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Estimation Of Expected Time To Recruitment For A Two Graded Manpower System With Two Thresholds Having Different Epochs For Decisions And Exits

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***Abstract:** In this paper, for a two graded manpower system, a mathematical model is constructed using a univariate policy of recruitment based on shock model approach. The expected time to recruitment is obtained based on the following assumptions , (i) the loss of manpower due to the attrition from a sequence of independent and identically distributed exponential random variables (ii) the exponential inter exit times form an ordinary renewal process and (iii) the threshold for each grade has two types of threshold namely optional and mandatory threshold. A different probabilistic analysis is used to derive the analytical results.*

***Keywords:** Two grade manpower system, decision and exit epochs, univariate policy of recruitment with two thresholds, renewal process and mean time to recruitment.*

I. INTRODUCTION

Exit of personnel, voluntary and involuntary, is a common phenomenon in any marketing organization. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. In fact, frequent recruitments may also be expensive due to the cost of recruitments and training. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. An univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: *Recruitment is made whenever the cumulative loss of man hours exceeds a breakdown threshold.* Several researchers have studied the problem of time to recruitment for a two grade man power system using shock model approach. In this context, in [8], [4], [7], [5] and [6] the authors have obtained the variance of the time to recruitment for a two grade man power system using several recruitment policies under different conditions on the loss of man power, breakdown thresholds when the inter – policy decision times form an ordinary renewal process. In [9], and [1] the authors have estimated the mean time to recruitment using geometric process for inter – decision times. In [2], the authors has studied the problem of time to recruitment with two sources of depletion under different conditions on the inter-policy decisions, inter-transfer decisions when the breakdown threshold for each grade has only one component. Recently, in [3] the author has extended

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this work for a two grade man power system according as the inter-policy decision times and inter-transfer decision times form the same or different ordinary renewal processes respectively.

In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points

Which may or may not coincide with decision points. This aspect is taken into account for the first time in [10] and variance of time to recruitment is obtained when inter-decision times and exit times are independent and identically distributed exponential random variables using univariate policy for recruitment and Laplace transform in the analysis. Recently, in [11] the author has studied the work in [10] by considering optional and mandatory thresholds which considering non-instantaneous exits at decision epochs. In the present paper, for a two grade manpower system, a mathematical model is constructed in which attrition due to policy decisions take place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the expected time to recruitment when the system has different epochs for policy decisions and exits and the inter-exit times form an ordinary renewal process.

II. NOTATIONS

X_i : The amount of depletion of manpower (loss of man hours) in the organization due to i^{th} exit point Let S_i be the total loss of manpower upto the i exit points.

X_i 's are i.i.d random variable with distribution function m(.), distribution function M(.) & Mean 1/α; α>0

Y_A, Y_B: exponential random variable denoting the optional threshold for grade A and grade B with parameters λ_A, λ_B respectively.

Z_A, Z_B –: exponential random variable denoting the mandatory threshold for grade A and grade B with parameters μ_A , μ_B respectively.

Assume that $Y_A < Z_A$ & $Y_B < Z_B$.

$Y = \max(Y_A, Y_B)$ & $Z = \max(Z_A, Z_B)$

p : probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower.

q : the probability that every decision has exit of personnel. When q=0 corresponds to the case where exits are impossible. It is assumed that q≠0.

T: random variable denoting the time to recruitment with distribution function L(.), density function l(.), mean E(T) and variance V(T).

$N_e(t)$: Number of exits points lying in (0,t].

$f_k(t)$, $F_k(t)$:k fold convolution of f(.) and F(.) respectively.

The univariate CUM policy of recruitment employed in this section is stated as follows: *Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of man hours exceeds the optional threshold.*

III. MAIN RESULT

P(T>t) = P{ there are exactly 'k' exits in [0,t] and cumulative loss of manpower does not exceed the optional threshold (or) In 'k' exits , no recruitment is made when the optional threshold is crossed with 'p' probability but cumulative loss of manpower does not exceeds the mandatory threshold Z}.

ie). P(T>t) =

$$\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k \le Y\}+p\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k > Y\}P\{S_k \le Z\} \quad (1)$$

From Renewal theory,

$$P\{N_e(t)=k\}=G_k(t)-G_{k+1}(t) \text{ and } G_0(t)=1 \quad \text{--- (2)}$$

∴(1) becomes,

P(T>t)

$$=\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k \le Y\}+p\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k > Y\}P\{S_k \le Z\} \quad (3)$$

Model-I:Y= max (Y_A , Y_B) & Z= max (Z_A , Z_B)

$$P\{S_k < Y\} = \int_0^{\infty} P\{Y > X\}g_k(x)dx$$

$$P(S_k \le Y) = a_1^k + a_2^k - a_3^k ,$$

Where $a_1 = E[e^{-\lambda_A x}]; a_2 = E[e^{-\lambda_B x}]$ & $a_3 = E[e^{-(\lambda_A+\lambda_B)x}]$.

$$P(S_k \le Z) = b_1^k + b_2^k - b_3^k ,$$

where $b_1 = E[e^{-\mu_A x}], b_2 = E[e^{-\mu_B x}]$ & $b_3 = E[e^{-(\mu_A+\mu_B)x}]$ --- (4)

P(T > t) =

$$\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}\{a_1^k + a_2^k - a_3^k\}+$$

$$p\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}\{1-a_1^k - a_2^k + a_3^k\}\{b_1^k + b_2^k - b_3^k\} \quad \text{--- (5)}$$

$$\text{L(t)} = \overline{a_1}\sum_{k=1}^{\infty}G_k(t)a_1^{k-1} + \overline{a_2}\sum_{k=1}^{\infty}G_k(t)a_2^{k-1} -$$

$$\overline{a_3}\sum_{k=1}^{\infty}G_k(t)a_3^{k-1} + p\,\{\,\overline{b_1}\sum_{k=1}^{\infty}G_k(t)b_1^{k-1} + \overline{b_2}\sum_{k=1}^{\infty}G_k(t)b_2^{k-1}$$

$$-\overline{b_3}\sum_{k=1}^{\infty}G_k(t)b_3^{k-1} + \overline{a_1b_3}\sum_{k=1}^{\infty}G_k(t)(a_1b_3)^{k-1} -$$

$$\overline{a_1b_1}\sum_{k=1}^{\infty}G_k(t)(a_1b_1)^{k-1} - \overline{a_1b_2}\sum_{k=1}^{\infty}G_k(t)(a_1b_2)^{k-1}$$

$$+\overline{a_2b_3}\sum_{k=1}^{\infty}G_k(t)(a_2b_3)^{k-1} - \overline{a_2b_1}\sum_{k=1}^{\infty}G_k(t)(a_2b_1)^{k-1} -$$

$$\overline{a_2b_2}\sum_{k=1}^{\infty}G_k(t)(a_2b_2)^{k-1} + \overline{a_3b_1}\sum_{k=1}^{\infty}G_k(t)(a_3b_1)^{k-1}$$

$$+\overline{a_3b_2}\sum_{k=1}^{\infty}G_k(t)(a_3b_2)^{k-1} - \overline{a_3b_3}\sum_{k=1}^{\infty}G_k(t)(a_3b_3)^{k-1}\,\}$$

Differentiating and taking Laplace transform on both sides,

$$l^*(s) = \overline{a_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_1^{k-1} + \overline{a_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_2^{k-1} -$$

$$\overline{a_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_3^{k-1} + p\{\overline{b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_1^{k-1} +$$

$$\overline{b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_2^{k-1} -$$

$$\overline{b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_3^{k-1} + \overline{a_1b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_1b_3)^{k-1} -$$

$$\overline{a_1b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_1b_1)^{k-1} -$$

$$\overline{a_1b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_1b_2)^{k-1} + \overline{a_2b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_2b_3)^{k-1} -$$

$$\overline{a_2b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_2b_1)^{k-1} - \overline{a_2b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_2b_2)^{k-1}$$

$$+ \overline{a_3b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_1)^{k-1} + \overline{a_3b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_2)^{k-1}$$

$$- \overline{a_3b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_3)^{k-1} \} \quad \text{--- (6)}$$

It can be shown that distribution function G(.) of the inter exit times satisfy the relation

$$G^*(s) = \sum_{n=1}^{\infty}(1-q)^{n-1}q_A F_n(x) + \sum_{n=1}^{\infty}(1-q)^{n-1}q_B F_n(x) \quad \text{--- (7)}$$

Taking laplace transform on both sides ,

$$G^*(s) = \frac{q_A f^*(s)}{1-\{(1-q)f^*(s)\}} + \frac{q_B f^*(s)}{1-\{(1-q)f^*(s)\}}$$

Mean E(T) = $-\{\frac{d}{ds} l^*(s)\}_{s=0}$

E(T) = $\frac{\overline{a_3}(q_A+q_B)}{\lambda[q-a_3(q_A+q_B)]^2} - \frac{\overline{a_1}(q_A+q_B)}{\lambda[q-a_1(q_A+q_B)]^2} -$

$$\frac{\overline{a_2}(q_A+q_B)}{\lambda[q-a_2(q_A+q_B)]^2} + p\{\frac{\overline{b_3}(q_A+q_B)}{\lambda[q-b_3(q_A+q_B)]^2}$$

$$-\frac{\overline{b_1}(q_A+q_B)}{\lambda[q-b_1(q_A+q_B)]^2} -$$

$$\frac{\overline{b_2}(q_A+q_B)}{\lambda[q-b_2(q_A+q_B)]^2} + \frac{\overline{a_1b_1}(q_A+q_B)}{\lambda[q-a_1b_1(q_A+q_B)]^2} +$$

$$\frac{\overline{a_1b_2}(q_A+q_B)}{\lambda[q-a_1b_2(q_A+q_B)]^2} -$$

$$\frac{\overline{a_1b_3}(q_A+q_B)}{\lambda[q-a_1b_3(q_A+q_B)]^2} + \frac{\overline{a_2b_1}(q_A+q_B)}{\lambda[q-a_2b_1(q_A+q_B)]^2} +$$

$$\frac{\overline{a_2b_2}(q_A+q_B)}{\lambda[q-a_2b_2(q_A+q_B)]^2} - \frac{\overline{a_2b_3}(q_A+q_B)}{\lambda[q-a_2b_3(q_A+q_B)]^2} -$$

$$\frac{\overline{a_3b_1}(q_A+q_B)}{\lambda[q-a_3b_1(q_A+q_B)]^2} -$$

$$\frac{\overline{a_3b_2}(q_A+q_B)}{\lambda[q-a_3b_2(q_A+q_B)]^2} + \frac{\overline{a_3b_3}(q_A+q_B)}{\lambda[q-a_3b_3(q_A+q_B)]^2} \} \quad \text{--- (8)}$$

This gives the mean of the time to recruitment for Model-I

Model-II: Y= min(Y_A, Y_B) & Z= min(Z_A, Z_B)

$$P\{S_k < Y\} = \int_0^{\infty} P\{Y > X\} g_k(x)dx$$

$$P(S_k \le Y) = a_3^k \text{ where } a_3 = E[e^{-(\lambda_A+\lambda_B)x}].$$

$$P(S_k \le Z) = b_3^k \text{ where } b_3 = E[e^{-(\mu_A+\mu_B)x}]$$

Equation (1) becomes,

P(T>t) =

$$\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k \le Y\} + p\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k > Y\}P\{S_k \le Z\}$$

$$P(T>t) = 1 + \overline{a_3}\sum_{k=1}^{\infty}G_k(t)a_3^{k-1} + p$$

$$\{\overline{a_3b_3}\sum_{k=1}^{\infty}G_k(t)(a_3b_3)^{k-1} - \overline{b_3}\sum_{k=1}^{\infty}G_k(t)b_3^{k-1}\} \quad \text{--- (9)}$$

$$L(t) = \overline{a_3}\sum_{k=1}^{\infty}G_k(t)a_3^{k-1} + p\{\overline{b_3}\sum_{k=1}^{\infty}G_k(t)b_3^{k-1} -$$

$$\overline{a_3b_3}\sum_{k=1}^{\infty}G_k(t)(a_3b_3)^{k-1}\}$$

$$l(t) = \overline{a_3}\sum_{k=1}^{\infty}g_k(t)a_3^{k-1} + p\{\overline{b_3}\sum_{k=1}^{\infty}g_k(t)b_3^{k-1} \quad \overline{a_3b_3}\sum_{k=1}^{\infty}g_k(t)(a_3b_3)^{k-1}\}$$

$$l^*(s) = \frac{\overline{a_3}f^*(s)(q_A+q_B)}{1-f^*(s)\{\overline{q}+a_3(q_A+q_B)\}} + p\{\frac{\overline{b_3}f^*(s)(q_A+q_B)}{1-f^*(s)\{\overline{q}+b_3(q_A+q_B)\}} - \frac{\overline{a_3b_3}f^*(s)(q_A+q_B)}{1-f^*(s)\{\overline{q}+a_3b_3(q_A+q_B)\}}\}$$

$$E(T) = \frac{\overline{a_3}(q_A+q_B)}{\lambda[q-a_3(q_A+q_B)]^2} + p\{\frac{\overline{b_3}(q_A+q_B)}{\lambda[q-b_3(q_A+q_B)]^2} - \frac{\overline{a_3b_3}(q_A+q_B)}{\lambda[q-a_3b_3(q_A+q_B)]^2}\} \quad (10)$$

This gives the mean of the time to recruitment for Model-II

IV. CONCLUSION

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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