International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 4 Issue 1, January 2017

ISSN: 2394-4404

Prime Labeling For Duplication Of Graph Elements In $K_n - e$

S. Chandrakala

Assistant Professor of Mathematics,
T.D.M.N.S College, T.Kallikulam, Tamilnadu, India

Abstract: ***A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,…\|V\| such that for edge xy the labels assigned to x and y are relatively prime . A graph which admits prime labeling is called a prime graph. In this paper we investigate prime labeling for K_n –e where e is an edge. We also discuss prime labeling in the context of graph operations namely duplication in K_n –e.***

Keywords: Prime labeling, complete graph, Duplication.

I. INTRODUCTION

In this paper, I consider only finite simple undirected graph. The graph G has vertex set V =V(G) and edge set E =E(G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H[3] has proved that the path P_n on n vertices is a prime graph. Deretsky et al [2] have proved that the cycle C_n on n vertices is a prime graph .Around 1980 Roger Entringer conjectured that all trees have prime labeling which is not settled today.

In [7] S.K. Vaidhya and K.K. Kanmani have proved that the graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle C_n admit prime labeling. In [5] Meena and Vaithilingam have proved the Prime labeling for some helm related graphs.

In this paper I prove that the graph K_n-e is a prime graph. Also I prove that the graph obtained by duplicating by a vertex, by an edge are all prime graphs in K_n- e.

DEFINITION 1.1

Let G = (V(G),E(G)) be a graph with p vertices . A bijection f:V(G)→ {1,2,…p} is called a prime labeling if for each edge e =uv, $\gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

DEFINITION 1.2

Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

DEFINITION 1.3

Duplication of a vertex v_k by a new edge e = $v_k' v_k''$ in a graph G produces a new graph G' such that $N(v_k') = \{v_k, v_k''\}$ and $N(v_k'') = \{v_k, v_k'\}$.

DEFINITION 1.4

Duplication of an edge e =uv by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.

DEFINITION 1.5

Duplication of an edge e =uv of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that
$N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

DEFINITION 1.6

Let $G_1, G_2, \ldots G_n, n \geq 2$ be n copies of a fixed graph G.The graph obtained by adding an edge between G_i and G_{i+1} for i= 1,2,….n -1 is called the path union of G.

DEFINITION 1.7

A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

RESULT 1.8

The complete graph K_n does not have a prime labeling for $n \geq 4$.

II. MAIN RESULTS

THEOREM 2.1

The graph K_n – e is a prime graph iff $n \leq 5$

PROOF:

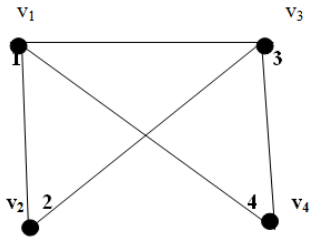
Let G be a K_n – e graph where $e = v_i v_{i+2}, i = 2$.
Then $|V(G)| = n$ and $|E(G)| = \frac{n(n-1)}{2} - 1$
Define a labeling f: V(G) → $\{1,2,\ldots,n\}$ as follows.

$$f(v_i) = i \text{ , for } 1 \leq i \leq n$$

Then f admits prime labeling.
Hence G is a prime graph.

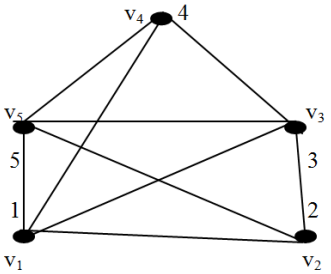
ILLUSTRATION OF 2.1

EXAMPLE 1



Prime labeling of K_4 – e (e = v_2v_4)

EXAMPLE 2



Prime labeling of K_5 –e (e= v_2v_4)

THEOREM 2.2

The graph obtained by duplicating a vertex v_n of K_n – e is a prime graph iff $n \leq 4$.

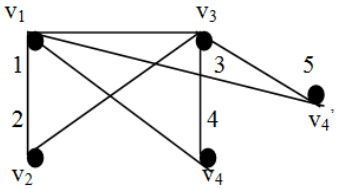
PROOF:

Let G be the graph obtained by duplicating the vertex v_n in K_n –e where $e = v_i v_{i+2}, i = 2$.
Then $|V(G)| = n + 1$. Let the new vertex be V_n'. The edge set
E(G) = $\{v_1v_i : i = 2 \ldots n\} \cup \{v_2v_i : i = 3,5,6 \ldots . n\} \cup \ldots \ldots \cup \{v_iv_n' : i = 1,3, \ldots . n\}$
Define a labeling $f : V(G) \to \{1,2, \ldots n+1\}$ *as follows*

$$f(v_i) = i, \quad 1 \leq i \leq n$$

$f(v_n') = n+1$.
Then f admits prime labeling.
Hence G is a prime graph.

ILLUSTRATION OF 2.2



Prime labeling for duplication of v_4 in K_4-e.

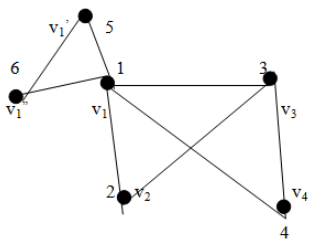
THEOREM 2.3

The graph obtained by duplication of a vertex v_1 by an edge in K_n –e is a prime graph iff $n \leq 5$.

PROOF:

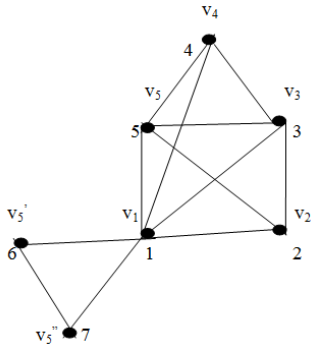
Let G be the graph obtained by duplication of a vertex v_1 by an edge $v_1'v_1''$ in K_n- e graph where $e = v_2v_{i+2}, i = 2$. Then |V(G)| = n+2. Then G contains a cycle C_3 whose vertices are v_1, v_1' and v_1''.
Define a labeling
$f : V(G) \to \{1,2,3, \ldots . n+2\}$ *as follows.*
$f(v_i) = i, 1 \leq i \leq n$
$f(v_1') = n+1$,
$f(v_1'') = n+2$.
Then f admits prime labeling .Thus G is a prime graph.

ILLUSTRATION OF 2.3



Prime labeling for duplication of v_1 by an edge $v_1'v_1''$ in K_4 – e.

ISSN: 2394-4404



Prime labeling for duplication of v_1 by an edge $v_1'v_1''$ in K_5- e

THEOREM 2.4

The graph obtained by duplication of every vertex by an edge in $K_n - e$ is not a prime graph.

PROOF:

Let $v_1, v_2, \ldots, v_n$ be the vertices of $K_n - e$ and G be the graph obtained by duplication of every vertex v_i by an edge $v_i'v_i''$ for i =1,2,…. ,n. Then G is a graph with 3n vertices and having n disjoint cycles each of length three. Any prime labeling of G must contains at the most one even label in each of these n cycles as it is not possible to assign even labels for two adjacent vertices. Hence G is not a prime graph.

THEOREM 2.5

The graph obtained by duplication of an edge by a vertex in $K_n - e, (n \leq 5)$ is a prime graph.

PROOF:

Let G be the graph obtained by duplication of an edge v_iv_{i+1} by a vertex v_i' in $K_n - e$ where $e = v_iv_{i+2}, i = 2$. Then $|V(G)| = n + 1$ and $|E(G)| = \frac{n(n-1)}{2} + 1$.

Define f:V(G) → {1,2,… $n + 1$} by

$f(v_i) = i,\ 1 \leq i \leq n$

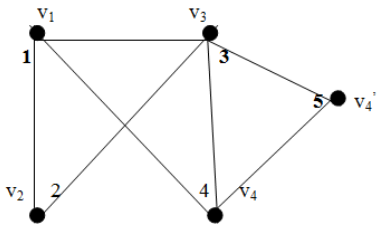
$f(v_i')$= n+1.

Then f admits a prime labeling for G.

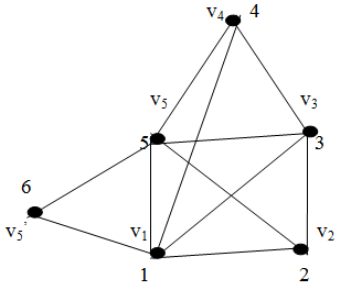
Hence G is a prime graph.

ILLUSTRATION OF 2.5

EXAMPLE 1



Prime labeling for duplication of an edge v_3v_4 by a new vertex v_4' in K_4- e.



Prime labeling for duplication of an edge v_1v_5 by a new vertex v_5' in K_5- e.

THEOREM 2.6

The graph obtained by duplication of an edge in K_n – e is a prime graph iff $n \leq 4$

PROOF:

Let $v_1, v_2, \ldots, v_n$ be the vertices of $K_n - e$.

Let G be the graph obtained by duplication of an edge e = $v_{n-1}v_n$ by an edge $v_{n-1}'v_n'$ in $k_n - e$

where e=v_iv_{i-2} ,i=2. Then |V(G)| = n+2.

Define $f: V(G) \to \{1,2,\ldots..n+2\}$ by

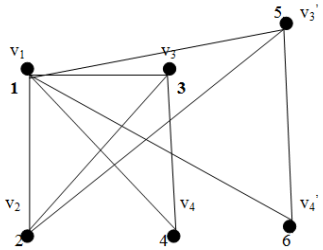
$f(v_i) = v_i$, $1 \leq i \leq$n

$f(v_{n-1}') = n + 1$

$f(v_n')$ = n +2.

Then f admits a prime labeling .Hence G is a prime graph.

ILLUSTRATION OF 2.6



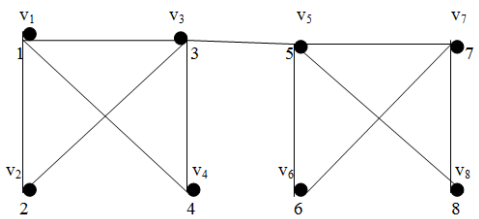
Prime labeling for duplication of an edge v_3v_4 by a new edge $v_3'v_4'$ in K_4- e.

REMARK 2.7

The graph obtained by duplication of an edge in K_5 – e is not a prime graph.

RESULT 2.8

The graph obtained by the path union of two pieces of K_4- e is a prime graph.



Prime labeling for path union of two copies of $K_4 - e$.

RESULT 2.9

The graph obtained by the path union of two copies of $K_5 - e$ is not a prime graph.

III. CONCLUSION

I have investigated some results on prime labeling for the graphs resulted from the duplication of graph elements. Extending the study to other families of graph is an open area of research.

REFERENCES

[1] Bondy.J.A and Murthy U.S.R, "Graph Theory and Application" (North Holland). New York (1976).

[2] Deretsky.T, Lee.S.M and Mitchem.J, "On Vertex Prime Labeling of Graphs in Graph Theory, Combinatorics and Applications", Vol.I (Ed.Alavi.J, Chartrand. G, Oellerman.O, and Schwenk. A) Proceedings of the 6th international conference Theory and Applications of Graphs, Wiley, New York,(1991) 359-369.

[3] Fu, H.C and Huany.K.C., "On prime Labelings Discrete Mathematics", 127(1994), 181-186.

[4] Gallian J.A, "A Dynamic survey of Graph labeling", The Electronic Journal of Combinatorics, 18(2011), 147, #DS6.

[5] Meena.S and Vaithilingam.K "Prime Labeling for some Helm Related Graphs", International Journal of innovative research in science, Engineering and Technology, Vol.2, 4 April 2009.

[6] Tout.A, Dabboucy.A.N and Howalla. K, "Prime Labeling of Graphs", Nat. Acad. Sci letters 11 (1982) 365-368

[7] Vaidya S.K and Kanmani K.K "Prime Labeling for some cycle Related Graphs", Journal of Mathematics Research vol.2. No.2.pp 98-104, May 2010.