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Flexural Vibrations Of Poroelastic Plates Resting On The Elastic Foundation

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Abstract: ***This paper deals with the flexural vibrations of poroelastic plates resting on the elastic foundation are investigated in the framework of Biot's theory. Non-dimensional phase velocity is computed as a function of shear foundation and wavenumber. For the numerical results, poroelastic solids namely sandstone and bone are employed, and the results are presented graphically.***

Keywords: Poroelasticity, Shear foundation, Flexural vibrations, Phase velocity, Wavenumber.

I. INTRODUCTION

The study of elastic foundations in poroelastic solids has wide applications in the fields of Engineering and Geophysics. Even in human body, a number of muscoskeletal models of knee joint and skin employ different forms of elastic wrinkler foundations. Plates resting on elastic foundation are frequently used structural elements in modelling of engineering problems such as concrete roads, mat foundations of buildings and reinforced concrete pavements of air port runways. Pasternak [1] studied a new method of analysis of an elastic foundation by means of two foundation constants. In the said paper the two parameter model is frequently adopted to described the mechanical behaviour of foundations. The well known Wrinkler model can be considered as a special case that ignores the shear deformation of the foundation. Hasini Baferani et al [2] investigated accurate solution for free vibration analysis of functionally graded thick rectangular plates resting on elastic foundation. In the said paper, free vibration analysis of moderately thick rectangular FG plates on elastic foundation with various combinations of simply supported and clamped boundary conditions are studied. Winkler model is considered to describe the reaction of elastic foundation on the plate. Governing equations of motion are derived based on the Mindlin plate theory. A semi-analytical solution is presented for the governing equations using the extended Kantorovich method together with infinite power series solution. Effects of elastic foundation, boundary conditions, material, and geometrical parameters on natural frequencies of the FG plates are investigated. Matsunsga [3] studied vibration and stability of thick plates on elastic foundations. Natural frequencies and buckling stresses of a thick isotropic plate on two-parameter elastic foundations are analyzed by taking into account the effect of shear deformation, thickness change, and rotatory inertia. Using the method of power series expansion of the displacement components, a set of fundamental dynamic equations of a two-dimensional, higher-order theory for thick rectangular plates subjected to in plane stress is derived through Hamilton principle. Hosseini Hashemi et al [4] investigated vibration analysis of rectangular Mindlin plates on elastic foundations and vertically in contact with stationary fluid by Ritz method. In the said paper, free vibration analysis of vertical rectangular Mindlin plates resting on Pasternak elastic foundation and fully or partially in contact with fluid on one side is investigated for different combinations of boundary conditions. In order to analyze the interaction of the Mindlin plate with the elastic foundation and fluid system, three displacement components of the plate are expressed in the Ritz method by adopting a set of static Timoshenko beam functions satisfying geometric boundary conditions in a Cartesian co-ordinate system. The method of separation of variables and the method of Fourier series expansion is used to model fluid and to obtain the exact expression of the motion of fluid in the form of integral equations. Exact solutions for rectangular Mindlin plates under in plane loads resting on Pasternak elastic foundation is studied by Akhavan et al. [5]. In the said paper, the effect of foundation stiffness parameters and loading factors on the natural frequencies of the plate, constrained by different combinations of classical boundary conditions, is presented for various values of aspect ratios and thickness to length ratios. Tajeddini et al [6] studied three dimensional free vibrations of variable thickness thick circular and annular isotropic and functionally graded plates on Pasternak foundation. In the said paper, the kinematic and the potential energy of the plate–foundation system are formulated and the polynomial-Ritz method is used to solve the eigenvalue problem. The fundamental solution of Mindlin plates resting on an elastic foundation in Laplace domain and its applications is investigated by Wen [7]. In the said paper, fundamental solution is applied to a shear deformable plate resting on the elastic foundation under either a static or a dynamic load. The complete expressions for internal point kernels, i.e. fundamental solutions by the boundary element method, for the Mindlin plate theory are derived in the Laplace transform domain. Zhou et al [8] studied three dimensional vibration analysis of rectangular thick plates on Pasternak foundations. In the said paper, the Ritz method is used to derive the eigenvalue equation of the rectangular plate by augmenting the strain energy of the plate with the potential energy of the elastic foundation. Free vibration of simply supported rectangular plates on Pasternak foundation: An exact and three dimensional solutions are studied by Dehghany and Farajpour [9]. In the said paper, the navier equations of motion are replaced by three decoupled equations in terms of displacement components. The equations are solved in semi-inverse method, and solution is formed for a double fourier sine series. Vibration characteristics of fluid filled cylindrical shells based on elastic foundations is studied by Abdul Ghafar Shah et al [10]. In the said paper, frequencies are strongly affected when a cylindrical shell is attached with elastic foundation.

Employing the Biot's theory [11], Deflection of poroelastic plates is studied by Taber [12]. In the said paper, governing equations are derived on linear consolidation theory and reduce to a single fourth order integro-partial differential equation is used to solved for the transverse displacement. The results are presented for a simply supported rectangular plate with a time dependent surface pressure. Edge waves in poroelastic plate under plane stress conditions are studied by Malla Reddy and Tajuddin [13]. In the said paper, the governing equations of plane stress problems in poroelastic solids are formulated. Three dimensional vibration analysis of an infinite poroelastic plate immersed in an inviscid elastic plate is studied by Shah and Tajuddin [14]. In the said paper, the frequency equation is obtained for each pervious and impervious surface of poroelastic plate in contact with an inviscid elastic fluid and poroelastic plate in vaccum as a particular case. Pradhan et al [15] investigated shear waves in a fluid saturated elastic plate. In the said paper, the frequency spectrum for SH in the plate is studied and the propagation is damped due to the two phase character of the porous medium. Vertical vibrations of elastic foundation resting on saturated half space is studied by Guo-Cai wang et al [16]. In the said paper, the foundation is subjected to time-harmonic vertical loadings and then the transform solutions for the governing equations of the saturated media are dervied. On flexural vibrations of poroelastic circular cylindrical shells immersed in an acoustic medium is studied by Shah and Tajuddin [17]. However to the best of author's knowledge flexural vibrations of poroelastic plates resting on the elastic foundation are not yet investigated. Therefore in this paper an attempt is made to investigate the same in the framework of Biot's theory. The pertinent governing equations in the case of flexural vibrations are derived. Phase velocity is computed as a function of shear foundation. Phase velocity is computed for three types of poroelastic materials and then discussed. It is observed that phase velocity is greater for material-1 than that of material-2 and material-3. It is also clear that as the shear foundation increases phase velocity decreases.

This paper is organized as follows. In section 2, governing equations and solutions of the problem are given. Numerical results are described in section 3. Finally, conclusions are given in section 4.

II. GOVERNING EQUATIONS AND SOLUTION OF THE PROBLEM

The dynamic equations in cartesian coordinate system in the absence of body forces [11] are as follows

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U),$$
$$\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V),$$
$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}w+\rho_{12}W),$$
$$\frac{\partial s}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U),$$
$$\frac{\partial s}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V),$$
$$\frac{\partial s}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w+\rho_{22}W), \tag{1}$$

In eq. (1), $\rho_{11},\rho_{12},\rho_{22}$ are the mass coefficients, (u,v,w) and (U,V,W) are the displacements of solid and fluid. s is the fluid pressure, σ_{ij} are the stress components are given by

$$\sigma_{xx}=2Ne_{xx}+Ae+Q\varepsilon,$$
$$\sigma_{yy}=2Ne_{yy}+Ae+Q\varepsilon,$$
$$\sigma_{zz}=2Ne_{zz}+Ae+Q\varepsilon,$$
$$s=Qe+R\varepsilon. \tag{2}$$

In eq. (2), e_{ij}'s are strain components, A,N,Q,R are poroelastic constants, e and ε are dilatations of solid and fluid. The strain e_{ij} are related the displacements are as follows

$$\sigma_{xz}=2Ne_{xz},\quad e_{xx}=\frac{\partial u}{\partial x},\qquad e_{xz}=\frac{1}{2}(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}),$$
$$\sigma_{yz}=2Ne_{yz},\quad e_{yy}=\frac{\partial v}{\partial y},\qquad e_{xy}=\frac{1}{2}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}),$$

$$\sigma_{xy} = 2Ne_{xy}, \quad e_{zz} = \frac{\partial w}{\partial z}, \quad e_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right). \quad (3)$$

In eq. (3) $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are the normal stresses, $\sigma_{xz}, \sigma_{yz}, \sigma_{xy}$ are the shear stresses, e_{xx}, e_{yy}, e_{zz} are the normal strains, e_{xy}, e_{yz}, e_{xz} are the shear strains. Substitution of eq. (2) and eq. (3) in eq. (1) and introducing the Wrinkler and Pasternak foundations $(-Kw + G\nabla^2 w)$ in the $z-$direction, the equations of motion for the flexural problem are as follows

$$N\nabla^2 u + (A+N)\frac{\partial e}{\partial x} + Q\frac{\partial \varepsilon}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U),$$

$$N\nabla^2 v + (A+N)\frac{\partial e}{\partial y} + Q\frac{\partial \varepsilon}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{11}v + \rho_{12}V),$$

$$N\nabla^2 w + (A+N)\frac{\partial e}{\partial z} + Q\frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{11}w + \rho_{12}W) - Kw + G\nabla^2 w,$$

$$Q\frac{\partial e}{\partial x} + R\frac{\partial \varepsilon}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U),$$

$$Q\frac{\partial e}{\partial y} + R\frac{\partial \varepsilon}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{12}v + \rho_{22}V),$$

$$Q\frac{\partial e}{\partial z} + R\frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{12}w + \rho_{22}W). \quad (4)$$

Now one assume the solution to the eq. (4) in the following form

$$u(x,y,z) = C_1 e^{j\omega t - j(k_1 x + k_2 y + k_3 z)},$$

$$v(x,y,z) = C_2 e^{j\omega t - j(k_1 x + k_2 y + k_3 z)},$$

$$w(x,y,z) = C_3 e^{j\omega t - j(k_1 x + k_2 y + k_3 z)},$$

$$U(x,y,z) = C_4 e^{j\omega t - j(k_1 x + k_2 y + k_3 z)},$$

$$V(x,y,z) = C_5 e^{j\omega t - j(k_1 x + k_2 y + k_3 z)},$$

$$W(x,y,z) = C_6 e^{j\omega t - j(k_1 x + k_2 y + k_3 z)}, \quad (5)$$

In all the above $C_1, C_2, C_3, C_4, C_5, C_6$ are arbitrary constants, j is the complex unity and $k_i (i = 1,2,3)$ is the wavenumber in the i^{th} direction such that the wavenumber $k = \sqrt{k_1^2 + k_2^2 + k_3^2}$. Substituting eq. (5) in the eq. (4), the equation of motion in terms of displacements are as follows.

$$(N(k_1^2 + k_2^2 + k_3^2) + (A+N)k_1^2 - \omega^2\rho_{11})C_1 + (A+N)k_1k_2C_2 + (A+N)k_1k_3C_3$$
$$+ (Qk_1^2 - \omega^2\rho_{12})C_4 + Qk_1k_2C_5 + Qk_1k_3C_6 = 0,$$
$$(A+N)k_1^2C_1 + (N(k_1^2 + k_2^2 + k_3^2) + (A+N)k_2^2 - \omega^2\rho_{11}))C_2 + (A+N)k_2k_3C_3 + Qk_1k_2C_4$$
$$+ (Qk_2^2 - \omega^2\rho_{12})C_5 + Qk_2k_3C_6 = 0,$$
$$(A+N)k_1k_3C_1 + (A+N)k_2k_3C_2 + (N(k_1^2 + k_2^2 + k_3^2) - \omega^2\rho_{11} - K - G(k_1^2 + k_2^2 + k_3^2))C_3$$
$$+ Qk_1k_3C_4 + Qk_2k_3C_5 + (Qk_3^2 - \omega^2\rho_{12})C_6 = 0,$$
$$(Qk_1^2 - \omega^2\rho_{12})C_1 + Qk_1k_2C_2 + Qk_1k_3C_3 + \left(Rk_1^2 - \omega^2\rho_{22}\right)C_4 + Rk_1k_2C_5 + Rk_1k_3C_6 = 0,$$
$$Qk_1k_2C_1 + (Qk_2^2 - \omega^2\rho_{12})C_2 + Qk_1k_3C_3 + Rk_1^2C_4 + \left(Rk_2^2 - \omega^2\rho_{22}\right)C_5 + Rk_2k_3C_6 = 0,$$
$$Qk_1k_3C_1 + Qk_2k_3C_2 + (Qk_3^2 - \omega^2\rho_{12})C_3 + Rk_1k_3C_4 + Rk_2k_3C_5 + (Rk_3^2 - \omega^2\rho_{22})C_6 = 0. \quad (6)$$

III. NUMERICAL RESULTS

For the numerical results, we consider the wave propagation in the z-direction. In this case $k_1 = k_2 = 0$ and wrinkler foundation $K = 0$. Then the equations of motion reduces to

$$\begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = 0, \quad (7)$$

where

$$A_{11} = Nk_3^2 - N\frac{\omega^2}{V_s^2},$$

$$A_{22} = Nk_3^2 - N\frac{\omega^2}{V_s^2},$$

$$A_{33} = (Pk_3^2 - Gk_3^2 - \omega^2\rho_{11}) - (Qk_3^2 - \omega^2\rho_{12})\left(\frac{Qk_3^2 - \omega^2\rho_{12}}{Rk_3^2 - \omega^2\rho_{22}}\right).$$

For a non-trivial solution, the determinant of above coefficient matrix is zero. This leads to the frequency equation:

$$\begin{vmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{vmatrix} = 0. \quad (8)$$

The frequency equation is investigated by introducing the non-dimensionalised quantities givenbelow:

$$a_1 = \frac{P}{H}, \quad a_2 = \frac{Q}{H}, \quad a_3 = \frac{R}{H}, \quad a_4 = \frac{N}{H},$$

$$d_1 = \frac{\rho_{11}}{\rho}, \quad d_2 = \frac{\rho_{12}}{\rho}, \quad d_3 = \frac{\rho_{22}}{\rho}, \quad \tilde{z} = \left(\frac{V_0}{V_s}\right)^2,$$

$$\rho = \rho_{11} + 2\rho_{12} + \rho_{22}, \quad H = P + 2Q + R, \quad V_0^2 = \frac{H}{\rho},$$

$$m = \frac{c}{c_0}, \quad c = \frac{\omega}{k}, \quad c_0^2 = \frac{N}{\rho}. \quad (9)$$

In eq. (9), c is phase velocity, k is wavenumber and m is non-dimensional phase velocity, c_0 and V_0 are the reference velocities. Employing the non-dimensional quantities in the frequency equation, we obtain implicit relation between non-dimensional phase velocity (m), shear foundation and non-dimensional wavenumber. For numerical process, three poroelastic solids are considered and then discussed. Of three poroelastic solids, two are sandstone saturated with water and kerosene, respectively [18, 19], and the third one is bony element. The physical parameter values of first two materials pertaining to the eq. (9) are given in the Table 1. Further, the values of bone poroelastic parameters A, N, Q, R and its

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mass coefficients ρ_{ij} are computed following the paper [20]. The values of Young's modulus and Poisson ratio are taken to be 3×10^{6} and 0.28, respectively as suggested in the paper [20]. Phase velocity is computed using the bisection method implemented in MATLAB, and the results are depicted in the figure-1. Figure.1 shows the plots of non dimensional phase velocity against the shear foundation for fixed wavenumber. From this figure it is clear that material-1 values are greater than that of material-2 this discrepancy is due to fluid present in the materials. The values of bone are much less than that of material-1 and material-2. Moreover, for fixed shear foundation, non-dimensional phase velocity value is computed against wavenumber, and the values found to be same for all the values of wavenumber. For example, when the shear foundation value is 2, the non-dimensional phase velocity is 1.5547 for all the values of the wavenumber in the range 1-100. From these results, we conclude that phase velocity is independent of wavenumber.

Material parameters	a_1	a_2	a_3	a_4	d_1	d_2	d_3	$\tilde{z}$
Material-1	0.843	0.65	0.28	0.234	0.901	-0.001	0.101	3.851
Material-2	0.96	0.006	0.028	0.412	0.887	0	0.123	2.129

Table 1

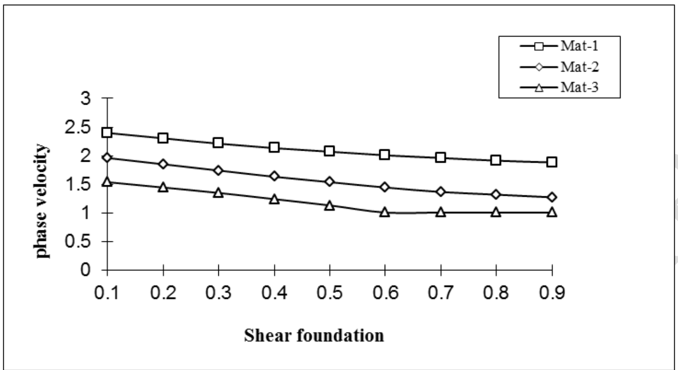

Figure 1: Variation of non-dimensional phase velocity with shear foundation

IV. CONCLUSION

Employing Biot's theory, flexural vibrations of poroelastic plates resting on elastic foundation are investigated. Non-dimensional phase velocity against shear foundation is computed for three types of poroelastic solids. From the results, it is clear that material-1 values are greater than that of material-2 and also as the shear foundation increases, phase velocity decreases in all the cases. When the shear foundation is fixed, it is observed that phase velocity is independent of wavenumber. This kind of analysis can be made for any poroelastic solid cylinder if the values of parameters are available.

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