International Journal of Innovative Research and Advanced Studies (IJIRAS)
Volume 3 Issue 4, April 2016

ISSN: 2394-4404

Intuitionistic Fuzzy Contra β Generalized Continuous Mappings

Saranya M
MSc Mathematics, Avinashilingam University,
Coimbatore, Tamil Nadu, India

Jayanthi D
Assistant Professor of Mathematics,
Avinashilingam University, Coimbatore,
Tamil Nadu, India

***Abstract:** The purpose of this paper is to introduce the notion of intuitionistic fuzzy contra β generalized continuous mappings and study their behaviour and properties in intuitionistic fuzzy topological spaces. Additionally we obtain some interesting theorems.*

***Keywords:** Intuitionistic fuzzy topology, intuitionistic fuzzy β generalized closed set, intuitionistic fuzzy β generalized continuous mappings, intuitionistic fuzzy contra β generalized continuous mappings.*

I. INTRODUCTION

Atanassov [1] introduced intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces. The purpose of this paper is to introduce the notion of intuitionistic fuzzy contra β generalized continuous mappings and study their behaviour and properties in intuitionistic fuzzy topological spaces. Additionally we obtain some interesting theorems.

II. PRELIMINARIES

DEFINITION 2.1: [1] An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}.$$

DEFINITION 2.2: [1] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

- ✓ $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
- ✓ $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
- ✓ $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
- ✓ $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,
- ✓ $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

DEFINITION 2.3: [2] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

- ✓ $0_\sim, 1_\sim \in \tau$,
- ✓ $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- ✓ $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

COROLLARY 2.4: [2] Let A, A_i ($i \in J$) be intuitionistic fuzzy sets in X and B, B_j($j \in K$) be intuitionistic fuzzy sets in Y and f: X → Y be a function. Then

- ✓ $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
- ✓ $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
- ✓ c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]
- ✓ d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]
- ✓ e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
- ✓ f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
- ✓ g) $f^{-1}(0\sim) = 0\sim$
- ✓ h) $f^{-1}(1\sim) = 1\sim$
- ✓ $f^{-1}(B^c) = (f^{-1}(B))^c$

DEFINITION 2.5: [12] Two IFSs are said to be q-coincident ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

DEFINITION 2.6: [3] An *intuitionistic fuzzy point* (IFP for short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \le \mu_A$ and $\beta \ge \nu_A$.

DEFINITION 2.7: [6] Let A be an IFS in an IFTS (X, τ). Then the β-interior and β-closure of A are defined as

$\beta int(A) = \cup\{G / G$ is an IFβOS in X and $G \subseteq A\}$,

$\beta cl(A) = \cap\{K / K$ is an IFβCS in X and $A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $\beta cl(A^c) = (\beta int(A))^c$ and $\beta int(A^c) = (\beta cl(A))^c$.

DEFINITION 2.8: [8] An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy β generalized closed set* (IFβGCS for short) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFβOS in (X, τ).

The complement A^c of an IFβGCS A in an IFTS (X, τ) is called an *intuitionistic fuzzy β generalized open set* (IFβGOS in short) in X [9].

DEFINITION 2.9: [4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy continuous* (IF continuous for short) mapping if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$

DEFINITION 2.10: [10] If every IFβGCS in (X, τ) is an IFβCS in (X, τ), then the space can be called as an intuitionistic fuzzy β generalized $T_{1/2}$ space ($IF\beta_gT_{1/2}$ in short).

DEFINITION 2.11: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

- ✓ *intuitionistic fuzzy contra continuous* (IFC continuous for short) *mapping* if $f^{-1}(B) \in IFO(X)$ for each IFCS B in Y
- ✓ *intuitionistic fuzzy contra α- continuous* (IFCα continuous for short) *mapping* if $f^{-1}(B) \in IF\alpha O(X)$ for each IFCS B in Y
- ✓ *intuitionistic fuzzy contra pre continuous* (IFCP continuous for short) *mapping* if $f^{-1}(B) \in IFPO(X)$ for each IFCS B in Y

III. INTUITIONISTIC FUZZY CONTRA β GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy contra β generalized continuous mappings and investigated some of their properties.

DEFINITION 3.1: A mapping f: (X, τ) → (Y, σ) is called an *intuitionistic fuzzy contra β generalized continuous* (IFCβG continuous for short) mapping if $f^{-1}(V)$ is an IFβGCS in (X, τ) for every IFOS V of (Y, σ).

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples.

Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

EXAMPLE 3.2: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping.

REMARK 3.3: Every IFC continuous mapping, IFCα continuous mapping, IFCP continuous mapping, IFCS continuous mapping, IFCβ continuous mapping and IFCSP continuous mapping are IFCβG continuous mapping but the converses are not true in general. This can be seen from the following examples.

EXAMPLE 3.4: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, $IF\beta C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping but since $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is not an IFCS in X, as $cl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$, f is not an IFC continuous mapping.

EXAMPLE 3.5: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $cl(int(cl(f^{-1}(G_2)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \not\subseteq f^{-1}(G_2)$. Hence $f^{-1}(G_2)$ is not an IFαCS in X. Hence f is not an IFCα continuous mapping.

EXAMPLE 3.6: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$ is an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $cl(int(f^{-1}(G_2))) = cl(G_1) = 1\sim \not\subseteq f^{-1}(G_2)$. Hence $f^{-1}(G_2)$ is not an IFPCS in X. Hence f is not an IFCP continuous mapping.

EXAMPLE 3.7: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $int(cl(f^{-1}(G_2))) = int(G_1^c) = G_1 \not\subseteq f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$. Hence $f^{-1}(G_2)$ is not an IFSCS in X. Hence f is not an IFCS continuous mapping.

EXAMPLE 3.8: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.8_v), (0.5_u, 0.2_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.5_u, 0.8_v), (0.5_u, 0.2_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$ is an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $int(cl(int(f^{-1}(G_2)))) = int(cl(G_1)) = int(1\sim) = 1\sim \not\subseteq f^{-1}(G_2) = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$. Hence $f^{-1}(G_2)$ is not an IFβCS in X. Hence f is not an IFCβ continuous mapping.

EXAMPLE 3.9: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping.

Since IFPC(X) = $\{0 \sim, 1 \sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$ and

There exists no IFPCS B in X such that $int(B) \subseteq f^{-1}(G_2) \subseteq B$, $f^{-1}(G_2)$ is not an IFSPCS in X. Hence f is not an IFCSP continuous mapping.

REMARK 3.10: IFCG continuous mappings and IFCβG continuous mappings are independent to each other.

EXAMPLE 3.11: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $G_2 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X.

Then, IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

Hence $f^{-1}(G_2)$ is an IFβGCS in (X, τ). Therefore f is an IFCβG continuous mapping. We have $cl(f^{-1}(G_2)) = G_1^c \not\subseteq G_1$. Hence $f^{-1}(G_2)$ is not an IFGCS in X. Hence f is not an IFCG continuous mapping.

EXAMPLE 3.12: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$, $G_2 = \langle x, (0.3_a, 0.1_b), (0.7_a, 0.8_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFCG continuous mapping, where G_3 is an IFOS in (Y, σ).

Now IFβC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ or $\mu_a < 0.3$ and $\mu_b < 0.1$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$.

We have $f^{-1}(G_3) \subseteq G_1$ but $\beta cl(f^{-1}(G_3)) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle = 1\sim \not\subseteq G_1$. Hence $f^{-1}(G_3)$ is not an IFβGCS in X. Hence f is not an IFCβG continuous mapping.

THEOREM 3.13: A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IFCβG continuous mapping if and only if the inverse image of each IFCS in Y is an IFβGOS in X.

PROOF: (NECESSITY): Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Then $f^{-1}(A^c)$ is an IFβGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβGOS in X.

(SUFFICIENCY): Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis $f^{-1}(A^c)$ is IFβGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFβGOS in X. Therefore $f^{-1}(A)$ is an IFβGCS in X. Hence f is an IFCβG continuous mapping.

THEOREM 3.14: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping and let $f^{-1}(A)$ be an IFβOS in X for every IFCS A in Y. Then f is an IFCβG continuous mapping.

PROOF: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFβOS in X, by hypothesis. Since every IFβOS is an IFβGOS [9], $f^{-1}(A)$ is an IFβGOS in X. Hence f is an IFCβG continuous mapping, by Theorem 3.13.

THEOREM 3.15: Let f: $(X, \tau) \to (Y, \sigma)$ be a bijective mapping. Suppose that one of the following properties hold:

- ✓ $f^{-1}(cl(B)) \subseteq int(\beta cl(f^{-1}(B)))$ for each IFS B in Y
- ✓ $cl(\beta int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ for each IFS B in Y
- ✓ $f(cl(\beta int(A))) \subseteq int(f(A))$ for each IFS A in X
- ✓ $f(cl(A)) \subseteq int(f(A))$ for each IFβOS A in X

Then f is an IFCβG continuous mapping.

PROOF: (i) ⇒ (ii) is obvious by taking complement of (i).

(ii) ⇒ (iii) Let $A \subseteq X$. Put $B = f(A)$ in Y. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(\beta int(A)) = cl(\beta int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(\beta int(A))) \subseteq f(f^{-1}(int(B))) = int(B) = int(f(A))$.

(iii) ⇒ (iv) Let $A \subseteq X$ be an IFβOS. Then $\beta int(A) = A$. By hypothesis, $f(cl(\beta int(A))) \subseteq int(f(A))$. Therefore $f(cl(A)) = f(cl(\beta int(A))) \subseteq int(f(A))$.

Suppose (iv) holds. Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFS in X and $\beta int(f^{-1}(A))$ is an IFβOS in X. Hence by hypothesis, $f(cl(\beta int(f^{-1}(A)))) \subseteq int(f(\beta int(f^{-1}(A)))) \subseteq int(f(f^{-1}(A))) = int(A) \subseteq A$. Therefore $cl(\beta int(f^{-1}(A))) = f^{-1}(f(cl(\beta int(f^{-1}(A))))) \subseteq f^{-1}(A)$. Now $cl(int(f^{-1}(A))) \subseteq cl(\beta int(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IFβGCS [8] in X. Thus f is an IFCβG continuous mapping.

THEOREM 3.16: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping. Suppose that one of the following properties hold:

- ✓ $f(\beta cl(A)) \subseteq int(f(A))$ for each IFS A in X
- ✓ $\beta cl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y
- ✓ $f^{-1}(cl(B)) \subseteq \beta int(f^{-1}(B))$ for each IFS B in Y

Then f is an IFCβG continuous mapping.

PROOF: (i) ⇒ (ii) Let $B \subseteq Y$. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(\beta cl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $\beta cl(f^{-1}(B)) \subseteq f^{-1}(f(\beta cl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$.

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq \beta int(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS [9] in X. Therefore f is an IFCβG continuous mapping.

THEOREM 3.17: Let f: $(X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then f is an IFCβG continuous mapping if $cl(f(A)) \subseteq f(\beta int(A))$ for every IFS A in X.

PROOF: Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. By hypothesis $cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Since f is an onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\beta int(f^{-1}(A)))) = \beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS [9] in X. Thus f is an IFCβG continuous mapping.

THEOREM 3.18: If f: $(X, \tau) \to (Y, \sigma)$ is an IFCβG continuous mapping, where X is an IF$\beta_g T_{1/2}$ space, then the following conditions hold:

- ✓ $\beta cl(f^{-1}(B)) \subseteq f^{-1}(int(\beta cl(B)))$ for every IFOS in Y,
- ✓ $f^{-1}(cl(\beta int(B))) \subseteq \beta int(f^{-1}(B))$ for every IFCS B in Y.

PROOF: (i) Let $B \subseteq Y$ be an IFOS. By hypothesis $f^{-1}(B)$ is an IFβGCS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(B)$ is an IFβCS in X. This implies $\beta cl(f^{-1}(B)) = f^{-1}(B) \subseteq f^{-1}(int(B)) \subseteq f^{-1}(int(\beta cl(B)))$.

(ii) can be proved easily by taking the complement of (i).

THEOREM 3.19: If f: $(X, \tau) \to (Y, \sigma)$ is an IFCβG continuous mapping and g: $(Y, \sigma) \to (Z, \delta)$ is an IF continuous mapping then $g \circ f : (X, \tau) \to (Z, \delta)$is an IFCβG continuous mapping.

PROOF: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFOS in Y, since g is an IF continuous mapping. Since f is an IFCβG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβGCS in X. Therefore $g \circ f$ an IFCβG continuous mapping.

THEOREM 3.20: If f: $(X, \tau) \to (Y, \sigma)$ is an IFCβG continuous mapping and g: $(Y, \sigma) \to (Z, \delta)$ is an IFC continuous mapping then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IFβG continuous mapping.

PROOF: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IFC continuous mapping. Since f is an IFCβG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβGOS in X. Therefore $g \circ f$ is an IFβG continuous mapping.

THEOREM 3.21: For a mapping f: $(X, \tau) \to (Y, \sigma)$, where X is an IF$\beta_g T_{1/2}$ space, the following are equivalent:

- ✓ f is an IFCβG continuous mapping
- ✓ For every IFCS A in Y and for every IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\, _q\, A$ then $p_{(\alpha,\beta)}\, _q\, \beta int(f^{-1}(A))$
- ✓ For every IFCS in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\, _q\, A$ then there exists an IFβGOS B such that $p_{(\alpha,\beta)}\, _q\, B$ and $f(B) \subseteq A$.

PROOF: (i) ⇒ (ii) Let f be an IFCβG continuous mapping. Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})\, _q\, A$ then $p_{(\alpha,\beta)}\, _q\, f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an IFβGOS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(A)$ is an IFβOS in X. Hence $\beta int(f^{-1}(A)) = f^{-1}(A)$. This implies $p_{(\alpha,\beta)}\, _q\, \beta int(f^{-1}(A))$.

(ii) ⇒ (i) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$ and let $f(p_{(\alpha,\beta)})\, _q\, A$ then $p_{(\alpha,\beta)}\, _q\, f^{-1}(A)$. By hypothesis this implies $p_{(\alpha,\beta)}\, _q\, \beta int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \beta int(f^{-1}(A))$. But $\beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\beta int(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS [9] in X. This implies f is an IFCβG continuous mapping.

(ii) ⇒ (iii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})\, _q\, A$ then $p_{(\alpha,\beta)}\, _q\, f^{-1}(A)$. By

ISSN: 2394-4404

hypothesis this implies $p_{(\alpha,\beta)}\ q\ \beta int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \beta int(f^{-1}(A))$. But $\beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\beta int(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFβOS in X and hence an IFβGOS [9] in X. Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)}\ q\ B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (ii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})\ q\ A$ then $p_{(\alpha,\beta)}\ q\ f^{-1}(A)$. By hypothesis there exists an IFβGOS B in X such that $p_{(\alpha,\beta)}\ q\ B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an $IF\beta_gT_{1/2}$ space, $f^{-1}(A)$ is an IFβOS in X and $\beta int(f^{-1}(A)) = f^{-1}(A)$. Therefore $p_{(\alpha,\beta)}\ q\ \beta int(f^{-1}(A))$.

THEOREM 3.22: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IFCβG continuous mapping if $f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

PROOF: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. Since every IFCS is an IFβCS, $\beta cl(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an IFC continuous mapping. Then by Remark 3.3, f is an IFCβG continuous mapping.

THEOREM 3.23: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IFCβG continuous mapping, where X is an $IF\beta_gT_{1/2}$ space if and only if $f^{-1}(\beta cl(B)) \subseteq \beta int(f^{-1}(cl(B)))$ for every IFS B in Y.

PROOF: NECESSITY: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis, $f^{-1}(cl(B))$ is an IFβGOS in X. Since X is an $IF\beta_gT_{1/2}$ space, $f^{-1}(cl(B))$ is an IFβOS in X. Therefore $f^{-1}(\beta cl(B)) \subseteq f^{-1}(cl(B)) = \beta int(f^{-1}(cl(B)))$.

SUFFICIENCY: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. By hypothesis, $f^{-1}(\beta cl(B)) \subseteq \beta int(f^{-1}(cl(B))) = \beta int(f^{-1}(B))$. But $\beta cl(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\beta cl(B)) \subseteq \beta int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFβOS in X and hence an IFβGOS [9] in X. Hence f is an IFCβG continuous mapping.

REFERENCES

[1] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy sets and systems, 1986, 87-96.

[2] Coker, D., An introduction to intuitionistic fuzzy topological space, Fuzzy sets and systems, 1997, 81-89.

[3] Coker, D., and Demirci, M., On intuitionistic fuzzy points, *Notes on Intuitionistic Fuzzy Sets* 1(1995), 79-84.

[4] Gurcay, H., Coker, D., and Haydar, Es. A., On fuzzy continuity in intuitionistic fuzzy topological spaces, The J. fuzzy mathematics, 1997, 365-378.

[5] Krsteska, B., E. Ekici, Intuitionistic fuzzy contra strong precontinuity , Faculty of Sciences and Mathematics, University of Nis, Siberia, 2007, 273-284.

[6] Jayanthi, D., Generalized β closed sets in intuitionistic fuzzy topological spaces, International Journal of Advance Foundation and research in Science & Engineering (IJAFRSE), December 2014.

[7] Jayanthi, D., Relation between semipre closed sets and beta closed sets in intuitionistic fuzzy topological spaces, Notes on Intuitionistic Fuzzy Sets, 2013, 53-56.

[8] Saranya, M., and Jayanthi, D., On Intuitionistic fuzzy β generalized closed sets, International Journal of Computational Engineering Research (IJCER), March 2016, 37-43.

[9] Saranya, M., and Jayanthi, D., On Intuitionistic fuzzy β generalized open sets, (accepted)

[10] Saranya, M., and Jayanthi, M., On Intuitionistic fuzzy β generalized $T_{1/2}$ spaces, (accepted)

[11] Saranya, M., and Jayanthi, D., On Intuitionistic fuzzy β generalized continuous mappings, International Journal of Advance Foundation and research in Science & Engineering (IJAFRSE), Vol 2, Issue 10, April 2016.

[12] Thakur, S. S., and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria: Mathematica, 2006, 257-272.

[13] Thakur, S.S., and Singh, S., On fuzzy semi-pre open sets and fuzzy semi-pre continuity, Fuzzy sets and systems, 1998, 383-391.

[14] Young Bae Jun & seok – Zun Song, Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings, jour. of Appl. Math & computing, 2005, 467-474.