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Mean Time To Recruitment For A Multigrade Manpower System With Two Sources Of Depletion When The Breakdown Threshold Distribution Follows Scbz Property

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***Abstract:** In this paper a multi graded organization in which depletion of manpowers occur due to policy and transfer decisions is considered when the breakdown threshold distribution follows SCBZ property. Mean time to recruitment is obtained by using an univariate CUM policy of recruitment (ie)"The organization survives iff atleast $r\ (1 \le r \le n)$ out of n grades survives in the sense that threshold crossing has not take place in these grades". The influence of the nodal parameter on the system characteristics is studied and relevant conclusions are presented.*

***Keywords:** Loss of man hours, policy decision, transfer decision, threshold, SCBZ property.*

I. MODEL DESCRIPTION AND ASSUMPTIONS

- ✓ An organization having n-grades in which decisions are taken at random epochs $(0,\infty)$ is considered.
- ✓ At every policy decision epoch a random number of person quit the organization and at every transfer decision epoch a random number of persons are transferred.
- ✓ It is assumed that the loss of manpower is linear and cumulative.
- ✓ The loss of manpower process, process of inter policy and inter transfer decision times are statistically independent.
- ✓ The thresholds for the n-grades are independent and identically distributed random variables following SCBZ property with same parameter.
- ✓ Univariate CUM policy of recruitment "The organization survives iff atleast $r\ (1 \le r \le n)$ out of n grades survives in the sense that threshold crossing has not take place in these grades".

II. NOTATIONS

x_i: Continuous random variable denoting the amount of depletion of manpower caused due to the i^th policy decision in organization.

t_i: time of occurrence of the i^th decision.

$\bar{x}_{m1}$: Cumulative loss of manpower due to the first $m1$ policy decisions in the Organization.

y_j: Continuous random variable denoting the amount of depletion of manpowers caused due to the j^th transfer decision in organization.

$\bar{y}_{n1}$: Cumulative loss of manpower due to the first $n1$ transfer decisions in the Organization.

$W_U(.)$:The distribution function of inter policy decision times with hyper exponential i.i.d random variable.

$W_V(.)$:The distribution function of inter transfer decision times with exponential i.i.d random variable.

$W_U{}^{m1}(.)$: $m1$ fold convolution of W_U with itself.

$W_V{}^{n1}(.)$: $n1$ fold convolution of W_V with itself.

$\bar{x}_{m1} + \bar{y}_{n1}$: The cumulative loss of manpower due to $m1$ policy decisions and $n1$ transfer decision.

$\bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}$: Distribution functions of cumulative loss of manpower due to $m1$ policy decisions and $n1$ transfer decision.

T: Time to recruitment.

$E(T)$ Mean time to recruitment.

$N_p(T)$: Number of policy decisions at time T.

$N_{Trans}(T)$ Number of transfer decisions at time T.

III. MAIN RESULT

From renewal theory, the survival function of T is

$$P(T > t) = P(\bar{x}_{N_p}(T) + \bar{y}_{N_{Trans}}(T) < Z) \quad (1)$$

Conditioning upon $N_p(T)$ and $N_{Trans}(T)$ and using law of total probability

$$P(T > t) = \sum_{m1=0}^{\infty} P(N_p(T) = m1) \sum_{n1}^{\infty} P(N_{Trans}(T) = n1)\, P(\bar{x}_{m1} + \bar{y}_{n1} \leq Z) \quad (2)$$

As $\{N_p(T)\}$ and $\{N_{Trans}(T)\}$ are two independent ordinary renewal process by hypothesis invoking the result.

$P(N_p(T) = m_1) = W_U^{m1}(t) - W_U^{m1+1}(t)$ and

$P(N_{Trans}(T) = n_1) = W_V^{n1}(t) - W_V^{n1+1}(t)$

$$P(T > t) = \sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)]\, P(\bar{X}_{m1} + \bar{Y}_{n1} \leq Z) \quad (3)$$

Where $W_U^{0}(t) = W_V^{0}(t) = 1$

Let $z_j, j = 1,2, \dots n$ follows SCBZ property with same parameter

$$H(z_j) = \begin{cases} 1 - e^{-\theta_1 z_j} & z_j \leq \tau_0 \\ 1 - e^{-\theta_1 z_j} e^{-\theta_2 (z_j - \tau_0)} & \tau_0 \leq z_j \end{cases}$$

$j = 1, \dots n$ for fixed $\tau = \tau_0$

Assuming the truncation level, itself a random variable such that τ follows exponential with parameter θ, we have by law of total probability,

$$H(z) = 1 - \left(\frac{\theta_1 - \theta_2}{\theta + \theta_1 - \theta_2}\right) e^{-(\theta+\theta_1)z} - \left(\frac{\theta}{\theta + \theta_1 - \theta_2}\right) e^{-\theta_2 z}$$

$$H(z) = 1 - pe^{-(\theta+\theta_1)z} - qe^{-\theta_2 z} \quad \dots\dots(4)$$

Where $p = \left(\frac{\theta_1-\theta_2}{\theta+\theta_1-\theta_2}\right)$ and $q = \left(\frac{\theta}{\theta+\theta_1-\theta_2}\right)$ with p+q=1(5)

Since Z is independent of $\bar{x}_{m1}$and$\bar{y}_{n1}$,by hypothesis conditioning upon z and using law of total probability

$$P(\bar{x}_{m1} + \bar{y}_{n1} \leq Z) = \int_0^{\infty} P(\bar{x}_{m1} + \bar{y}_{n1} < z) h(z) dz \quad \dots(6)$$

where

$$H(z) = 1 - \sum_{i=r}^{n} nC_i \left\{ [pe^{-(\theta+\theta_1)z} + qe^{-\theta_2 z}]^i [1 - pe^{-(\theta+\theta_1)z} - qe^{-\theta_2 z}]^{n-i} \right\}$$

$$\begin{aligned} h(z) = \sum_{i=r}^{n} nC_i \Big\{ &\{p^i i(\theta+\theta_1) e^{-i(\theta+\theta_1)z} + \cdots q^i i\theta_2 e^{-i(\theta_2)z}\} \\ &- (n-i)C_1 \{p^{i+1}(i+1)(\theta+\theta_1) e^{-(i+1)(\theta+\theta_1)z} + \cdots q^{i+1}(i+1)(\theta_2) e^{-(i+1)(\theta_2)z}\} \\ &+ \dots (-1)^{n-i} \{p^n n(\theta+\theta_1) e^{-n(\theta+\theta_1)z} + \cdots q^n n\theta_2 e^{-n(\theta_2)z}\} \Big\} \end{aligned} \quad (7)$$

$$P(\bar{x}_{m1} + \bar{y}_{n1} \leq Z)$$

$$= \int_0^{\infty} P(\bar{x}_{m1} + \bar{y}_{n1} < z) \times \sum_{i=r}^{n} nC_i \left\{ \begin{matrix} \{p^i i(\theta+\theta_1) e^{-i(\theta+\theta_1)z} + \cdots q^i i\theta_2 e^{-i(\theta_2)z}\} \\ -(n-i)C_1 \{p^{i+1}(i+1)(\theta+\theta_1) e^{-(i+1)(\theta+\theta_1)z} + \cdots q^{i+1}(i+1)(\theta_2) e^{-(i+1)(\theta_2)z}\} + \\ \dots (-1)^{n-i} \{p^n n(\theta+\theta_1) e^{-n(\theta+\theta_1)z} + \cdots q^n n\theta_2 e^{-n(\theta_2)z}\} \end{matrix} \right\} dz \quad (8)$$

Substituting equation (8) in (3)

$$\begin{aligned} P(T > t) = &\sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)] \int_0^{\infty} P(\bar{x}_{m1} + \bar{y}_{n1} < z) \\ &\times \sum_{i=r}^{n} nC_i \Big\{ \{p^i i(\theta+\theta_1) e^{-i(\theta+\theta_1)z} + \cdots q^i i\theta_2 e^{-i(\theta_2)z}\} \\ &- (n-i)C_1 \{p^{i+1}(i+1)(\theta+\theta_1) e^{-(i+1)(\theta+\theta_1)z} + \cdots q^{i+1}(i+1)(\theta_2) e^{-(i+1)(\theta_2)z}\} \\ &+ \dots (-1)^{n-i} \{p^n n(\theta+\theta_1) e^{-n(\theta+\theta_1)z} + \cdots q^n n\theta_2 e^{-n(\theta_2)z}\} \Big\} dz \end{aligned} \quad (9)$$

$$P(T > t) = \sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)]$$

$$\times \sum_{i=r}^{n} nC_i \left\{ \begin{matrix} \{p^i(\theta+\theta_1)\bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}(i(\theta+\theta_1)) + \cdots q^i i\theta_2 \bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}(i(\theta_2))\} - \\ (n-i)C_1\{p^{i+1}(i+1)(\theta+\theta_1)\bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}((i+1)(\theta+\theta_1)) + \cdots q^{i+1}(i+1)\theta_2 \bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}((i+1)\theta_2)\} + \\ \dots(-1)^{n-i}\{p^n n(\theta+\theta_1)\bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}(n(\theta+\theta_1)) + \cdots + q^n n\theta_2 \bar{W}_{\bar{x}_{m1}+\bar{y}_{n1}}(n(\theta_2))\} \end{matrix} \right\}$$

$$P(T > t) = \sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)]$$

$$\times \sum_{i=r}^{n} nC_i \left\{ \begin{matrix} \left\{p^i i(\theta+\theta_1)\frac{\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(i(\theta+\theta_1))}{i(\theta+\theta_1)} + \cdots q^i i\theta_2 \frac{\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(i(\theta_2))}{i\theta_2}\right\} - \\ (n-i)C_1\left\{p^{i+1}(i+1)(\theta+\theta_1)\frac{\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}((i+1)(\theta+\theta_1))}{(i+1)(\theta+\theta_1)} + \cdots q^{i+1}(i+1)\theta_2 \frac{\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}((i+1)\theta_2)}{(i+1)\theta_2}\right\} + \\ \dots(-1)^{n-i}\left\{p^n n(\theta+\theta_1)\frac{\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(n(\theta+\theta_1))}{n(\theta+\theta_1)} + \cdots + q^n n\theta_2 \frac{\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(n(\theta_2))}{n\theta_2}\right\} \end{matrix} \right\}$$

$$P(T > t) = \sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)]$$

$$\times \sum_{i=r}^{n} nC_i \left\{ \begin{matrix} \{p^i \bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(i(\theta+\theta_1)) + \cdots + q^i \bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(i(\theta_2))\} - \\ (n-i)C_1\{p^{i+1}\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}((i+1)(\theta+\theta_1)) + \cdots q^{i+1}\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}((i+1)\theta_2)\} + \\ \dots(-1)^{n-i}\{p^n \bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(n(\theta+\theta_1)) + \cdots + q^n \bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(n(\theta_2))\} \end{matrix} \right\} \quad (10)$$

Where $\bar{w}_{\bar{x}_{m1}+\bar{y}_{n1}}(\theta) = \{\bar{w}_{\bar{x}_i}(\theta)\}^{m1} \{\bar{w}_{\bar{y}_i}(\theta)\}^{n1}$ (11)

Using (11) in (10)

$$P(T > t) = \sum_{i=r}^{n} nC_i \sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)]$$

$$\times \left\{ \begin{matrix} \{p^i\{\bar{w}_{\bar{x}_i}(i(\theta+\theta_1))\}^{m1}\{\bar{w}_{\bar{y}_i}(i(\theta+\theta_1))\}^{n1} + \cdots q^i\{\bar{w}_{\bar{x}_i}(i\theta_2)\}^{m1}\{\bar{w}_{\bar{y}_i}(i\theta_2)\}^{n1}\} \\ -(n-i)C_1 \times \left\{ \begin{matrix} p^{i+1}\{\bar{w}_{\bar{x}_i}((i+1)(\theta+\theta_1))\}^{m1}\{\bar{w}_{\bar{y}_i}((i+1)(\theta+\theta_1))\}^{n1} + \cdots \\ q^{i+1}\{\bar{w}_{\bar{x}_i}((i+1)\theta_2)\}^{m1}\{\bar{w}_{\bar{y}_i}((i+1)\theta_2)\}^{n1} \end{matrix} \right\} \\ + \dots (-1)^{n-i} \left\{ \begin{matrix} p^n\{\bar{w}_{\bar{x}_i}(n(\theta+\theta_1))\}^{m1}\{\bar{w}_{\bar{y}_i}(n(\theta+\theta_1))\}^{n1} + \cdots \\ +q^n\{\bar{w}_{\bar{x}_i}(n\theta_2)\}^{m1}\{\bar{w}_{\bar{y}_i}(n\theta_2)\}^{n1} \end{matrix} \right\} \end{matrix} \right\}$$

$$P(T > t) = \sum_{i=r}^{n} nC_i \left\{ \begin{matrix} \{p^i D_{i(\theta+\theta_1)}(t) + \cdots + q^i D_{i(\theta_2)}(t)\} - \\ (n-i)C_1\{p^{i+1}D_{(i+1)(\theta+\theta_1)}(t) + \cdots + q^{i+1}D_{(i+1)(\theta_2)}(t)\} + .. \\ (-1)^{n-i}\{p^n D_{n(\theta+\theta_1)}(t) + \cdots + q^n D_{n(\theta_2)}(t)\} \end{matrix} \right\} \quad (12)$$

Where

$$D_\theta(t) = \sum_{m1=0}^{\infty} [W_U^{m1}(t) - W_U^{m1+1}(t)] \times \{\bar{w}_{\bar{x}_i}(\theta)\}^{m1} \sum_{n1}^{\infty} [W_V^{n1}(t) - W_V^{n1+1}(t)] \times \{\bar{w}_{\bar{y}_i}(\theta)\}^{n1} \quad (13)$$

Expanding and simplifying the equation (13)

$$D_\theta(t) = \left\{1 - [1 - \bar{W}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\} \times \left\{1 - [1 - \bar{W}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} W_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\}$$

$$G_\theta(t) = 1 - D_\theta(t)$$

$$\begin{aligned} &= 1 - \left\{1 - [1 - \bar{W}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\}\left\{1 - [1 - \bar{W}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} W_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\} \\ &= \left\{[1 - \bar{W}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\} + \left\{[1 - \bar{W}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} W_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\} \\ &- \left\{[1 - \bar{W}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\}\left\{[1 - \bar{W}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} W_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\} \end{aligned} \quad (14)$$

$$g_\theta(t) = \frac{d}{dt}(G_\theta(t))$$

$$\begin{aligned} &= \left\{[1 - \bar{w}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} w_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\} + \left\{[1 - \bar{w}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} w_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\} \\ &- \left\{[1 - \bar{w}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\}\left\{[1 - \bar{w}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} w_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\} \\ &- \left\{[1 - \bar{w}_{\bar{x}_i}(\theta)] \sum_{m1=0}^{\infty} w_U^{m1}(t)\{\bar{w}_{\bar{x}_i}(\theta)\}^{m1-1}\right\}\left\{[1 - \bar{w}_{\bar{y}_i}(\theta)] \sum_{n1=0}^{\infty} W_V^{n1}(t)\{\bar{w}_{\bar{y}_i}(\theta)\}^{n1-1}\right\} \end{aligned} \quad (15)$$

Since $\bar{w}_V(t)$ is exponential with parameter μ_2,$\bar{w}_V^{n1}(t)$ is a gamma distribution with parameter $\mu_2, n1$

Therefore,

$$[1-w_{y_i}(\theta)]\sum_{n1=0}^{\infty} W_V^{n1}(t)\{\bar{w}_{y_i}(\theta)\}^{n1-1} = [1-\bar{w}_{y_i}(\theta)]\sum_{n1=0}^{\infty}\int_0^t \frac{\mu_2^{n1}u^{n1-1}e^{-\mu_2 u}}{\Gamma(n1)}\{\bar{w}_{y_i}(\theta)\}^{n1-1}du$$

$$= [1-\bar{w}_{y_i}(\theta)]\int_0^t \mu_2 e^{-\mu_2 u}\sum_{n1=0}^{\infty}\frac{[\mu_2 u\bar{w}_{y_i}(\theta)]^{n1}}{(n1-1)!}du$$

$$= [1-\bar{w}_{y_i}(\theta)]\int_0^t \mu_2 e^{-\mu_2 u}e^{\mu_2 u\bar{w}_{y_i}(\theta)}du = \mu_2[1-\bar{w}_{y_i}(\theta)]\int_0^t e^{-\mu_2 u[1-\bar{w}_{y_i}(\theta)]}du$$

$$= \mu_2[1-\bar{w}_{y_i}(\theta)]\left[\frac{e^{-\mu_2 u[1-\bar{w}_{y_i}(\theta)]}}{\mu_2[1-\bar{w}_{y_i}(\theta)]}\right]_0^t = 1-e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t} \qquad (16)$$

Therefore

$$[1-\bar{w}_{y_i}(\theta)]\sum_{n1=0}^{\infty} w_V^{n1}(t)\{\bar{w}_{y_i}(\theta)\}^{n1-1} = \mu_2[1-\bar{w}_{y_i}(\theta)]e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t} \qquad (17)$$

Substituting equation (16) and (17) in equation (15)

$$g_\theta(t) = \left\{[1-\bar{w}_{x_i}(\theta)]\sum_{m1=0}^{\infty} w_U^{m1}(t)\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\} + \mu_2[1-\bar{w}_{y_i}(\theta)]e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t}$$

$$-\left\{[1-\bar{w}_{x_i}(\theta)]\sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\}\mu_2[1-\bar{w}_{y_i}(\theta)]e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t}$$

$$-\left\{[1-\bar{w}_{x_i}(\theta)]\sum_{m1=0}^{\infty} w_U^{m1}(t)\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\}\left[1-e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t}\right]$$

$$= \mu_2[1-\bar{w}_{y_i}(\theta)]e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t} - \left\{[1-\bar{w}_{x_i}(\theta)]\sum_{m1=0}^{\infty} W_U^{m1}(t)\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\}\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t}$$

$$+\left\{[1-\bar{w}_{x_i}(\theta)]\sum_{m1=0}^{\infty} w_U^{m1}(t)\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\}\left[e^{-\mu_2[1-\bar{w}_{y_i}(\theta)]t}\right] \qquad (18)$$

Taking Laplace steljies transform on both side

$$\bar{g}_\theta(s) = \frac{\mu_2[1-\bar{w}_{y_i}(\theta)]}{s+\mu_2[1-\bar{w}_{y_i}(\theta)]} - \left\{\mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)]\times\sum_{m1=0}^{\infty}\frac{[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]^{m1}}{s+\mu_2[1-\bar{w}_{y_i}(\theta)]}\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\}$$

$$+\left\{[1-\bar{w}_{x_i}(\theta)]\sum_{m1=0}^{\infty}[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]^{m1}\{\bar{w}_{x_i}(\theta)\}^{m1-1}\right\}$$

$$\bar{g}_\theta(s) = \frac{\mu_2[1-\bar{w}_{y_i}(\theta)]}{s+\mu_2[1-\bar{w}_{y_i}(\theta)]} - \left\{\frac{\mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)][\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\sum_{m1=0}^{\infty}[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]^{m1-1}}{s+\mu_2[1-\bar{w}_{y_i}(\theta)]}\right\}$$

$$+\left\{[1-\bar{w}_{x_i}(\theta)][\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\sum_{m1=0}^{\infty}[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]^{m1-1}\right\}$$

$$\bar{g}_\theta(s) = \frac{\mu_2[1-\bar{w}_{y_i}(\theta)]}{s+\mu_2[1-\bar{w}_{y_i}(\theta)]} - \frac{\mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)][\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]}{\{s+\mu_2[1-\bar{w}_{y_i}(\theta)]\}\{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}} + \frac{[1-\bar{w}_{x_i}(\theta)][\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]}{\{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}}$$

$$\bar{g}_\theta(s) = \frac{\mu_2[1-\bar{w}_{y_i}(\theta)]}{s+\mu_2[1-\bar{w}_{y_i}(\theta)]} - \mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)]A(s) + [1-\bar{w}_{x_i}(\theta)]B(s) \qquad (19)$$

Where

$$\left.\begin{aligned} A(s) &= \frac{[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]}{[s+\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]} \\ B(s) &= \frac{[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]}{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]} \end{aligned}\right\} \qquad (20)$$

From equation (20)

$$A(s) = \frac{[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]}{[s+\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]}$$

$$\frac{d}{ds}(A(s)) = \frac{\left\{\begin{aligned}&\{[s+\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\,[\bar{w}_U{}'(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\} - \\ &[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\left\{\begin{aligned}&[s+\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[\bar{w}_U{}'(s+\mu_2[1-\bar{w}_{y_i}(\theta)])] + \\ &[1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\end{aligned}\right\}\end{aligned}\right\}}{\{[s+\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$\left[\frac{d}{ds}(A(s))\right]_{s=0}$$

$$= \frac{\left\{\begin{aligned}&\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\,[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\} - \\ &[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\left\{\begin{aligned}&[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])] + \\ &[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\end{aligned}\right\}\end{aligned}\right\}}{\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$= \frac{\left\{\begin{aligned}&\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\} - \\ &[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}\end{aligned}\right\}}{\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2} \qquad (21)$$

Again from equation (20)

$$B(s) = \frac{[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]}{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]}$$

$$\frac{d}{ds}[B(s)] = \frac{\left\{\begin{aligned}&\{\{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}\{[\bar{w}_U{}'(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\}\} - \\ &\{[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\}\{-[\bar{w}_U{}'(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}\end{aligned}\right\}}{\{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}^2}$$

$$\frac{d}{ds}[B(s)] = \frac{\{[\bar{w}_U{}'(s+\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{1-[\bar{w}_U(s+\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}^2}$$

$$\left[\frac{d}{ds}[B(s)]\right]_{s=0} = \frac{\{[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}^2} \qquad (22)$$

Using equations (21) and (22) in (19)

$$\left[\frac{d}{ds}[\bar{g}_\theta(s)]\right]_{s=0} = \frac{-\mu_2[1-\bar{w}_{y_i}(\theta)]}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}^2}$$

$$-\mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)]\times\frac{\left\{\begin{aligned}&\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\} - \\ &[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}\end{aligned}\right\}}{\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$+\frac{\{[1-\bar{w}_{x_i}(\theta)][\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}^2}$$

$$\left[\frac{d}{ds}[\bar{g}_\theta(s)]\right]_{s=0} = \frac{-1}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}}$$

$$\left[\frac{d}{ds}[\bar{g}_\theta(s)]\right]_{s=0} = \frac{-1}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}} - \frac{\mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)]\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$+\frac{\mu_2[1-\bar{w}_{y_i}(\theta)][1-\bar{w}_{x_i}(\theta)][\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}}{\{[\mu_2[1-\bar{w}_{y_i}(\theta)]]\,[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$+\frac{\{[1-\bar{w}_{x_i}(\theta)][\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]\}^2}$$

$$\left[\frac{d}{ds}[\bar{g}_\theta(s)]\right]_{s=0} = \frac{-1}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}} - \frac{[1-\bar{w}_{x_i}(\theta)]\{[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$+\frac{[1-\bar{w}_{x_i}(\theta)][\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$+\frac{[1-\bar{w}_{x_i}(\theta)]\{[\bar{w}_U{}'(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}^2}$$

$$\left[\frac{d}{ds}[\bar{g}_\theta(s)]\right]_{s=0} = \frac{-1}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}} + \frac{[1-\bar{w}_{x_i}(\theta)][\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}}$$

$$= \frac{-1+[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]+[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]-[\bar{w}_{x_i}(\theta)][\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}}$$

$$\left[\frac{d}{ds}[\bar{g}_\theta(s)]\right]_{s=0} = \frac{\{-1+[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}}$$

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$$-\left[\frac{d}{ds}[\bar{g}_{\theta}(s)]\right]_{s=0}=\frac{\{1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])]\}}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}\{[1-[\bar{w}_U(\mu_2[1-\bar{w}_{y_i}(\theta)])\{\bar{w}_{x_i}(\theta)\}]]\}} \quad (23)$$

$$\left.\begin{array}{l} w_U(t)=p\mu_h e^{-\mu_h t}+(1-p)\mu_l e^{-\mu_l t} \\ \bar{w}_U(s)=\frac{p\mu_h}{s+\mu_h}+\frac{(1-p)\mu_l}{s+\mu_l} \\ \bar{w}_U{}'(s)=\frac{p\mu_h}{(s+\mu_h)^2}+\frac{(1-p)\mu_l}{(s+\mu_l)^2} \end{array}\right\} \quad (24)$$

$$-\left[\frac{d}{ds}[\bar{g}_{\theta}(s)]\right]_{s=0}=\frac{1-\frac{p\mu_h}{\mu_2[1-\bar{w}_{y_i}(\theta)]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2[1-\bar{w}_{y_i}(\theta)]+\mu_l}}{\{\mu_2[1-\bar{w}_{y_i}(\theta)]\}\left\{\left[1-\left[\frac{p\mu_h}{\mu_2[1-\bar{w}_{y_i}(\theta)]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2[1-\bar{w}_{y_i}(\theta)]+\mu_l}\right]\{\bar{w}_{x_i}(\theta)\}\right]\right\}} \quad (25)$$

Where
$$\left.\begin{array}{l} \bar{w}_{x_i}(\theta)=\frac{\lambda_1}{\lambda_1+\theta} \\ \bar{w}_{y_i}(\theta)=\frac{\lambda_2}{\lambda_2+\theta} \end{array}\right\} \quad (26)$$

We know that

$$E(T)=-\left[\frac{d}{ds}[\bar{w}_T(s)]\right]_{s=0}$$

From(12)

$$E(T)=-\sum_{i=r}^{n} nC_i\left\{\begin{array}{l} \left\{p^i\left[\frac{d}{ds}[\bar{g}_{i(\theta+\theta_1)}(s)]\right]_{s=0}+\cdots+q^i\left[\frac{d}{ds}[\bar{g}_{i(\theta_2)}(s)]\right]_{s=0}\right\}- \\ (n-i)C_1\left\{\begin{array}{l} p^{i+1}\left[\frac{d}{ds}[\bar{g}_{(i+1)(\theta+\theta_1)}(s)]\right]_{s=0}+\cdots \\ +q^{i+1}\left[\frac{d}{ds}[\bar{g}_{(i+1)(\theta_2)}(s)]\right]_{s=0} \end{array}\right\}+\ldots \\ (-1)^{n-i}\left\{p^n\left[\frac{d}{ds}[\bar{g}_{n(\theta+\theta_1)}(s)]\right]_{s=0}+\cdots+q^n\left[\frac{d}{ds}[\bar{g}_{n(\theta_2)}(s)]\right]_{s=0}\right\} \end{array}\right\} \quad (27)$$

IV. NUMERICAL ILLUSTRATION

The behavior of the performance measure due to the change in parameter is analyzed numerically for different values of n and r.

Sub Case (i) n=3, r=1

From (27) the mean time to recruitment is given by,

$$E(T)=-\left\{\left\{p^3\left[\frac{d}{ds}[\bar{g}_{3(\theta+\theta_1)}(s)]\right]_{s=0}+\cdots+q^3\left[\frac{d}{ds}[\bar{g}_{3(\theta_2)}(s)]\right]_{s=0}\right\}-3\left\{p^2\left[\frac{d}{ds}[\bar{g}_{2(\theta+\theta_1)}(s)]\right]_{s=0}+\cdots+q^2\left[\frac{d}{ds}[\bar{g}_{2(\theta_2)}(s)]\right]_{s=0}\right\}+3\left\{p^1\left[\frac{d}{ds}[\bar{g}_{(\theta+\theta_1)}(s)]\right]_{s=0}++q^1\left[\frac{d}{ds}[\bar{g}_{(\theta_2)}(s)]\right]_{s=0}\right\}\right\}$$

$$E(T)=\{\{p^3E_1+3p^2q^1E_2+3p^1q^2E_3+q^3E_4\}-3\{p^2E_5+2pqE_6+q^2E_7\}+3\{p^1E_8+q^1E_9\}\} \quad (28)$$

Sub Case (ii) n=3, r=2

From (27) the mean time to recruitment is given by

$$E(T)=\{\{-2\{p^3E_1+3p^2q^1E_2+3p^1q^2E_3+q^3E_4\}\}+3\{\{p^2E_5+2pqE_6+q^2E_7\}\}\} \quad (29)$$

Sub Case (iii) n=3, r=3

From (27) the mean time to recruitment is given by

$$E(T)=\{p^3E_1+3p^2q^1E_2+3p^1q^2E_3+q^3E_4\} \quad (30)$$

Where

$$E_1=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta+\theta_1)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta+\theta_1)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta+\theta_1)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta+\theta_1)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta+\theta_1)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+3(\theta+\theta_1)}\right\}\right\}}$$

$$E_2=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(2\theta+2\theta_1+\theta_2)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(2\theta+2\theta_1+\theta_2)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(2\theta+2\theta_1+\theta_2)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(2\theta+2\theta_1+\theta_2)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(2\theta+2\theta_1+\theta_2)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+(2\theta+2\theta_1+\theta_2)}\right\}\right\}}$$

$$E_3=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+2\theta_2)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+2\theta_2)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+2\theta_2)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+2\theta_2)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+2\theta_2)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+(\theta+\theta_1+2\theta_2)}\right\}\right\}}$$

$$E_4=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta_2)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta_2)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta_2)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta_2)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+3(\theta_2)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+3(\theta_2)}\right\}\right\}}$$

$$E_5=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta+\theta_1)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta+\theta_1)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta+\theta_1)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta+\theta_1)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta+\theta_1)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+2(\theta+\theta_1)}\right\}\right\}}$$

$$E_6=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+\theta_2)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+\theta_2)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+\theta_2)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+\theta_2)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1+\theta_2)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+(\theta+\theta_1+\theta_2)}\right\}\right\}}$$

$$E_7=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta_2)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta_2)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta_2)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta_2)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+2(\theta_2)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+2(\theta_2)}\right\}\right\}}$$

$$E_8=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta+\theta_1)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+(\theta+\theta_1)}\right\}\right\}}$$

$$E_9=\frac{1-\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta_2)}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta_2)}\right]+\mu_l}}{\left\{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta_2)}\right]\right\}\left\{1-\left[\frac{p\mu_h}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta_2)}\right]+\mu_h}+\frac{(1-p)\mu_l}{\mu_2\left[1-\frac{\lambda_2}{\lambda_2+(\theta_2)}\right]+\mu_l}\right]\left\{\frac{\lambda_1}{\lambda_1+(\theta_2)}\right\}\right\}} \quad (31)$$

COMPARISON TABLE

The influence of parameters on the performance measures namely the mean time for recruitment is studied numerically. In the following tables these performance measures are calculated by varying the parameter 'λ_1' ,'λ_2','μ_h','μ_l','μ_2','θ_1'and'θ_2' one at a time and taking the parameters $p = 0.3, 1-p = q = 0.7$ $\theta = 0.2$

λ_1	λ_2	μ_h	μ_l	μ_2	θ_1	θ_2	Case(i) E(T)	Case(ii) E(T)	Case(iii) E(T)
0.2	0.3	0.4	0.2	0.5	0.3	0.4	2.8662	2.0285	1.6113
0.2	0.3	0.4	0.2	0.5	0.3	0.5	2.6356	1.9276	1.5716
0.2	0.3	0.4	0.2	0.5	0.3	0.6	2.4868	1.8571	1.5429
0.2	0.3	0.4	0.2	0.5	0.4	0.5	2.5732	1.8966	1.5587
0.2	0.3	0.4	0.2	0.5	0.5	0.5	2.5319	1.8726	1.5480
0.2	0.3	0.4	0.2	0.5	0.6	0.5	2.5031	1.8535	1.5389
0.2	0.3	0.4	0.2	0.6	0.3	0.5	2.3382	1.7056	1.3578
0.2	0.3	0.4	0.2	0.7	0.3	0.5	2.1016	1.5299	1.2427
0.2	0.3	0.4	0.2	0.8	0.3	0.5	1.9087	1.3871	1.1252
0.2	0.3	0.4	0.3	0.6	0.3	0.5	2.1111	1.5501	1.2674
0.2	0.3	0.4	0.4	0.6	0.3	0.5	1.9388	1.4294	1.1724
0.2	0.3	0.4	0.5	0.6	0.3	0.5	1.8036	1.3330	1.0956
0.2	0.3	0.5	0.2	0.6	0.3	0.5	2.2874	1.6671	1.3558
0.2	0.3	0.6	0.2	0.6	0.3	0.5	2.2473	1.6359	1.3296
0.2	0.3	0.7	0.2	0.6	0.3	0.5	2.2150	1.6102	1.3077
0.2	0.5	0.4	0.2	0.6	0.3	0.5	2.8499	1.9557	1.4925
0.2	0.6	0.4	0.2	0.6	0.3	0.5	3.0734	2.0709	1.5424
0.2	0.7	0.4	0.2	0.6	0.3	0.5	3.2787	2.1802	1.5908
0.3	0.3	0.4	0.2	0.6	0.3	0.5	2.4796	1.7673	1.4118

0.4	0.3	0.4	0.2	0.6	0.3	0.5	2.5936	1.8213	1.4340
0.5	0.3	0.4	0.2	0.6	0.3	0.5	2.6871	1.8689	1.4545

Table 1

V. FINDINGS

- ✓ As 'λ_1' and 'λ_2'the parameter for loss of manpower increases the mean time to recruitment increases.
- ✓ As 'μ_h','μ_l' and 'μ_2' the parameter for policy and transfer decision increases the mean time to recruitment decreases.
- ✓ As 'θ_1'and'θ_2'the parameter for threshold level of loss of manpower increases the mean time to recruitment decreases.

VI. CONCLUSIONS

In the context of providing scope for future work, it is worthwhile to mention that the present work can be studied by considering different types of loss in manpower also.

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