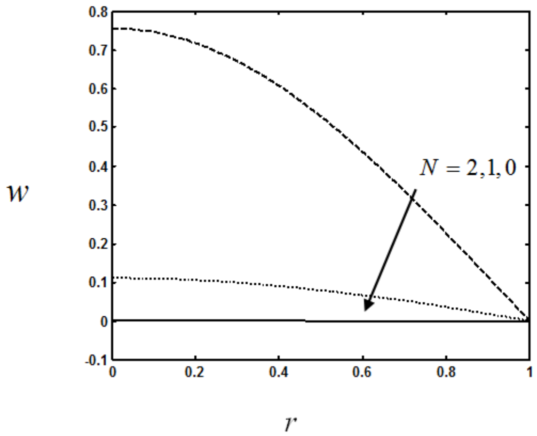

Figure 3: Effects of radiation parameter N *on* w *for* $p_0=1$, $\omega=10, \lambda=0.5$, $Pr=0.7, Gr=1$, $\mathrm{Re}=1$ *and* $t=0.1$

Fig.4 depicts the effects of Grashof number Gr on w for $N=1$, $p=1$, $\omega=1$, $\lambda=0.1, \mathrm{Pr}=0.7, \mathrm{Re}=1$ and $t=0.1$. It is noted that, the axial velocity w increases at the axis of tube on increasing Grashof number Gr, while it decreases near the tube wall with increasing Gr.

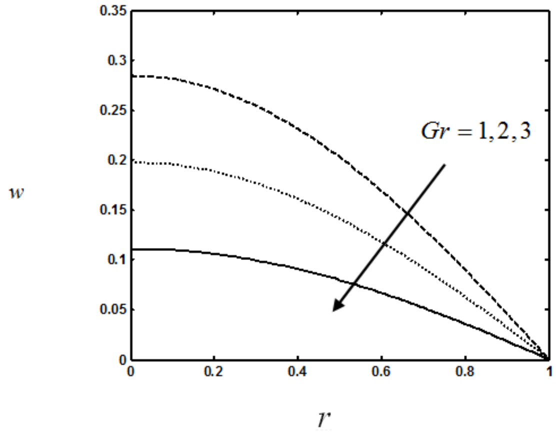

Figure 4: Effects of Grashof number Gr *on* w *for* $p_0=1$, $\omega=10, \lambda=0.5$, $Pr=0.7, N=1$, $\mathrm{Re}=1$ *and* $t=0.1$

The effect of Reynolds number Re on w for $N=1$, $p=1, \omega=1$, $\lambda=0.1$, $Pr=0.7, \mathrm{Re}=1$ and $t=0.1$ is shown in Fig. 5. It is found that, the axial velocity w decreases with an increase in Reynolds number Re.

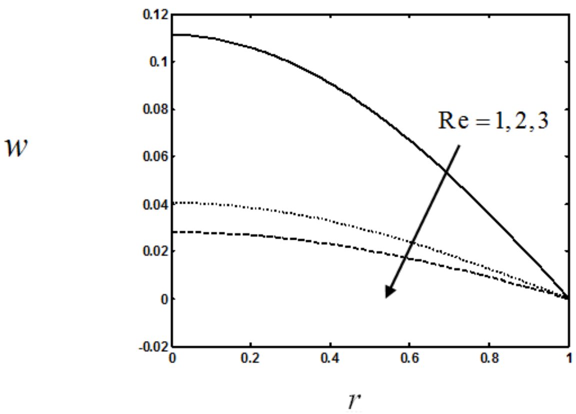

Figure 5: Effects of Reynolds number Re *on* w *for* $p_0=1$, $\omega=10, \lambda=0.5$, $Pr=0.7, Gr=1$, $N=1$, *and* $t=0.1$

Fig. 6 shows the effect of λ on w for $N=1$, $Pr=0.7$, $Gr=1$, $p=1$, $\omega=1$, $\mathrm{Re}=1$ and $t=0.1$. It is observed that, the axial velocity w increases on increasing λ.

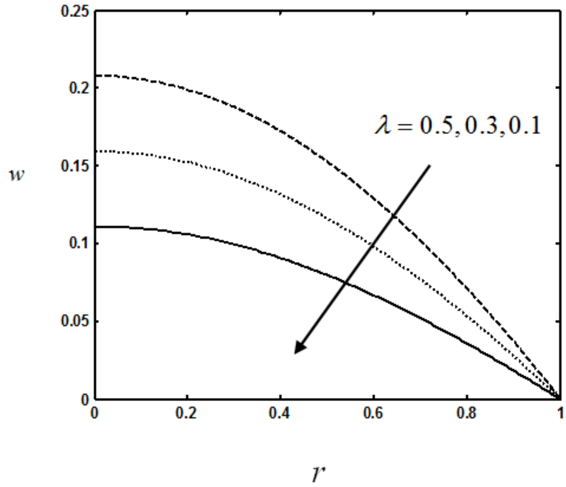

Figure 6: Effects of pressure constant λ *on* w *for* $p_0=1$, $\omega=10, Pr=0.7, Gr=1$, $N=1$, $\mathrm{Re}=1$ *and* $t=0.1$.

Effect of oscillating parameter ω on w for $p_0=1$, $\lambda=0.1$, $Pr=0.7$, $Gr=1$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$ is depicted in Fig. 7. It is found that, the axial velocity w increases with increasing ω.

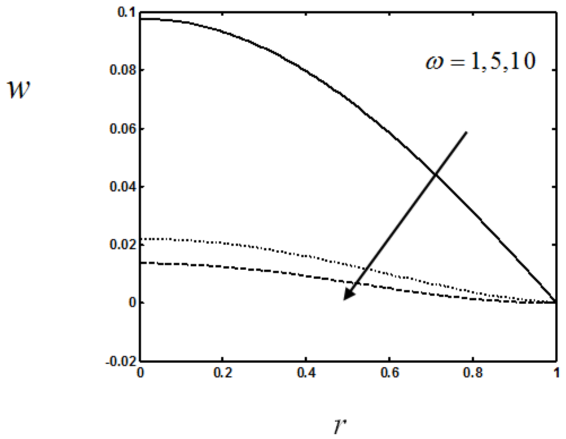

Figure 7: Effects of oscillating parameter ω *on* w *for* $M=1$, $p_0=1$, $\lambda=0.5$, $Pr=0.7, Gr=1$, $N=1$, $\mathrm{Re}=1$ *and* $t=0.1$

Fig. 8 shows the effect of Prandtl number Pr on θ for $\omega=10$, $N=1$, $\mathrm{Re}=1$ and $t=0.1$. It is found that, the temperature θ decreases with increasing Prandtl number Pr.

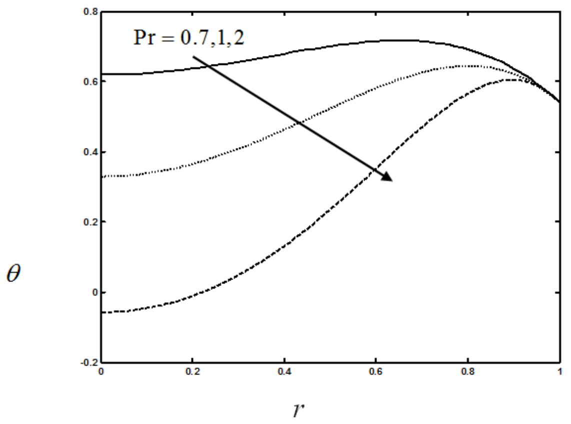

Figure 8: Effects of Prandtl number $\Pr$ *on* θ *for* $\omega = 10$, $N = 1$, $\mathrm{Re} = 1$ *and* $t = 0.1$

Effect of radiation parameter N on θ for $\omega = 10$, $Pr = 0.7$, $\mathrm{Re} = 1$ and $t = 0.1$ is shown in Fig. 9. It is noted that, the temperature θ increases with increasing N.

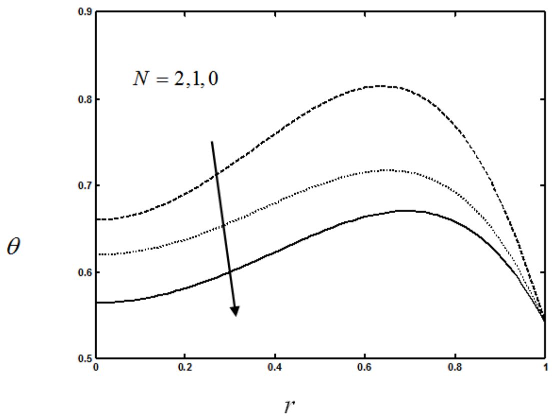

Figure 9: Effects of radiation parameter N *on* θ *for* $\omega = 10$, $Pr = 0.7$, $\mathrm{Re} = 1$ *and* $t = 0.1$

Fig. 10 depicts the effect of Reynolds number Re on θ for $\omega = 10$, $Pr = 0.7$, $N = 1$ and $t = 0.1$. It is observed that, the temperature θ decreases with increasing Reynolds number Re.

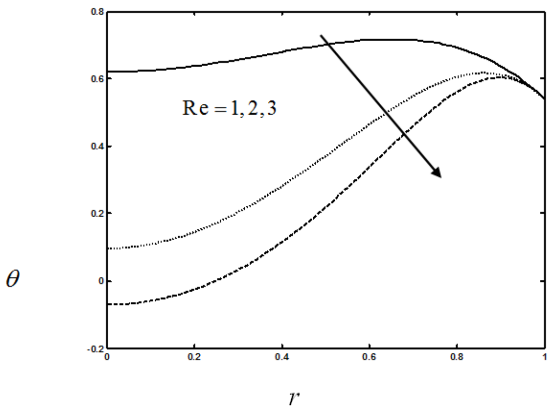

Figure 10: Effects of Reynolds number Re *on* θ *for* $\omega = 10$, $Pr = 0.7$, $N = 1$ *and* $t = 0.1$

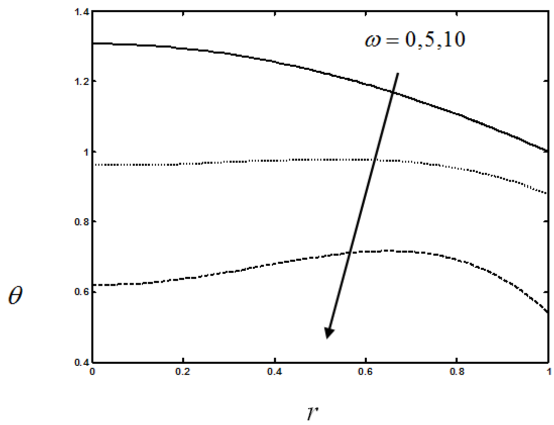

Figure 11: Effects of oscillating parameter ω *on* θ *for* $Pr = 0.7$, $N = 1$, $\mathrm{Re} = 1$ *and* $t = 0.1$

Effects of oscillating parameter ω on θ for $Pr = 0.7$, $N = 1$, $\mathrm{Re} = 1$ and $t = 0.1$ is depicted in Fig. 11. It is found that, the temperature θ increases with increasing ω.

V. CONCLUSIONS

In this chapter, we studied the effects of radiation on oscillatory flow of viscous incompressible fluid in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. It is found that the velocity w increases with increasing $\Pr, N, Gr, \omega$ and λ, while it decreases with increasing Re; the temperature increases with increasing $\Pr, N$ and ω, while it decreases with increasing Re.

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