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Properties Of Fuzzy Ψ** – Continuous Functions

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Abstract: ***In this paper, we introduced the concept of fuzzy ψ** – irresolute functions and fuzzy almost ψ** – irresolute continuous functions and some of their properties were investigated. Also we introduced fuzzy ψ** – homeomorphism and its properties were analyzed.***

Keywords: Fuzzy ψ** – continuous functions, fuzzy ψ** – irresolute functions, fuzzy g* – closed sets, fuzzy ψ** – homeomorphism.

I. INTRODUCTION

Ever since the introduction of fuzzy sets by zadeh[7] and fuzzy topological spaces by chang[4] various notions in classical topology have been extended to fuzzy topological spaces. The concept of generalized fuzzy semi – closed sets was introduced by Balasubramanian, Chandrasekar[2]. The concept of fuzzy semi – generalized closed sets in fuzzy toplogical space was introduced by H. Maki, T.Fukutake, M.Kojima and H.Harada[5]. In 2000, M.Caldas[3] defined and studied weak and strong forms of irresolute maps in general topology.

The class of fuzzy sets on a universe X will be denoted by I^X and will be denoted by Greek letters λ, μ, ε, η, δ, γ etc.

A family τ of fuzzy sets in X is called fuzzy topology for X if

- ✓ $0, 1 \in \tau$
- ✓ If $\lambda, \mu \in \tau$ then $\lambda \cap \mu \in \tau$
- ✓ If $\delta_i \in \tau$ for each i then $\bigcup_{i \in I} \delta_i$

Moreover the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a fuzzy open set.

II. PRELIMINARIES

In this paper X and Y are fuzzy topological spaces. Let μ be a fuzzy set in X. We denote the interior and closure of a fuzzy set μ be int(μ) and cl(μ) respectively.

A fuzzy set μ in a space X is called fuzzy pre – open (resp. fuzzy semi – open) if $\mu \leq int(cl(\mu))$ (resp. $\mu \leq cl(int(\mu))$). The complement of a fuzzy pre – open (resp. fuzzy semi - open) set is said to be fuzzy pre closed (resp. fuzzy semi - closed).

A fuzzy set μ in a space X is called fuzzy β – open (fuzzy semi – preopen) if $\mu \leq cl(int(cl(\mu)))$. The complement of a fuzzy β – open set is said to be fuzzy β – closed.

Let μ be a fuzzy set in a fuzzy topological space X. The fuzzy β – closure and fuzzy β – interior of μ are defined as $\bigcap\{\eta : \mu \leq \varepsilon, \varepsilon \text{ is } \beta\text{ – closed}\}$, $\bigcup\{\eta : \mu \geq \varepsilon, \varepsilon \text{ is } \beta\text{ – open}\}$ and denoted by β – cl(μ) and β-int(μ) respectively.

A fuzzy set in X is called a fuzzy singleton if and only if it takes the value 0, for all $y \in X$ except one $x \in X$. If its value at x is ε $(0 < \varepsilon \leq 1)$ we denote this fuzzy singleton by x_ε, where the pt. x is called its support. For any singleton x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ if and only if $\varepsilon \leq \mu(x)$.

Let $f : X \to Y$ a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$ defined by $g(x_o) = (x_\varepsilon, f(x_\varepsilon))$ is called the fuzzy graph function of f. The subset $\{x_\varepsilon, f(x_\varepsilon);\ x_\varepsilon \in X\} \leq X \times Y$ is called the fuzzy graph of f and is denoted by G(f).

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A fuzzy set λ of (X, τ) is called fuzzy ψ – closed if scl(λ) ≤ μ, λ ≤ μ and μ is fuzzy semi – generalised open in X.

III. FUZZY Ψ** – CLOSED SETS

DEFINITION 3.1

A fuzzy set λ in a fuzzy topological space (X, τ) is called fuzzy ψ^* – closed if scl(λ) ≤ int μ, λ ≤ μ and μ is fuzzy semi generalised open in X.

DEFINITION 3.2

A fuzzy set λ in (X, τ) is called fuzzy ψ** – closed if scl(λ) ≤ int(cl(μ)), λ ≤ μ and μ is fuzzy semi generalised open in X.

DEFINITION 3.3

A fuzzy set λ in a fuzzy topological space is said to be fuzzy ψ** – open if int(cl(μ)) ≤ scl (λ), where λ ≤ μ and μ is fsg – open in X.(fuzzy semi-generalized open).

EXAMPLE 3.1

Let X = $\{x_1, x_2, x_3\}$. Define $f_i : X \to [0, 1]$ as follows; i = 1, 2, 3 $f_1(x) = 0_x$, $f_2(x) = 1_x$; $f_3(x) = \begin{cases} 0, \text{if } x = x_2, x_3 \\ 1, \text{ if } x = x_1 \end{cases}$. Clearly T = $\{f_1, f_2, f_3\}$ is a fuzzy topology on X. Define $\lambda_1, \lambda_2 : X \to [0, 1]$ as $\lambda_1(x) = \begin{cases} 0, \text{if } x = x_3 \\ 1, \text{ if } x = x_1, x_2 \end{cases}$, $\lambda_2(x) = \begin{cases} 0, \text{if } x = x_2 \\ 1, \text{ if } x = x_1, x_3 \end{cases}$. Hence λ_1 and λ_2 are fuzzy ψ** – open map.

THEOREM 3.1

The union of two fuzzy ψ** – closed sets is fuzzy ψ** – closed.

PROOF:

Let λ_1 and λ_2 are two fuzzy ψ** – closed sets in a fuzzy topological space (X, τ). By definition(3.2) $scl(\lambda_1) \leq int(cl(\mu))$ and $scl(\lambda_2) \leq int(cl(\mu))$ whenever $\lambda_1, \lambda_2 \leq \mu$ and μ is fuzzy semi generalised open. Therefore $scl(\lambda_1 \cup \lambda_2) \leq int(cl(\mu))$. This implies that the union of two fuzzy ψ** – closed sets is fuzzy ψ** – closed.

REMARK 3.1

The intersection of two fuzzy ψ** – closed sets need not be a fuzzy ψ** – closed set.

THEOREM 3.2

Every fuzzy ψ** – closed set is fuzzy ψ* – closed, but the converse is not true.

PROOF:

Let λ be a fuzzy ψ** – closed set. Therefore scl(λ) ≤ int(cl(μ)) where μ is fuzzy semi – generalised open. This implies that scl(λ) ≤ int(μ) and hence λ is fuzzy ψ* – closed sets. Conversely, suppose that λ is fuzzy ψ* – closed. Then scl(λ) ≤ int(μ). If λ is fuzzy ψ** – closed then scl(λ) ≤ int(cl(μ)). But this is not true for all fuzzy ψ* – closed sets.

DEFINITION 3.4

A fuzzy set λ is said to be fuzzy g* – closed if cl(λ) ≤ μ whenever λ ≤ μ and μ is fuzzy generalised open in X.

DEFINITION 3.5

A fuzzy function f : X → Y is said to be fuzzy g^* – continuous if the inverse image of each fuzzy open set is fuzzy g* – open in Y.

DEFINITION 3.6

Let X and Y be fuzzy topological spaces. A function f : X → Y is said to be fuzzy ψ** – continuous if the inverse image of each fuzzy ψ** – open set is fuzzy ψ** – open.

REMARK 3.2

Also the inverse image of each fuzzy ψ** – closed set is fuzzy ψ** – closed[6].

THEOREM 3.3

The following statement are equivalent for a fuzzy function f : X → Y;

- ✓ f is fuzzy ψ** – continuous,
- ✓ for every fuzzy ψ** – open set λ in Y $f^{-1}(\lambda)$ is fuzzy ψ – open.
- ✓ $f^{-1}(int(cl(\mu)))$ is fuzzy ψ** – open for every fuzzy open set.
- ✓ f is fuzzy g* – open.

PROOF:

(i)$\Rightarrow$(ii): Let f be fuzzy ψ** – continuous. Let λ be a fuzzy ψ** – open set. Then Y-λ is fuzzy ψ** – closed. Then $f^{-1}(Y-\lambda) = X-f^{-1}(\lambda)$ is fuzzy ψ** – open. Thus $f^{-1}(\lambda)$ is fuzzy ψ** – open. This implies $f^{-1}(\lambda)$ is fuzzy ψ – open.

(ii)$\Rightarrow$(iii): For every fuzzy ψ** – open set μ in Y, $f^{-1}(\mu)$ is fuzzy ψ – open. Let μ be a fuzzy ψ** – open set in Y. This implies that int(cl(μ)) ≤ scl(λ), whenever μ is fuzzy semi generalised open. Hence $f^{-1}(scl(\lambda))$ is fuzzy ψ** – open for every fuzzy open set.

(iii)$\Rightarrow$(iv): If f is fuzzy ψ^{**} – open then the inverse image of each fuzzy ψ^{**} – open set in Y is fuzzy ψ^{**} – open in X. Also $f^{-1}(scl(\lambda))$ is fuzzy ψ^{**} – open for every fuzzy open set. Then $f^{-1}(int(cl(\lambda)))$ is fuzzy g^{*} – open, since λ is fuzzy g^{*} – open. This implies that $int(cl(\mu)) \le scl(\lambda)$. This implies $cl(\mu) \le scl(\lambda)$, that is $\mu \le cl(\lambda)$. Hence f is fuzzy g^{*} – open.

DEFINITION 3.5

A fuzzy function $f : X \to Y$ is called fuzzy ψ^{**} – irresolute if the inverse image of each fuzzy ψ^{**} – open set is fuzzy ψ^{**} – open.

DEFINITION 3.6

A function $f : X \to Y$ is called fuzzy ψ^{**} – open if the image of each fuzzy ψ^{**} – open set is fuzzy ψ^{**} – open.

THEOREM 3.4

Let X, Y, Z be fuzzy topological spaces and $f : X \to Y$ be a fuzzy function. If f is fuzzy ψ^{**} – irresolute and g is fuzzy almost ψ^{**} – continuous, then gof is fuzzy almost ψ^{**} – continuous.

PROOF:

Let $\lambda \le \mu$ be fuzzy almost ψ^{**} – continuous set and $(gof)(x_\varepsilon) \in \lambda$. This implies $g(f(x_\varepsilon)) \in \lambda$. Since g is fuzzy almost ψ^{**} – continuous function then it follows that their exists a fuzzy ψ^{**} – open set δ containing $f(x_\varepsilon)$ such that $g(\delta) \le \lambda$. Since f is a fuzzy ψ^{**} – irresolute function it follows that their exists a fuzzy ψ^{**} – open set γ containing x_ε such that $f(\gamma) \le \delta$. Therefore $(gof)(\gamma) = g(f(\gamma)) \le g(\delta) \le \lambda$. Thus gof is a fuzzy almost ψ^{**} – continuous function.

DEFINITION 3.7

A fuzzy topological space is said to be fuzzy weakly ψ_ε if for any fuzzy ψ^{*} – open set μ and each $x_\varepsilon \in \mu$ then exists a fuzzy ψ^{**} – open set δ containing x_ε such that $x_\varepsilon \in \delta \le \mu$.

DEFINITION 3.8

A fuzzy function $f : X \to Y$ is said to be fuzzy almost ψ^{**} – continuous at $x_\varepsilon \in X$ if for each fuzzy ψ^{**} – open set γ containing $f(x_\varepsilon)$ then exists a fuzzy ψ^{**} – open set μ containing x_ε such that $f(\mu) \le int(cl(\gamma))$.

THEOREM 3.5

Let $f : X \to Y$ be a fuzzy almost ψ^{**} – continuous function. If Y is fuzzy weakly ψ_ε then f is fuzzy almost ψ^{**} – continuous.

PROOF:

Let μ be any fuzzy ψ^{**} – open set of Y. Since Y is fuzzy weakly ψ_ε then exists a family P whose members are fuzzy ψ^{**} – open set of Y such that $\mu = \{\delta: \delta \in P\}$. Since f is fuzzy almost ψ^{**} – continuous, $f^{-1}(\delta)$ is fuzzy ψ^{**} – open in X for each $\delta \in P$ and $f^{-1}(\mu)$ is fuzzy ψ^{**} – open in X. Therefore f is fuzzy almost ψ^{**} – continuous.

THEOREM 3.6

If $f : X \to Y$ is a surjective fuzzy ψ^{**} – open and $g : Y \to Z$ is a fuzzy ψ^{**} – function such that $gof : X \to Z$ is fuzzy almost ψ^{**} – continuous then g is fuzzy almost ψ^{**} – continuous.

PROOF:

Let f is a surjective fuzzy ψ^{**} – open function. Suppose that x_ε is a fuzzy singleton in X. Let δ be a fuzzy ψ^{**} – open set containing x_ε such that $g(f(\mu)) \le \delta$. Since f is fuzzy ψ^{**} – open, $f(\mu)$ is fuzzy ψ^{**} – open set in Y containing $f(x_\varepsilon)$ such that $g(f(\mu)) \le \delta$. Therefore g is fuzzy almost ψ^{**} – continuous.

DEFINITION 3.9

A map $f : X \to Y$ is said to be fuzzy ψ^{**} – open map if f(U) is fuzzy ψ^{**} – open in Y for every fuzzy open set U in x.

DEFINITION 3.10

A bijection map $f : (X, T) \to (Y, S)$ is called fuzzy ψ^{**} – homeomorphism if f is both ψ^{**} – continuous and ψ^{**} – open.

THEOREM 3.7

For any fuzzy bijection map $f : X \to Y$ the following statements are equivalent:

(i) The inverse map $f^{-1}: Y \to X$ is fuzzy ψ^{**} – continuous

(ii) f is a fuzzy ψ^{**} – open map

(iii) f is fuzzy ψ^{**} – closed map

PROOF:

(i)$\Rightarrow$(ii): Let μ be any fuzzy ψ^{**} – open set in X. Since f^{-1} is fuzzy ψ^{**} – continuous the inverse image of μ and f^{-1} namely $f(\mu)$ is fuzzy ψ^{**} – open in Y. Hence f is fuzzy ψ^{**} – open map.

(ii)$\Rightarrow$(iii): Let λ be a fuzzy ψ^{**} – closed set in X. Then λ' is fuzzy ψ^{**} – open in X. Since f is a fuzzy ψ^{**} – open map $f(\lambda')$ is fuzzy ψ^{**} – open map in Y. But $f(\lambda') = Y - f(\lambda)$ implies $f(\lambda)$ is fuzzy ψ^{**} – closed in Y. Therefore f is a fuzzy ψ^{**} – closed map.

(iii)$\Rightarrow$(i):Let λ be any fuzzy ψ^{**} – closed set in X. Then the inverse image of f under f^{-1} namely $f(\lambda)$ is fuzzy ψ^{**} – closed in Y. Since f is a fuzzy ψ^{**} – closed map, f^{-1} is fuzzy ψ^{**} – continuous.

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THEOREM 3.8

Let f : (X, T) → (Y, S) be fuzzy bijective, and fuzzy ψ** – continuous map. Then the following statements are equivalent,

(i) f is fuzzy ψ** – open map.
(ii) f is fuzzy ψ** – homeomorphism.
(iii) f is fuzzy g* – closed map.

PROOF:

(i)$\Rightarrow$(ii): By assumption f is bijective fuzzy ψ** – continuous and fuzzy ψ** – open then by def [3.10], f is a fuzzy ψ** – homeomorphism.

(ii)$\Rightarrow$(iii): Since f is ψ** – open and bijective, then by theorem[3.7] f is a fuzzy ψ** – closed map.

(iii)$\Rightarrow$(i): Since f is fuzzy ψ** – closed and bijective and by theorem[3.7] f is a fuzzy ψ** – open map.

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