

A Mathematical Study Of Two Phase (One Phase Is Newtonian And Other Is Non-Newtonian) Coronary Blood Flow In Venules Using Herschel – Bulkley Model During Angina

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Abstract: This mathematical study focuses on the behaviour of blood flow through venules during Angina. Here blood is represented as Herschel-Bulkley fluid model and flow model is shown by the equation of motion and the continuity equations. Using appropriate boundary conditions, numerical expression for volumetric flow rate, pressure drop and wall shear stress have been derived. Affect the coronary circulation directly as an externally viscous force compressing the blood vessels or they may act indirectly by producing changes in the blood-pressure drop, that is, by altering the pressure perusing the coronary blood vessels. Any of these factors may alter the coronary blood flow or change the distribution of the flow between the two phases of the heart beat. The role of hematocrit in explicit in the determination of blood pressure in the case of angina heart diseases. The hematocrit increase the blood pressure drop is also increase. The overall presentation is in tensorial form and solution technique adapted is analytical as well as numerical.

Keywords: Two phase blood flow, Coronary blood flow ,coronary circulation, Angina, Plasma, Hematocrit, Herschel-Bulkley Model, Non-Newtonian, Blood Pressure Drop, Behavior of blood.

I. INTRODUCTION

A. STRUCTURE & FUNCTION OF HEART

The human heart is a muscular organ containing four chambers that is situated just to the left of the midline of the thoracic cavity. It is approximately the size of a man's closed fist. The upper two chambers (atria) are divided by a wall like structure called the interatrial septum. The lower two chambers (ventricles) are divided by a similar structure called the interventricular septum. Between each atrium and ventricle, valves allow blood to flow in one direction, preventing backflow[1].

Blood flow through the heart is shown in figure-1 Blood that is low in oxygen flows into the right atrium from the veins known as the superior vena cava and inferior vena cava. The superior vena cava carries blood from the remainder of the

trunk and the legs. Blood in the right atrium then flows through the right atrioventricular (tricuspid) valve into the right ventricle[2]. The pulmonary circuit, with deoxygenated blood flowing into the right and left pulmonary arteries and their smaller branches. The blood becomes oxygenated while moving through the lungs capillary beds. Also in this part of the system carbon dioxide is released[3].

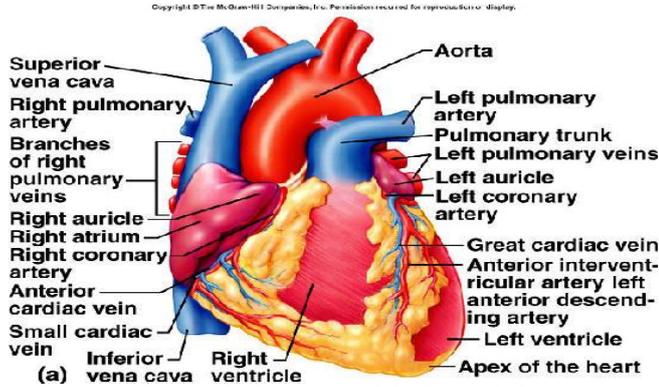


Figure 1: Structure of the Heart [4]

B. CORONARY CIRCULATION

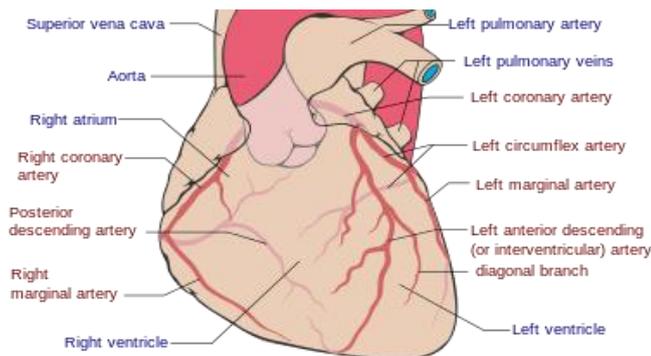


Figure 2: Coronary Circulation [5]

As the left and right coronary arteries run on the surface of the heart, they can be called epicardial coronary arteries. These arteries, when healthy, are capable of auto regulation to maintain coronary blood flow at levels appropriate to the needs of the heart muscle. These relatively narrow vessels are commonly affected by atherosclerosis and can become blocked, causing angina or a heart attack. (See also: circulatory system.) The coronary arteries that run deep within the myocardium are referred to as subendocardial[5].

The coronary arteries are classified as "end circulation", since they represent the only source of blood supply to the myocardium; there is very little redundant blood supply, which is why blockage of these vessels can be so critical[6],[7].

C. STRUCTURE AND FUNCTION OF VENULES

Venules are small blood vessels that collect spent blood from capillary beds and transport it to the larger veins for transport back to the heart. Apart from their small size and narrow interior lumens, venules are structurally similar to veins, and several venules often merge together to form a vein.

Venule walls consist of three layers. The inner layer is a membrane built from endothelial cells that, in certain specialized venules, permit the passage of fluid and white blood cells through the vessel wall. The middle layer of a venule wall is an extremely thin sheet of smooth muscle and elastic tissue that helps the vessel maintain its shape. This middle layer is much thinner in venules than it is in other blood vessels. The outer layer of the venule is composed of a

tough, fibrous sheath of connective tissue that binds the entire structure together.

Venules do more than simply transport blood from capillary beds to the veins. At sites where an infection has developed, venules release white blood cells to fight the foreign cells. By slowly releasing fluid through their semipermeable membranes, venules also play a role in maintaining the balance of the extracellular interstitial fluid. This fluid is then drained away by the lymphatic system[8].

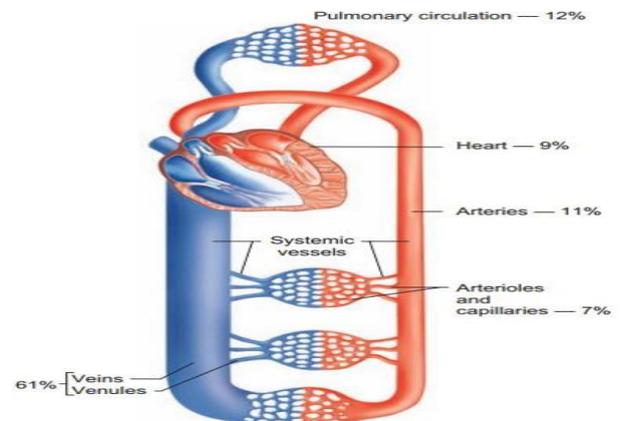


Figure 2: Structure of Venules [9]

D. CONSTITUTION OF BLOOD

Blood is a complex fluid consisting of particulate solids suspended in a non-Newtonian fluid. The particulate solids are red blood cells {RBCs}, white blood cells {WBCs} and platelets. The fluid is plasma, which itself is a complex mixture of proteins and other intergradient in an aqueous base 50% of plasma and 45% of the blood cells and 45% of the blood is RBCs and there is a few parts of the other cells. Which are ignorable So one phase of the blood is plasma and second phase of the blood is RBCs. Two phase coronary blood flow is study of measuring the blood pressure if haemoglobin known. The percentage of volume covered by blood cells in the whole blood is called hematocrit.

E. ANGINA

Angina (an-JI-nuh or AN-juh-nuh) is chest pain or discomfort that occurs if an area of your heart muscle doesn't get enough oxygen-rich blood. Angina may feel like pressure or squeezing in your chest. The pain also can occur in your shoulders, arms, neck, jaw, or back. Angina pain may even feel like indigestion. Angina isn't a disease; it's a symptom of an underlying heart problem. Angina usually is a symptom of coronary heart disease (CHD). CHD is the most common type of heart disease in adults. It occurs if a waxy substance called plaque builds up on the inner walls of your coronary arteries [10].

TYPES OF ANGINA

The major types of angina are stable, unstable, variant (Prinzmetal's), and microvascular. Knowing how the types differ is important. This is because they have different symptoms and require different treatment [11].

II. REAL MODEL

A. CHOICE OF FRAME OF REFERENCE

We have to select a frame of reference for mathematical modeling of the state of a moving blood. Keeping in view the difficulty and generality of the problem of blood flow, we select generalized three-dimensional orthogonal curvilinear co-ordinate system, [14] briefly prescribed as E^3 , called as 3-dim Euclidean space. We interpret the quantities related to blood flow in tensorial form which is comparatively more realistic. The biophysical laws thus expressed fully hold good in any co-ordinate system, which is compulsion for the truthfulness of the law. Now, let the co-ordinate axes be Ox^i where O is origin and superscript $i = 1, 2, 3$ let X^i be the co-ordinates of any point P in space. The mathematical description of the state if a moving blood is affected by means of functions which give the distribution of the blood velocity $V^k = V^k(X^i, t)$, $k = 1, 2, 3$ and of any two thermodynamic quantities pertaining to the blood, for instance the pressure $p = p(X^i, t)$ and the density $\rho = \rho(X^i, t)$. As is well known, all the thermodynamic quantities are determined by the values of any two of them, together with the equation of state. Hence, if we are given five quantities, namely the three components of velocity V^k , the pressure p and the density ρ , the state of moving blood is completely determined. All these quantities are, in general, functions of the co-ordinates X^i , $i = 1, 2, 3$ and of the time t . We emphasize that $V^k(X^i, t)$ is the velocity of the blood at a given point X^i in space and at a given t , i.e. it refers to fixed points in space and not to fixed particles of the blood; in the course of time, the latter move about in space [12].

B. TWO PHASE DESCRIPTION

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the bloods plasma and 2nd phase of blood is RBCs [21], [15].

C. CONSTITUTIVE EQUATIONS

Generally blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows: [13]

a. NEWTONIAN EQUATION

$\tau = \eta e^n$ when $n = 1$ then the nature of fluid is Newtonian.

Where η is viscosity coefficient. This is found to hold good in the broad blood vessels where there is low hematocrit.

b. THE NON-NEWTONIAN HERSCHEL – BUCKLEY EQUATION

$\tau = e^n + \tau_0$ ($\tau \geq \tau_0$) when $n \neq 1$ then the nature of fluid is Non-Newtonian.

$e = 0$ ($\tau < \tau_0$)

It holds good when blood shows yield stress. We notice that the yield stress arises because blood cells form aggregates in the form of rouleaux at low strain rate.

D. BOUNDARY CONDITIONS ARE AS FOLLOWS

- ✓ The velocity of blood flow on the axis of venules at $r=0$ will be maximum and finite, say $V_0 =$ maximum velocity, $V = V_0$ then $A = 0$
- ✓ The velocity of blood flow on the wall of blood venules at $r=R$, where, R is the radius of venules, will be zero. This condition is well known as no-slip condition. $V = 0$ At $r = R$

III. MATHEMATICAL MODELING

A. BASIC BIO-FLUID EQUATION FOR TWO PHASE BLOOD FLOW

B. EQUATION OF CONTINUITY FOR TWO PHASE BLOOD FLOW

The blood flow is effected by the pressure of blood and directly proportional to the volume occupied by blood cells. Let the volume portion covered by the blood cells in unit volume be X , this X be replaced by $\frac{H}{100}$, where H is the hematocrit the volume percentage of blood cells (16). If the mass of blood cells to plasma is r then clearly.

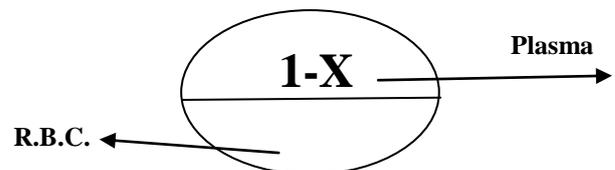


Figure 3: Unit volume of blood

$$r = \frac{X \rho_c}{(1-X) \rho_p} \dots\dots\dots(1)$$

Where ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. But in present context we have treated it constant. The both phase of blood, i.e. blood cells and plasma move with the common velocity (15). Campbell and Pitcher has presented a model for two phase of blood. The in flow of blood in coronary venules is equal to out flow of that since heart behavior just like a pumping station. Hence the principle of conservation mass of coronary venules holds good.



Figure 4: venules blood vessel

We get the equation of continuity for two phases as follows:

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c V^i)_{,i} \text{ for blood cells phase.....(2) and}$$

$$\frac{\partial(1-X)\rho_p}{\partial t} + (1-X)\rho_p V^i_{,i} = 0 \text{ for plasma phase.....(3)}$$

Where, V is the common velocity of two phase blood cells and plasma. If we define the uniform density of the blood ρ_m as follow

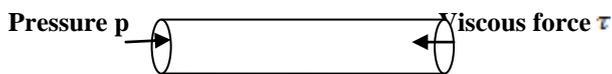
$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \text{.....(4)}$$

Combined the equation (2) and (3)

$$\frac{\partial\rho_m}{\partial t} + (\rho_m V^i)_{,i} = 0 \text{.....(5)}$$

C. EQUATION OF MOTION FOR TWO PHASE BLOOD FLOW

The hydro dynamical pressure p be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum in coronary venules (17).



$$\frac{dp}{dt} + p - \tau = F_{ext}(0) = 0$$

$$\frac{dp}{dt} = -p + \tau$$

We get equation of motion for the two phase of blood cells as follows:

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c V^j) V_{,j}^i = -X p_{,j} g^{ij} + X \eta_c (g^{jk} V_{,k}^i)_{,j} \text{.....(6)}$$

Similarly, taking the viscosity coefficient of plasma to be. The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p V^j\} V_{,j}^i = -(1-X) p_{,j} g^{ij} + (1-X) (\eta_p g^{jk} V_{,k}^i)_{,j} \text{.....(7)}$$

Now adding equation (6) and (7) and using relation (4), the equation of motion for blood flow with the both phases will be as follows:-

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m V^j) V_{,j}^i = -p_{,j} g^{ij} + \eta_m (g^{jk} V_{,k}^i)_{,j} \text{.....(8)}$$

Where $\eta_m = X \eta_c + (1-X) \eta_p$ is viscosity coefficient of blood as a mixture of two phase.

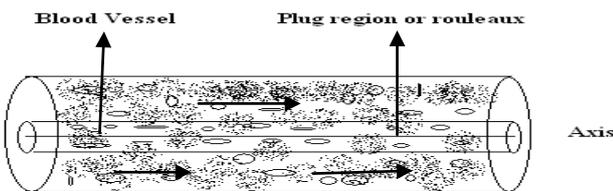


Figure 4: Herchel-Bulkley blood flow

The constitutive equation for test part of the blood vessel

is

$$T = \eta_m e^n + T_p$$

$$\text{or } T - T_p = \eta_m e^n = T_e$$

where, $T_e =$ effective stress

whose generalized form will be as follows
 $T^{ij} = -p g^{ij} + T_e^{ij}$ where, $T_e^{ij} = \eta_m (e^{ij})^n$ 'n \neq 1(Non-Newtonian)

While $e^{ij} = g^{ik} V_{,k}^j$

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m V^j) V_{,j}^i = -p_{,j} g^{ij} + \eta_m (g^{jk} V_{,k}^i)_{,j} \text{.....(9)}$$

Where the symbols have their usually meanings.

IV. SOLUTION

Since, the blood vessels are cylindrical. The above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier:-

$$x^1 = r, x^2 = \Theta, x^3 = z$$

Matrix of metric tensor in cylindrical co-ordinates is as follows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate matrix tensor is as follow-

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where as the chritoffel's symbol of 2nd kind is as follow:-

$$\left\{ \begin{matrix} 1 \\ 2 \\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 1 \\ 2 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 1 \\ 2 \\ 2 \end{matrix} \right\} = \frac{1}{r}$$

Remaining others are zero.

The equation of continuity -

$$\frac{\partial v}{\partial z} = 0$$

The equation of motion -

$$r\text{-component } 0 = -\frac{\partial p}{\partial z}, \Theta = \text{component } = 0$$

$$z \text{ component } 0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial v}{\partial r} \right)^n \right]$$

Here, this fact has been taken in view that the blood flow is axially Symmetric in venules concerned, i.e.

$V_\Theta = 0$ and V_r, V_z and p do not depend upon Θ .

We get $V_z = v(r)$ and $p = p(z)$ and

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \left[r \left(\frac{dv}{dz} \right)^n \right]$$

Since, pressure gradient $-\frac{dp}{dz} = p$

$$r \left(\frac{dv}{dz} \right)^n = -pr^2/2\eta_m + A, \text{ we apply}$$

Boundary condition: at $r = 0, v = v_0$ then $A = 0$

$$-\frac{dv}{dr} = \left(\frac{pr}{2\eta_m} \right)^{1/n} \text{.....(11)}$$

Replace r from r- r_p

$$-\frac{dv}{dr} = \left(\frac{p(r-r_p)}{2\eta_m} \right)^{1/n} \text{.....(12)}$$

Integrating above equation (12) under the no slip boundary condition- $v = 0$ at $r = R$ so as to get:

$$V = \left(\frac{p}{2\eta_m} \right)^{1/n} \cdot \frac{n}{n+1} [(R-r_p)^{1/n+1} - (r-r_p)^{1/n+1}] \text{.....(13)}$$

This is the formula for velocity of blood flow in arterioles.

Putting $r = r_p$ to get the velocity V_p of plug flow as follows:-

$$V_p = \frac{n}{n+1} \left(\frac{p}{2\eta_m} \right)^{1/n} (R-r_p)^{1/n+1} \text{.....(14)}$$

Where the value of r_p is taken from-(7)

V. RESULT & DISCUSSION (BIO-PHYSICAL INTERPRETATION)

Hematocrit vs. blood pressure from an authorized City Hospital & Research centre Jabalpur. By Dr Abhishek Dubey Patient case history(Age-48 years old)

PATIENT CASE HISTORY

S.No	Blood Pressure (mmhg)	Hemoglobin	Hematocrit
1.	130/80	11.0	33
2.	130/90	11.1	33.3
3.	120/80	11.2	33.6
4.	110/70	11.3	33.9

Table 1

The flow flux phase blood flow in coronary venules

$$Q = \int_0^{r_p} 2\pi r v_p dr + \int_{r_p}^R 2\pi r v dr = \int_0^{r_p} 2\pi \frac{n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} [(R-r_p)^{1/n} + 1] dr + \int_0^{r_p} 2\pi \frac{n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} [(R-r_p)^{1/n} - (R-r_p)^{1/n+1}] dr$$

Using(12) and (14)

$$= \frac{2\pi n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} (R-r_p)^{1/n} + 1 \left[\frac{r^2}{2} r_p + \frac{2\pi n}{(n+1)} \left(\frac{p}{2\eta}\right)^{1/n} \left[\frac{r^2}{2} (R-r_p)^{1/n} + 1 - \frac{r(r-r_p)^{1/n}}{\frac{1}{n}+2} + \frac{(r-r_p)^{1/n+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right] r_p \right]$$

$$= \frac{\pi n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} (R)^{1/n+3} \left[\frac{r_p^2}{R^2} \left(1 - \frac{r_p}{R}\right)^{1/n} + 1 + \left(1 + \frac{r_p}{R}\right) \left(1 - \frac{r_p}{R}\right)^{1/n} + 2 - \frac{2(1 - \frac{r_p}{R})^{1/n+2}}{(\frac{1}{n}+2)} + \frac{2(1 - \frac{r_p}{R})^{1/n+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right] \dots\dots\dots(15)$$

Q = 250 ml/min = 0.00416666 m³/sec.(18)
R=1, r_p=1/3

$\eta_p = 0.0015$ (pascal-sec), $\eta_m = 0.035$ (pascal-sec)(19)
Length of coronary venules $z_f - z_i = 0.2$ cm. = 0.002 m.(20)

P_i = Initial Blood pressure
 $\frac{1}{3} \left(\frac{S+D}{3} \right) = \frac{1}{3} \left(\frac{130+80}{3} \right) = 20.5555 \text{ mmhg} = 2736.602223 \text{ pas. sec.}$

P_f = Final Blood pressure
 $\frac{2}{3} \left(\frac{S+D}{3} \right) = \frac{2}{3} \left(\frac{130+80}{3} + 80 \right) = 13.70366 \text{ mmhg} = 1824.3965504 \text{ pas. sec.}$

Pressure drop (P_f - P_i)= 1824.3965504 - 2736.6022223 = -912.2056719 pascal second
H = 33, X = H/100 = 33/100 = 0.33

By using relation $\eta_m = \eta_c X + \eta_p (1 - X)$
0.035 = η_c (0. 33) + 0.0015 (1- 0.33) $\Rightarrow \eta_c = 0.1030151515 \text{ pas. sec.}$

Again using this relation and change in to the hematocrit
 $\eta_m = \eta_c X + \eta_p (1 - X) \Rightarrow \eta_m = 0.001015151515H + 0.0015$
Now substituting the value of r_p and R in equation (15)
We get equation

$$\frac{27 Q}{2\pi} = \left(\frac{P_f - P_i}{3\eta_m (Z_f - Z_i)} \right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

Substituting the value of Q, η_m , (P_f - P_i) and (Z_f - Z_i) and solve by numerical method

$$0.0178976986 = \left(\frac{912.2056719}{3 \times 0.35 \times 0.002} \right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$0.0178976986 = (13031439.152)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

We get n = -3.072

Again, $\Delta P = 3\eta_m (z_f - z_i) \left(\frac{27 Q}{2\pi A} \right)^n$,

Where A = $\left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$

$$= 3[\eta_c X + \eta_p (1-X)] (z_f - z_i) \left(\frac{27 Q}{2\pi A} \right)^n = 3[0.00101515H + 0.0015] (0.002) (0.0178976986)^{-3.072}$$

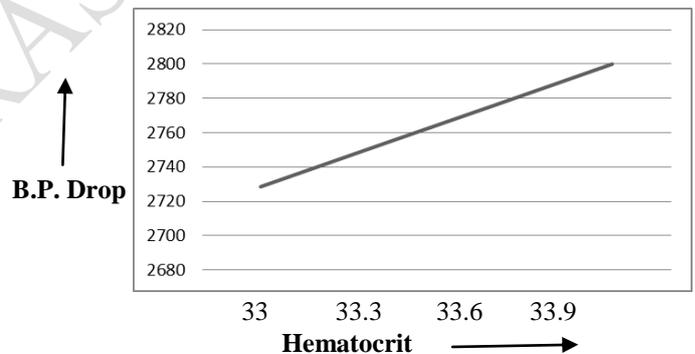
$$3.072 \left[\frac{6n^3 + 11n^2 + 6n + 1}{26n^3 + 33n^2 + 9n} \right]^{-3.072}$$

$$\Delta P = 79.14016947H + 116.9386339$$

Hematocrit	33	33.3	33.6	33.9
Blood pressure drop	2728.564226	2752.306277	2776.048328	2799.790379

Table 2: Hematocrit v/s Pressure drop

Bio-physical interpretation (Graphical Presentation of clinical data)



VI. CONCLUSION

A simple survey of the graph between blood pressure drop and hematocrit in cardiac patient shows that when hematocrit is increased the blood pressure drop is also increased.

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