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Motion Of Porous Cylinder In Cylindrical Container

Abhishek Kumar Singh

Pankaj Shukla

Sanjit Das

VIT University, Chennai Campus,
Chennai, India

***Abstract:** This paper concern the creeping motion of a porous circular cylinder at the instant it passes the center of cylindrical container. The flow inside the porous cylindrical particle is governed by Brinkman equation. However, the flow inside the cylindrical container is governed by Stokes equation. The drag force acting on porous cylindrical particle is evaluated.*

***Keywords:** Stokes equation; Brinkman equations; Modified Bessel functions; Stress-tensor; Drag.*

MSC (2000): 76D07

I. INTRODUCTION

The study of the motion of a particle at the instant it passes the centre of the cylindrical container serves as a model of interaction in multi particle system. This class of problems is important because it provide some information on wall effects. The container used to hold a packing of particles will induce a local area of order of the container wall in an otherwise random packing, and which will make both the micro and macro structural properties near the wall different from those far away from the wall. This is known as the wall effect and often characterized in terms of porosity for spheres packed in cylindrical container. The side wall effect has been studied by many researchers for many years. Drag experienced by porous cylinder in a viscous fluid at low Reynolds number was evaluated by Strechkina[1]. An analytical study of the steady incompressible flow past a circular cylinder embedded in porous medium based on the Brinkman model has been reported by Pop and Cheng [2]. The problem of stokes flow through a swarm of circular cylinders with Happel and Kuwabara boundary condition was discussed by Deo[3]. Flow of a viscous fluid through a porous circular pipe and its surrounding porous medium and the flow around nano spheres and nano cylinder were studied by Mathews and Hill[4]. Singh and gupta[5] had discussed the problem of uniform flow past a permeable inhomogeneous circular cylinder by assuming that the flow in the porous cylinder is governed by Darcy law. Gupta[6] had solved the problem of flow past a porous cylinder matched asymptotic expansion as done by Kapulan[7] for an impervious circular cylinder. In this paper he evaluates the drag force experienced by porous spherical particle and wall correction factor. In this paper we deal the creeping motion of a porous circular cylinder at the instant it passes the center of cylindrical container. The flow inside the porous cylindrical particle is governed by Brinkman equation. However, the flow inside the cylindrical container is governed by Stokes equation. The drag force acting on porous cylindrical particle is evaluated. The variation of drag with different parameter is graphically presented.

II. STATEMENT AND MATHEMATICAL FORMULATION OF THE PROBLEM

Here we have considered a porous circular cylindrical particle of radius a passing through the centre of a cylindrical container of radius b containing an axi-symmetric Stokes flow of a viscous incompressible fluid. This is equivalent to the inner particle at rest while the outer cylindrical container moves with a constant velocity U in the negative Z direction. We assume that the flow within the cylindrical container is Stokesian, and for the flow inside the porous circular cylinder is governed by Brinkman's equation.

GOVERNING EQUATIONS: The governing equations for the creeping flow of an incompressible Newtonian fluid, which lies in the region outside the porous cylindrical particle be governed by Stokes equation [Happel and Brenner [30]] as

$$\mu_1 \nabla^2 v^{(1)} = \nabla p^{(1)} \qquad (1)$$

Also, we assume that the flow inside the porous cylindrical particle is governed by the Brinkman equation [Zlatanovski [31]]

$$\nabla^2 v^{(2)} - \left(\frac{\sigma^2}{a^2}\right) v^{(2)} = \frac{1}{\mu_2} \nabla p^{(2)} \qquad (2)$$

Here μ_1 is the viscosity of the clear fluid, μ_2 denotes the effective viscosity of porous medium, k being the permeability of porous medium. The viscosity coefficients μ_1 and μ_2 are, in general, different. Here, $\mathbf{v}^{(i)}, p^{(i)}, i=1,2$ be the velocity vector and pressure outside and inside the porous cylindrical particle respectively. Equation (2) reduces to the Stokes equation for large permeability k, i.e. for small σ, ($\mu_2 \nabla^2 \mathbf{v}^{(2)} = \nabla p^{(2)}$) however, for low permeability this equation resembles with Darcy empirical equation ($-(\mu_1/k)\mathbf{v}^{(2)} = \nabla p^{(2)}$).

In addition, the equations of continuity for incompressible fluids must be satisfied in both regions:

$$\nabla . \mathbf{v}^{(i)} = 0, \qquad i = 1, 2. \qquad (3)$$

These equations of continuity for axi-symmetric, incompressible viscous fluid in cylindrical polar coordinates (r, θ, z) in both regions can be written as

$$\frac{\partial}{\partial r}(r v_r^{(i)}) + \frac{\partial}{\partial \theta}(v_\theta^{(i)}) = 0, \qquad (4)$$

where, $v_r^{(i)}$ and $v_\theta^{(i)}$, i =1, 2 are component of velocities in the direction of r and θ, respectively. The stream functions $\psi^{(i)}(r, \theta)$ in both regions, satisfying equations of continuity (5) may be defined as

$$v_r^{(i)} = \frac{1}{r}\frac{\partial \psi^{(i)}}{\partial \theta} \quad ; \quad v_\theta^{(i)} = -\frac{\partial \psi^{(i)}}{\partial r}. \qquad (5)$$

Let the index in the superscript under bracket of an entity $\chi^{(i)}$, $i = 1, 2$ indicates clear and porous fluid regions, respectively.

Using the following variables

$$\psi = Ub\psi^{(i)}, \quad p = \frac{\mu U}{b} p^{(i)}, \quad i = 1, 2 \qquad (6)$$

$$r' = br, \qquad (7)$$

and eliminating the pressures from both equations (1) and (2) and using (5), we get the following fourth order partial differential equations, respectively as

$$\nabla^4 \Psi^{(1)} = 0, \qquad (8)$$

$$\nabla^2 \left(\nabla^2 - \frac{\sigma^2}{a^2}\right) \Psi^{(2)} = 0 \qquad (9)$$

where, $\alpha^2 = \sigma^2 / \lambda^2$ with $\lambda^2 = \mu_2 / \mu_1$, $\sigma^2 = a^2/k$ and ∇^2 being the dimensionless operator defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \qquad (10)$$

The range of r *and* θ in the above equations (9) and (10), with in a cylinder is given by:

$$0 < r < \infty, \qquad 0 \le \theta \le 2\pi. \qquad (11)$$

Furthermore, the expressions for tangential and normal stresses $T_{r\theta}^{(i)}, T_{rr}^{(i)}$, $i = 1, 2$ respectively are given by

$$T_{r\theta}^{(i)} = \mu_i \left[\frac{1}{r^2}\frac{\partial^2 \psi^{(i)}}{\partial \theta^2} + \frac{1}{r}\frac{\partial \psi^{(i)}}{\partial r} - \frac{\partial^2 \psi^{(i)}}{\partial r^2}\right] \qquad (12)$$

$$T_{rr}^{(i)} = -p^{(i)} + \frac{2\mu_i}{r}\left[\frac{\partial^2 \psi^{(i)}}{\partial r \partial \theta} - \frac{1}{r}\frac{\partial \psi^{(i)}}{\partial \theta}\right] \qquad (13)$$

Also, the pressure may be obtained in both regions (Happel and Brenner, [8]) by integrating the following relations respectively as

$$\frac{\partial p^{(i)}}{\partial r} = \mu_i \left[\nabla^2 v_r^{(i)} - \frac{v_r^{(i)}}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta^{(i)}}{\partial \theta} - \delta_{2i}\alpha^2 v_r^{(i)}\right] \qquad (14)$$

$$\frac{1}{r}\frac{\partial p^{(i)}}{\partial \theta} = \mu_i \left[\nabla^2 v_\theta^{(i)} - \frac{v_\theta^{(i)}}{r^2} + \frac{2}{r^2}\frac{\partial v_r^{(i)}}{\partial \theta} - \delta_{2i}\alpha^2 v_\theta^{(i)}\right] \qquad (15)$$

where, $\delta_{21} = 0$ *and* $\delta_{22} = 1$.

A suitable stream function solution of the Stokes equation (8) can be expressed as

$$\psi^{(1)}(r, \theta) = [A_1 r + B_1 r^3 + C_1 / r + D_1 r \ln r] \sin\theta \qquad (16)$$

A suitable stream function solution of the Brinkman equation (9) can be expressed as

$$\Psi^2(r, \theta = [A_2 r + C_2 I_1(\sigma r)] \sin\theta. \qquad (17)$$

Here, $I_1(\sigma r)$ is the modified Bessel's functions of the order one of the first kind.

III. SOLUTION OF THE PROBLEM

The mathematically consistent boundary conditions for the concerned problem are as follows: On the porous surface:

$$v_{r'}^{(2)}(1, \theta) = v_{r'}^{(1)}(1, \theta), \qquad (18)$$

$$v_\theta^{(2)}(1, \theta) = v_\theta^{(1)}(1, \theta), \qquad (19)$$

$$p^{(1)}(1, \theta) = p^{(2)}(1, \theta) \qquad (20)$$

$$T_{r'\theta}^{(1)}(1, \theta) = T_{r'\theta}^{(2)}(1, \theta), \qquad (21)$$

On the outer cylinder, the condition of impenetrability leads to

$$v_{r'}^{(1)}(\eta, \theta) = -U \sin\theta \quad and \quad v_\theta^{(1)}(\eta, \theta) = -U \sin\theta \qquad (22)$$

Here, we have taken $\eta = \frac{a}{b}$.

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DETERMINATION OF ARBITRARY CONSTANTS

As a result of application of the boundary conditions (18) – (22), we find that

$$A_1 + B_1 + C_1 - A_2 - C_2 I_1(\sigma) = 0 \quad (23)$$

$$A_1 + 3B_1 - C_1 + D_1 - A_2 - C_2(\sigma I_0(\sigma) - I_1(\sigma)) = 0 \quad (24)$$

$$-4\mu_1 B_1 - \mu_2\left(2\sigma I_0(\sigma) - (\sigma^2+4)I_1(\sigma)\right)C_2 = 0 \quad (25)$$

$$2\mu_1(4B_1 - D_1) + \mu_2\sigma^2 A_2 = 0 \quad (26)$$

$$\eta^2 A_1 + B_1 + \eta^4 C_1 - D_1\eta^2 \log\eta = \eta^2 \quad (27)$$

$$\eta^2 A_1 + 3B_1 - \eta^4 C_1 + D_1\eta^2(1-\log\eta) = \eta^2 \quad (28)$$

Solving the above equations ((23)-(28)), we get all the constants which are cumbersome.

IV. EVALUATION OF DRAG FORCE

Integrating the normal and tangential stresses over the porous cylindrical shell of radius a in a cell, yields the drag force experienced per unit length F as given below.

$$F = \int_0^{2\pi} (T_{rr}^{(1)}\cos\theta - T_{r\theta}^{(1)}\sin\theta)_{r=a}\, a d\theta. \quad (29)$$

On evaluation of stress components from equations (13) and (14), one can find that

$$T_{rr}^{(1)} = -\frac{4\mu_1 U}{a}\left[r' B_1 + \frac{1}{r'^3}C_1 - \frac{1}{r'}D_1\right]\cos\theta \quad (30)$$

$$T_{r\theta}^{(1)} = -\frac{4\mu_1 U}{a}[r' B_1]\sin\theta \quad (31)$$

Inserting the values of (30) and (31) in equation (29) and integrating, we get

$$F = 4\pi\mu_1 U D_1 \quad (32)$$

where the value of constant D_1 is given in Appendix

DRAG FORCE ON A SOLID CYLINDER IN CELL MODEL (K→0)

When permeability k vanishes, i.e. permeability parameter $\sigma \to \infty$, then the porous circular cylinder behaves like a solid cylinder of radius b. In this case, we get the value of the constant D_1 as

$$D_1 = \frac{1+\eta^2}{1-\eta^2+\log\eta+\eta^2\log\eta} \quad (33)$$

Thus the value of drag force, from the equation (32), experienced by the porous circular cylinder in a cell comes out as

$$F = 4\pi\mu_1 U \frac{1+\eta^2}{1-\eta^2+\log\eta+\eta^2\log\eta}$$

$$= \frac{8\pi\mu_1 U(1+\gamma)}{2\gamma - 2 - \gamma\log\gamma - \log\gamma} \quad (34)$$

where $\gamma = \frac{\pi a^2}{\pi b^2} = \frac{1}{m^2}$,also, the drag coefficient C_D can be written as

$$C_D = \frac{16\pi(1+\gamma)}{\mathrm{Re}(2\gamma - 2 - \gamma\log\gamma - \log\gamma)} \quad (35)$$

This result of the drag force agrees with the earlier result as reported by Happel.

VELOCITY COMPONENTS, PRESSURE, VORTICITY AND STRESS TENSOR

Using the values of $\Psi^{(1)}(r,\theta)$ and $\Psi^{(2)}(r,\theta)$ from Eqs. (16) and (17),in Eq.(5),we have the following expressions for the velocity components for the outside and inside region of porous cylindrical particle as

$$v_r^{(1)} = U\left[A_1 + B_1(r')^2 + C_1\left(\frac{1}{r'}\right)^2 + D_1\ln(r')\right]\cos\theta, \quad (36)$$

$$v_\theta^{(1)} = -U\left[A_1 + 3B_1(r')^2 - C_1\left(\frac{1}{r'}\right)^2 + D_1(1+\ln(r'))\right]\sin\theta, \quad (37)$$

$$v_r^{(2)} = U\left[A_2 + C_2\left(\frac{1}{r'}\right)I_1(\sigma r')\right]\cos\theta, \quad (38)$$

$$v_\theta^{(2)} = -U\left[A_2 + C_2\left\{\sigma I_0(\sigma r') - \left(\frac{1}{r'}\right)I_1(\sigma r')\right]\sin\theta \quad (39)$$

Again, substituting the values of velocity components $v_r^{(i)}, v_\theta^{(i)}, i = 1,2$,from (36)-(39) in Equations (14) and (15) and then integrating the resulting equations:

$$dp^{(i)} = \frac{\partial p^{(i)}}{\partial r}dr + \frac{\partial p^{(i)}}{\partial \theta}d\theta, \qquad i = 1,2 \quad (40)$$

we obtain,

$$p^{(1)} = \frac{2\mu_1 U}{a}\left[4r' B_1 - \frac{1}{r'}D_1\right]\cos\theta, \quad (41)$$

$$p^{(2)} = \frac{\mu_2\sigma^2 U}{a}[-r' A_2]\cos\theta \quad (42)$$

where $p^{(1)}$and $p^{(2)}$are the pressures in the outside and inside the porous cylindrical particle, respectively.

The vorticity $\omega^{(1)}$ and $\omega^{(2)}$for outside and inside regions of the porous cylindrical particle can be expressed in terms of velocity component as

$$\omega^{(i)} = \frac{\partial v_\theta^{(i)}}{\partial r} - \frac{1}{r}\frac{\partial v_r^{(i)}}{\partial \theta} + \frac{v_\theta^{(i)}}{r} = -\nabla^2\Psi^{(i)}, \qquad i = 1,2 \quad (43)$$

Therefore,

$$\omega^{(1)} = -\frac{2U}{a}\left[4r' B_1 + \left(\frac{1}{r'}\right)D_1\right]\sin\theta, \quad (44)$$

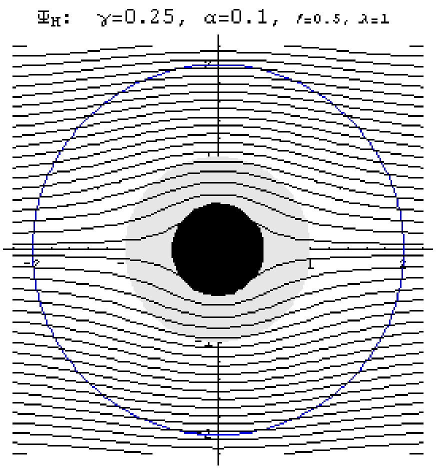
$$\omega^{(2)} = -\frac{U\sigma^2}{a}[I_1(\sigma r')C_2]\sin\theta. \quad (45)$$

Components of the stress tensor for the outside region of the porous cylindrical particle is given by Equations (30) and (31) and the component of the stress tensor for the inside region of the porous cylindrical particle can be obtained by using the equations (12) and (13) which comes out as

$$T_{r\theta}^{(2)} = \frac{\mu_2 U}{a}\left[\{\left(\frac{2\sigma}{r'}\right)I_0(\sigma r') - \left(\sigma^2 + \frac{4}{r'^2}\right)I_1(\sigma r')\}C_2\right]\sin\theta,$$

$$T_{rr}^{(2)} = \frac{\mu_2 U}{a}\left[\sigma^2 r' A_2 + \left\{\sigma I_0(\sigma r') - \left(\frac{2}{r'}\right)I_1(\sigma r')\right\}(2/r')C_2\right]\cos\theta \quad (47)$$

STREAM LINES PRESENTATION:



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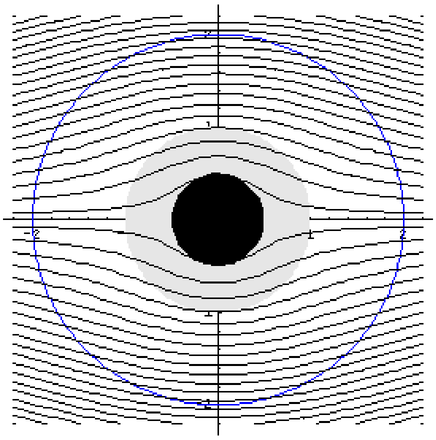
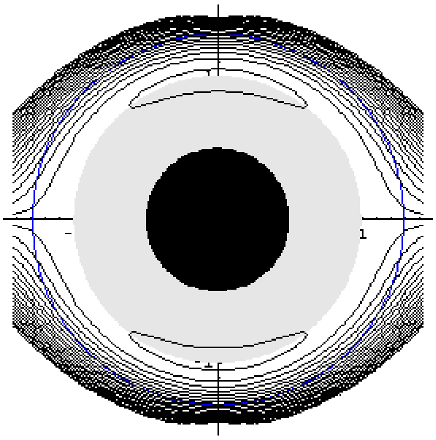
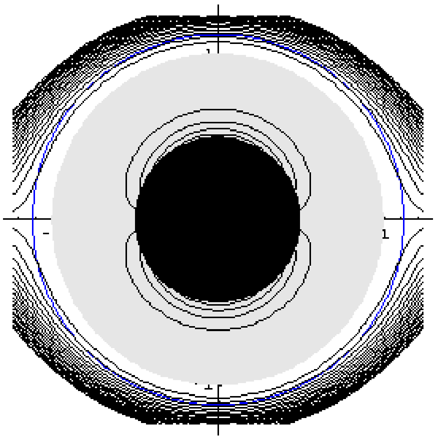
Ψ_H: $\gamma=0.25$, $\alpha=10$, $r=0.5$, $\lambda=1$



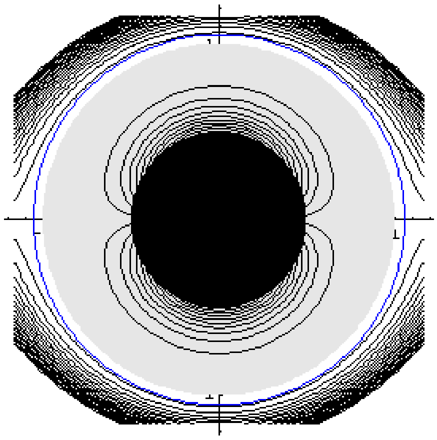
Ψ_H: $\gamma=0.6$, $\alpha=1$, $r=0.5$, $\lambda=3$



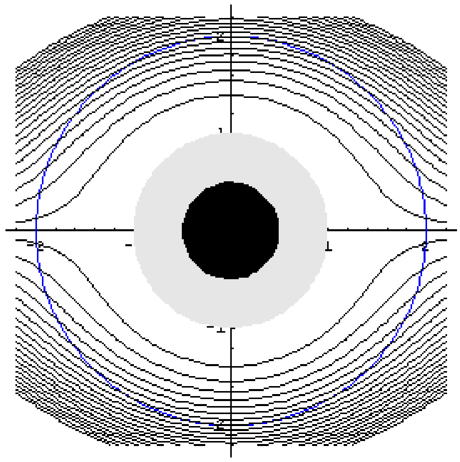
Ψ_H: $\gamma=0.8$, $\alpha=1$, $r=0.5$, $\lambda=3$



Ψ_H: $\gamma=0.9$, $\alpha=1$, $r=0.5$, $\lambda=3$



Ψ_H: $\gamma=0.25$, $\alpha=10$, $r=0.5$, , $\lambda=3$



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