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Certain Studies On Exponential Distribution Through Quality Levels And Relative Slopes

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Abstract: This paper deals with Repetitive Group Reliability Sampling Scheme (RGRSS) through the designing of relative slopes and ratio of relative slopes indexed through the basic quality levels namely Acceptable Quality Level (AQL), Limiting Quality Level (LQL) and indifference Quality Levels (IQL) using exponential distribution. Necessary tables and procedures were given for designing the systems with illustrations.

Keywords: Reliability sampling, Acceptable Quality Level (AQL), Limiting Quality Level (LQL), Indifference Quality Level (IQL), Relative Slopes.

I. INTRODUCTION

In Modern industrial companies have been characterized by the ever-increasing step of new products appearing on the market. There is also an increasing reliability requirement for these newly developed products. To deliver a new product with high reliability, it is necessary for the companies or manufacturers to go after and manage its reliability throughout its life cycle. In safety-critical industries failure is near unacceptable, means that the design process continues until the level of reliability required is achieved, even if additional design costs are incurred. The reliability methods currently available are effective in reducing product failure, but a need still exists for methods tailored towards the conceptual stage of design (Stephens 1966). The concept of Repetitive Group Sampling (RGS) plans was proposed by Sherman (1965) in which acceptance or rejection of a lot is based on the repeated sample results of the same lot given by Soundararajan and Ramaswamy (1986).

Reliability sampling is an important approach to control product quality, which in turn affects consumers' beliefs about the product and hence sales. Because Reliability sampling involves various costs, such as those due to sampling, rejection, and acceptance, a decision maker may look for as much information as possible to plan the test in order to minimize the average aggregate cost. As such, Repetitive Group Reliability Sampling Scheme (RGRSS) through the designing of relative slopes and ratio of relative slopes indexed through the basic quality levels. The steepness of the OC curve is measured about their slopes h along with their corresponding p may be used to design any sampling plan. Further, in practice the sharpness of inspection as measured by h_0 must be made a function of both the lot size and point of control.

In recent decades, the complexity of equipment used for military, space, commercial applications such as computers, automated production process, communication networks have increased manifold. The more complex a piece of equipment the less reliable it is and the more likely it is that one or more of its parts may fail. The Reliability Sampling Plans are widely used as a product control tool in such conditions.

Reliability sampling plans are applicable when the quality variable follows the lifetime of an item and their corresponding tests are called life tests. In reliability plans, the objective is to determine whether the lifetimes of items follows exponential which full fills the specified standard and to establish the minimum sample size for testing. It becomes important when quality of product follows lifetime given by RRL Kantam *et al* (2012).

Aslam *et al* (2013) has proposed RGS plan for the Weibull distribution using average life time as median for skewed distribution here an attempt is made for exponential distribution using average life time. This distribution is widely applicable in the area of reliability sampling plan to study the average life of the product, to improve a group sampling procedures by incorporating the repetitive scheme and to develop a new reliability sampling plan for assuring life of the product. Hence this plan is called Repetitive Group Reliability Sampling Scheme (RGRSS).

The principal effect on sampling inspection, which are filter and incentive effect are proposed a new criteria based on AQL, IQL and LQL. Hamaker (1960) has made elaborate studies about the slope h_0, which along with p_0, may be used to design any sampling plan. Such as (p_1, h_1), (p_2, h_2) and (p_*, h_*) can also be considered for selection of such plans. Cameron (1952) had provided tables for Constructing and for computing the Operating Characteristics for Single Sampling Plan.

Vadaldi (1986) has studied two principal effects of sampling inspection, which are filter and incentive effect and has proposed a new criterion based on the AQL and LQL points on the OC Curve. Suresh (1993) has presented and constructed tables for the selection of QSS with Single sampling plan as reference plan indexed through (p_1, h_1) and (p_2, h_2) involving incentive and filter effects.

Kaviyarasu (2012) has designed various plans for given values of (p_1, h_1), (p_2, h_2) and (p_0, h_0) for the selection of QSS-2 and QSS-3 with Repetitive Group Sampling plan as reference plan. Further they studied the QSS-1, QSS- 2 and QSS-3 with Conditional Repetitive Group Sampling (CRGS), Multiple Repetitive Group Sampling (MRGS) and Two-Stage Repetitive Group Sampling (TSRGS) plan as reference plan.

Kaviyarasu *et al* (2014) has Proposed Designing of repetitive Group Reliability Sampling Scheme (RGRSS) Through Basic Quality Levels. It follows the acceptance sampling methodologies in life time of the product through exponential distribution and parameters involved should be satisfying the producer risks and consumer risks.

GLOSSARY OF SYMBOLS

N = lot size
n = sample size
p = lot quality or process quality
$Pa\ (p)$ = Probability of Acceptance for given p
α = producers risk
β = consumers risk
p_0 = Indifference Quality Level (IQL) at which $Pa\ (p)$ = 0.50
h_0 = relative slope of the OC curve at p_0
h_1 = relative slope of the OC curve at p_1
h_2 = relative slope of the OC curve at p_2
n_1 = first sample size
n_2 = second sample size
OR = Operating Ratio = h_2/h_1

II. DESIGNING OF RGRSS WITH VARIOUS PARAMETER

A. (P_1, H_1), (P_2, H_2) AND (P_0, H_0) SPECIFIED

Designing plans for given values of p_1 p_2, p_0 and h_1, h_2, h_0 using the Table-1 for designing the parameters of RGRSS. For given h_1, h_2, h_0, scan the column headed h_1, h_2, h_0 using the Table 1 which is equal to or just greater than the desire value which locates the corresponding values of c_1, c_2 with np_1 and np_2.

EXAMPLE 1, for given p_1=0.004 and h_1=0.013 from Table-1 .Column headed h_1, locate the value which is equal to or just greater than the specified h_1 which is 0.114687, corresponding to this h_1 and np_1. Which associated with c_1=4, c_2=4. From this, one can obtain the sample size n=28.6≅ 29, thus, the selected plan parameters of RGRSS is n=29, c_1=4and c_2=4.

EXAMPLE 2, for given p_2=0.002 and h_2=0.002 from Table-1. Column headed h_2, locate the value which is equal to or just greater than the specified h_2 which is 0.050071, corresponding to this h_2 and np_2. Which associated with c_1=1, c_2=2. From this, one can obtain the sample size n=25, thus, the selected plan parameters of RGRSS is n=25, c_1=1 and c_2=1.

EXAMPLE 3, for given p_0=0.001 and h_0=0.002from Table-1. Column headed h_0, locate the value which is equal to or just greater than the specified h_0 which is 0.044745, corresponding to this h_0 and np_0. Which associated with c_1=5, c_2=9. From this, one can obtain the sample size n=44.7≅45, thus, the selected plan parameters for RGRSS is n=45, c_1=5 and c_2=9.

III. DESIGNING PLANS THROUGH THE RATIO RELATIVE SLOPES

Designing plans for specified AQL (or LQL) with the ratio relative slopes h_2/h_1. by using Table-1 under the column headed h_2/h_1, one can locate the values which is equal to or just greater than the desired ratio, corresponding to this located value one can find n,c_1, c_2 with np_1 values.

Example, p_1=0.07, h_2=0.2104, h_1=0.21049, one can obtain the ratio h_2/h_1=0.011. By using the Table-1 under the column headed h_2/h_1, locate the values which is equal to or just greater than the desired ratio, which is 0.01191 corresponding to this located value is 0.458792 which is associated with c_1, and c_2. From this one can obtain the sample size n=6.55≅7, Thus the selected plan parameter of RGRSS is n=7, c_1= 1 and c_2=1.

A. SELECTIONS AT AQL AND RELATIVE SLOPES AT LQL (P_1, H_2)

Table-1 is used to determine the parameter for RGRSS indexed by p_1 $(\alpha=0.05)$ and h_2 $(\beta=0.05)$. For example given p_1 =0.05 and h_2=0.02377 from Table-1 under column headed h_2, the value of h_2 is equal to or just greater than the value specified for np_1=0.367903.Which associated with a parameters c_1=1 and c_2=2, one can obtain the sample size n=np_1/p_1=7.35≅7 Thus, the selected plan parameters RGRSS is n=7, c_1= 1, and c_2=2.

B. SELECTIONS AT IQL AND RELATIVE SLOPES AT AQL (P_2, H_1)

Table-1 is used to determine the parameter for RGRSS indexed by h_1 *(α=0.05)* and p_2 *(β=0.05)*. For example given h_1 =0.07 and p_2=0.008 from Table-1 under column headed h_1, the value of h_1 is equal to or just greater than the value specified for np_2=0.046. And it is associated with its parameters are c_1=1 and c_2=4, one can obtain the sample size $n=np_2/p_2$=5.8≅6.Thus, the selected plan parameters RGRSS are n= 6, c_1=1, and c= 4.

IV. CONSTRUCTION OF TABLE-1

Let $P_a(p')$ and $P_r(p')$ be the properties for acceptance and rejection respectively in a particular group-sample, when $p = p'$.

Let $P_A(p')$ and $P_R(p')$be the probabilities for eventually accepting and rejecting the lot respectively, when $p = p'$. By definition one gets

$$P_a(p') = P[d \le c_1 / p = p'] \quad (4.1)$$

and $$P_r(p') = P[d > c_2 / p = p'] \quad (4.2)$$

The according to Sherman (1965)

$$P_A = P_a / (P_a + P_r) \quad (4.3)$$

and $$P_R = P_r / (P_a + P_r) \quad (4.4)$$

It's clear that $P_A + P_R = 1$.

For given p_1 (AQL), p_2 (LQL),α and β, one may write ,

$$P_A(p_1) = P_a(p_1)/[P_a(p_1) + P_r(p_1)] = 1 - \alpha \quad (4.5)$$

By the definition of a $P_a$andP_r, equation (5) may be written as,

$$\alpha P[d \le c_1/p = p_1] = (1 - \alpha)\, P[d > c_2/p = p_1 \quad (4.6)$$

When the sample size is not more than one-tenth of the lot size when the quality is measured in terms of defectives, the OC curve can be computed using the Binomial model. In addition to the condition of sample size being not more than one-tenth of the lot size, if the lot quality p (measured in terms of defectives) is less than or equal to 0.10, the OC curve can be based on the exponential model. When the quality is measured in terms of defects, the appropriate model is also the exponential model.

Under the condition of the exponential model, equation (4.6) become,

$$\alpha \int_0^{C1} np\ e^{(-np)x} = (1 - \alpha) \int_0^{c2} np\, e^{(-np)x} \quad (4.7)$$

For fixed α, the LHS of equation (7) is a monotonic decreasing function of np_1 and the RHS is a monotonic increasing function of np_1. Therefore every c_1 and c_2 there is an exactly one value of np_1.for different values of c_1 and $c_2$and when α=0.05, the np_1 values have been obtained using Newton's method of successive approximation and they are furnished in the second column of Table-1. By similar procedure for given (p_2, β), one has,

$$P_A(p_2) = P_a(p_2)/[P_a(p_2) + P_r(p_2)] = \beta \quad (4.8)$$

Which on substituting the exponential expression reduces to

$$(1 - \beta) \int_0^{C1} np\ e^{(-np)x} = \beta \int_0^{c2} np\, e^{(-np)x} \quad (4.9)$$

By the same argument followed in the earlier case the values for np_2 are obtained for different combinations of c_1 and c_2 and when β=0.10.

The value of np_0 and p_0 being the indifference quality level, are taken from Sherman (1965) and are furnished in Table-1.

The relative slope at $p = p_0$ is given by

$$h_0 = \frac{-p}{P_A(p)} \frac{dP_A(p)}{dp}\Big]\, p = p_0 \quad (4.10)$$

By substituting the values for $p_0 S(p_0)$ in the equation (4.10),h_0, similarly h_1,and h_2 values are obtained . Those are furnished in Table-1.

V. CONCLUSION

Reliability is a measure of uncertainty and therefore estimating reliability means using statistics and probability theory. In Reliability sampling a decision maker only focus on decreasing the sample size and minimizing the average aggregate sampling cost. As such, Repetitive Group Reliability Sampling Scheme (RGRSS) through the designing of relative slopes and ratio of relative slopes indexed through the basic quality levels, which is to improve a group sampling procedures by incorporating with the RGRSS for assuring life of the product. Relative slopes has obtained for the basic quality levels for h_0, h_1 and h_2 the readymade selection of the plan parameters which is used to support in decision making situation for both the producer as well as the consumer. Tables are provided here which tailor-made, handy and ready-made use to the industrial shop-floor condition.

c_1	c_2	h_0	np_0	h_1	np_1	h_2	np_2	$OR=h_2/h_1$
1	1	0.06127	0.247527	0.21049	0.458792	0.002507	0.050071	0.01191
1	2	0.04779	0.21861	0.135353	0.367903	0.002377	0.048754	0.017562
1	3	0.038689	0.196696	0.0972	0.311769	0.002267	0.047608	0.023323
1	4	0.032189	0.179412	0.074479	0.272909	0.002164	0.046524	0.029055
1	5	0.027352	0.165383	0.059567	0.244063	0.002069	0.045491	0.034734
2	2	0.015318	0.123764	0.052626	0.229404	0.000627	0.025035	0.011914
2	3	0.013452	0.115983	0.04141	0.203494	0.000609	0.024677	0.014707
2	4	0.011948	0.109305	0.033822	0.183908	0.000594	0.024377	0.017562
2	5	0.010708	0.10348	0.028373	0.168443	0.000580	0.024086	0.020442
2	6	0.009672	0.098348	0.024288	0.155847	0.000567	0.023804	0.023345
3	3	0.006808	0.082509	0.02339	0.152937	0.000279	0.01669	0.011928
3	4	0.006234	0.078957	0.019847	0.14088	0.000278	0.016673	0.014007
3	5	0.005741	0.075767	0.017151	0.130963	0.000268	0.016384	0.015626
3	6	0.00531	0.07287	0.015039	0.122632	0.000264	0.016251	0.017555
3	7	0.004932	0.070225	0.013342	0.115508	0.000260	0.016121	0.019487
4	4	0.003829	0.061882	0.013153	0.114687	0.000157	0.012518	0.011936
4	5	0.003583	0.059855	0.011611	0.107753	0.000156	0.012509	0.013436
4	6	0.003363	0.057992	0.010354	0.101753	0.000152	0.012339	0.014681
4	7	0.003166	0.056263	0.009321	0.096543	0.000150	0.012263	0.016093
4	8	0.002987	0.054652	0.008459	0.091972	0.000149	0.012188	0.017615
5	5	0.002451	0.049505	0.008418	0.09175	0.000100	0.010014	0.011879
5	6	0.002323	0.048197	0.007611	0.087239	0.000100	0.010009	0.013139
5	7	0.002207	0.046976	0.006936	0.08328	0.000097	0.009896	0.014116
5	8	0.0021	0.045828	0.006347	0.079667	0.000097	0.009847	0.015283
5	9	0.002002	0.044745	0.005847	0.076466	0.000096	0.009798	0.016419
6	6	0.001702	0.041255	0.005852	0.076498	0.000069	0.008345	0.011893
6	7	0.001627	0.040341	0.005371	0.073285	0.000069	0.008342	0.012959
6	8	0.001559	0.039479	0.004961	0.070436	0.000069	0.008337	0.014009
6	9	0.001495	0.038661	0.004606	0.067868	0.000067	0.008226	0.014698
6	10	0.001435	0.037883	0.004286	0.065468	0.000067	0.008192	0.015655

Table 1: Relative Slope values for RGRS scheme and it's Ratio

REFERENCES

[1] American national standards, Institute/American Society for Quality Control (ANSI/ASQC) STANDARD A2 (1987): "Terms, symbols and Definition for Acceptance Sampling", American Society for Quality Control, Milwaukee, Wisconsin. USA.

[2] Cameron JM (1952).Tables for constructing and computing the Operating Characteristics of Single

Sampling Plans. Industrial Quality Control. Vol.11, pp.37 - 39.

[3] Dodge, H. F. (1967): Notes on the Evolution of Acceptance Sampling Plans, Journal of Quality Technology. Vol.1, No.2, pp. 77-88.

[4] Hamaker, H. C. (1960). Attribute Sampling in Operation, Bulletin of the International Statistical Institute, Vol. 37, pp. 265 - 281.

[5] Kantam RRL *et al* (2012): Reliability test plans for Type-II Exponentiated log logistic Distribution, Journal of Reliability and Statistical Studies, Vol.5 Issue.1,pp.55-64.

[6] Kaviyarasu, V. and Suresh, K. K. (2012): Certain Results and Tables Relating to QSS-1 with Conditional RGS Plan, Journal of Mathematical Research, Canadian Center of Science and Education, Vol.3, No.4, pp.158-167, Nov 2011.

[7] Kaviyarasu, V and Fawaz, P (2014) Designing of repetitive Group Reliability Sampling Scheme (RGRSS) Through Basic Quality Levels, journal of Management in Manufacturing and Services, Vol.1 ,No.17, pp.35-47, Dec 2014.

[8] Muhammad Aslam *et al* (2013) A Repetitive Type of Group Acceptance Sampling Plan for assuring the Percentile Life, Vol.29 (4), 389-400.

[9] Sherman, R. E. (1965): "Design and Evaluation of Repetitive Group Sampling Plans", Technometrics, Vol.7, No.1, pp.11-21

[10] Soundararajan, V. and Ramasamy, V. (1986). Designing Repetitive Group Sampling Plan indexed by AQL and LQL, IAPQR Transaction, Vol.9, No.1, pp.9-14.

[11] Soundararajan, V. and Ramasamy, M. M. (1986): Procedures and Tables for Construction and Selection of Repetitive Group Sampling (RGS) Plan, the QR Journal, Vol.13, No.3, pp.19-21.

[12] Stephens, K. S. (1966): A class of Cumulative Results Sampling Inspection Plans and their evaluation, Ph.D., Thesis, Rutgers, The State University, New Brunswick, New Jersey.

[13] Suresh K.K, A Study on Acceptance Sampling using Acceptable and Limiting Quality Levels, Ph.D. Thesis, Bharathiar University, Tamilnadu (1993).

[14] Vedaldi, R. (1986): A new Criterion for the Construction of Single Sampling inspection plans by Attributes, Rivistadi Statistica Applicata,Vol.19,No.3, pp.235-244.