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Invariant Submanifold Of $\tilde{\psi}(6,4,2)$ Structure Manifold

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***Abstract:** In this paper, we have studied various properties of a $\tilde{\psi}(6,4,2)$ structure manifold and its invariant submanifold. Under two different assumptions, the nature of induced structure ψ, has also been discussed.*

***Keywords:** Invariant submanifold, Nijenhuis tensor, projection operators and complementary distributions.*

I. INTRODUCTION

Let V^m be a C^∞ m-dimensional Riemannian manifold imbedded in a C^∞ n-dimensional Riemannian manifold M^n, where $m<n$. The imbedding being denoted by

$$f : V^m \longrightarrow M^n$$

Let B be the mapping induced by f i.e. $B=df$

$$df : T(V) \longrightarrow T(M)$$

Let $T\ (V,M)$ be the set of all vectors tangent to the submanifold $f(V)$. It is well known that

$$B: T(V) \longrightarrow T(V,M)$$

Is an isomorphism. The set of all vectors normal to $f(V)$ forms a vector bundle over $f(V)$, which we shall denote by $N(V,M)$. We call $N(V,M)$ the normal bundle of V^m. The vector bundle induced by f from $N(V,M)$ is denoted by $N(V)$. We denote by $C: N(V) \longrightarrow N(V,M)$ the natural isomorphism and by $\eta_s^r(V)$ the space of all C^∞ tensor fields of type (r,s) associated with N (V). Thus $\zeta_0^0(V) = \eta_0^0(V)$ is the space of all C^∞ functions defined on V^m while an element of $\eta_0^1(V)$ is a C^∞ vector field normal to V^m and an element of $\zeta_0^1(V)$ is a C^∞ vector field tangential to V^m.

Let $\bar{X}$ and $\bar{Y}$ be vector fields defined along $f(V)$ and $\tilde{X}, \tilde{Y}$ be the local extensions of $\bar{X}$ and $\bar{Y}$ respectively. Then $[\tilde{X}, \tilde{Y}]$ is a vector field tangential to M^n and its restriction $[\tilde{X}, \tilde{Y}]/f(V)$ to $f(V)$ is determined independently of the choice of these local extension $\tilde{X}$ and $\tilde{Y}$. Thus $[\bar{X}, \bar{Y}]$ is defined as

(1.1) $$[\bar{X}, \bar{Y}] = [\tilde{X}, \tilde{Y}]/f(V)$$

Since B is an isomorphism

(1.2) $$[BX, BY] = B[X,Y] \quad \text{for all} \quad X,Y \in \zeta_0^1(V)$$

Let $\bar{G}$ be the Riemannain metric tensor of M^n, we define g and g^* on V^m and $N\ (V)$ respectively as

(1.3) $$g(X_1, X_2) = \tilde{G}(BX_1, BX_2)\ f, \quad \text{and}$$

(1.4) $$g^*(N_1, N_2) = \tilde{G}(CN_1, CN_2)$$

For all $X_1, X_2 \in \zeta_0^1(V)$ and $N_1, N_2 \in \eta_0^1(V)$

It can be verified that g and g^* are the induced metrics on V^m and $N\ (V)$ respectively.

Let $\tilde{\nabla}$ be the Riemannian connection determined by $\tilde{G}$ in M^n, then $\tilde{\nabla}$ induces a connection ∇ in $f(V)$ defined by

(1.5) $\nabla_{\bar{X}}\bar{Y} = \tilde{\nabla}_{\tilde{X}}\tilde{Y} / f(V)$

where $\bar{X}$ and $\bar{Y}$ are arbitrary C^∞ vector fields defined along $f(V)$ and tangential to $f(V)$.

Let us suppose that M^n is a C^∞ $\tilde{\psi}(6,4,2)$ structure manifold with structure tensor $\tilde{\psi}$ of type (1,1) satisfying

(1.6) $\tilde{\psi}^6 + \tilde{\psi}^4 + \tilde{\psi}^2 = 0$

Let $\tilde{L}$ and $\tilde{M}$ be the complementary distributions corresponding to the projection operators

(1.7) $\tilde{l} = \tilde{\psi}^6, \qquad \tilde{m} = I - \tilde{\psi}^6$

where I denotes the identity operator.

From (1.6) and (1.7), we have

(1.8) (a) $\tilde{l} + \tilde{m} = I$ (b) $\tilde{l}^2 = \tilde{l}$

(c) $\tilde{m}^2 = \tilde{m}$ (d) $\tilde{l}\,\tilde{m} = \tilde{m}\,\tilde{l} = 0$

Let D_l and D_m be the subspaces inherited by complementary projection operators l and m respectively.

We define

$$D_l = \{X \in T_p(V):\ lX = X,\ mX = 0\}$$
$$D_m = \{X \in T_p(V):\ mX = X,\ lX = 0\}$$

Thus $T_p(V) = D_l + D_m$

Also $Ker\ l = \{X : lX = 0\} = D_m$

$Ker\ m = \{X : mX = 0\} = D_l$

at each point p of $f(V)$.

II. INVARIANT SUBMANIFOLD OF $\tilde{\psi}(6,4,2)$ STRUCTURE MANIFOLD

We call V^m to be invariant submanifold of M^n if the tangent space $T^p(f(V))$ of $f(V)$ is invariant by the linear mapping $\tilde{\psi}$ at each point p of $f(V)$. Thus

(2.1) $\tilde{\psi}BX = B\psi X$, for all $X \in \zeta_0^1(V)$, and ψ being a (1,1) tensor field in V^m.

THEOREM (2.1): Let $\tilde{N}$ and N be the Nijenhuis tensors determined by $\tilde{\psi}$ and ψ in M^n and V^m respectively, then

(2.2) $\tilde{N}(BX, BY) = BN(X,Y)$, for all $X, Y \in \zeta_0^1(V)$

PROOF: We have, by using (1.2) and (2.1)

(2.3)

$$\tilde{N}(BX,BY) = [\tilde{\psi}BX, \tilde{\psi}BY] + \tilde{\psi}^2[BX,BY] - \tilde{\psi}[\tilde{\psi}BX, BY] - \tilde{\psi}[BX, \tilde{\psi}BY]$$
$$= [B\psi X, B\psi Y] + \tilde{\psi}^2 B[X,Y] - \tilde{\psi}[B\psi X, BY] - \tilde{\psi}[BX, B\psi Y]$$
$$= B[\psi X, \psi Y] + B\psi^2[X,Y] - \tilde{\psi}B[\psi X, Y] - \tilde{\psi}B[X, \psi Y]$$
$$= B\{[\psi X, \psi Y] + \psi^2[X,Y] - \psi[\psi X, Y] - \psi[X, \psi Y]\}$$
$$= B\,N(X,Y)$$

III. DISTRIBUTION $\tilde{M}$ NEVER BEING TANGENTIAL TO $f(V)$

THEOREM (3.1): if the distribution $\tilde{M}$ is never tangential to $f(V)$, then

(3.1) $\tilde{m}(BX) = 0$ for all $X \in \zeta_0^1(V)$ and the induced structure ψ on V^m satisfies

(3.2) $\psi^4 + \psi^2 + I = 0$. Thus ψ is (4,2,0)

PROOF: if possible $\tilde{m}(BX) \neq 0$. From (2.1) We get

(3.3) $\tilde{\psi}^2 BX = B\psi^2 X$; etc from (1.7) and (3.3)

$$\tilde{m}(BX) = (I - \tilde{\psi}^6)BX$$
$$= (I + \tilde{\psi}^2 + \tilde{\psi}^4)BX$$
$$= BX + B\psi^2 X + B\psi^4 X$$

(3.4) $\tilde{m}(BX) = B(X + \psi^2 X + \psi^4 X)$

This relation shows that $\tilde{m}(BX)$ is tangential to $f(V)$ which contradicts the hypothesis. Thus $\tilde{m}(BX) = 0$. Using this result in (3.4) and remembering that B is an isomorphism, We get

(3.5) $\psi^4 + \psi^2 + I = 0$

THEOREM (3.2:) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.6) $\tilde{N}_{\tilde{m}}(BX, BY) = 0$

PROOF : We have

(3.7) $\tilde{N}_{\tilde{m}}(BX,BY) = [\tilde{m}BX, \tilde{m}BY] + \tilde{m}^2[BX,BY] - \tilde{m}[\tilde{m}BX, BY] - \tilde{m}[BX, \tilde{m}BY]$

Using (1.2), (1.8) (c) and (3.1), we get (3.6).

THEOREM (3.3): Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.8) $\tilde{N}_{\tilde{l}}(BX, BY) = 0$

PROOF: We have

(3.9) $\tilde{N}_{\tilde{l}}(BX,BY) = [\tilde{l}BX, \tilde{l}BY] + \tilde{l}^2[BX,BY] - \tilde{l}[\tilde{l}BX, BY] - \tilde{l}[BX, \tilde{l}BY]$

Using (1.2), (1.8) (a), (b) and (3.1) in (3.9); we get (3.8)

THEOREN (3.4): Let $\tilde{M}$ be never tangential to $f(V)$. Define

(3.10) $\tilde{H}(\tilde{X}, \tilde{Y}) = \tilde{N}(\tilde{X}, \tilde{Y}) - \tilde{N}(\tilde{m}\tilde{X}, \tilde{Y}) - \tilde{N}(\tilde{X}, \tilde{m}\tilde{Y}) + \tilde{N}(\tilde{m}\tilde{X}, \tilde{m}\tilde{Y})$

For all $\tilde{X}, \tilde{Y} \in \zeta_0^1(M)$, then

(3.11) $\tilde{H}\left(BX,BY\right)=BN\left(X,Y\right)$

PROOF: Using $\tilde{X}=BX$, $\tilde{Y}=BY$ and (2.2), (3.1) in (3.10) We get (3.11).

IV. DISTRIBUTION $\tilde{M}$ ALWAYS BEING TANGENTIAL TO $f\left(V\right)$

THEOREM (4.1): Let $\tilde{M}$ be always tangential to $f\left(V\right)$, then

(4.1) (a) $\tilde{m}\left(BX\right)=Bm\,X$ (b) $\tilde{l}\left(BX\right)=Bl\,X$

PROOF: from (3.4), We get (4.1) (a). Also

(4.2) $l=\psi^6$

$lX=\psi^6\,X$

(4.3) $BlX=B\psi^6\,X$

Using (2.1) in (4.3)

(4.4) $BlX=\tilde{\psi}^6\,BX=\tilde{l}\left(BX\right)$,

which is (4.1) (b).

THEOREM (4.2): Let $\tilde{M}$ be always tangential to $f\left(V\right)$, then l and m satisfy

(4.5) (a) $l+m=I$ (b) $lm=ml=0$ (c) $l^2=l$ (d) $m^2=m$.

PROOF: Using (1.8) and (4.1) We get the results.

THEOREM (4.3): If $\tilde{M}$ is always tangential to $f\left(V\right)$, then

(4.6) $\psi 6+\psi^4+\psi^2=0$

PROOF: From (2.1)

(4.7) $\tilde{\psi}^6\,BX=B\psi^6\,X$

Using (1.6) in (4.7)

$\left(-\tilde{\psi}^4-\tilde{\psi}^2\right)BX=B\psi^6\,X$

$-B\psi^4X-B\psi^2X=B\psi^6\,X$

Or $\psi^6+\psi^4+\psi^2=0$ which is (4.6)

THEOREM (4.4): If $\tilde{M}$ Is always tangential to $f\left(V\right)$ then as in (3.10)

(4.8) $\tilde{H}\left(BX,BY\right)=BH\left(X,Y\right)$

PROOF: from (3.10) we get

(4.9) $\tilde{H}\left(BX,BY\right)=\tilde{N}\left(BX,BY\right)-\tilde{N}\left(\tilde{m}BX,BY\right)-\tilde{N}\left(BX,\tilde{m}BY\right)+\tilde{N}\left(\tilde{m}BX,\tilde{m}BY\right)$

Using (4.1) (a) and (2.2) in (4.9) we get (4.8).

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